—研究論文—
Scientific Papers

Ray-paths and Attenuation of Infrasonic Waves Generated at 100 km Altitude

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高度 100 km より発射される超低周波音波の伝搬路と減衰

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要旨: CIRA (1972 年発行) モデルを使って南緯 70° における音の伝搬を研究した. 初めに伝搬可能な音の低, 高域における周波数限界および線型性を仮定できる音圧の範囲について検討する. さらに, オーロラ超低周波音波の伝搬を検討する目的で, 高度 100 km より発射される音波の伝搬路および伝搬時間を計算して, 音の流れと 100 秒ごとの音波の位置を図示する. また上記の伝搬路に沿って (1/40) Hz の音波の減衰を計算した.

下向きに発射される音波, あるいは上向きに発射される音波でも 150 km 以下で反射される音波はほとんど減衰されないで地表へ到達する. 他方 200 km 以上に到達する音波の減衰は非常に大きい.

Abstract: Ray paths of sound waves generated at 100 km altitude are calculated by using model atmosphere given by CIRA 1972 at 70°S latitude. The wave position and the time required for the wave to arrive there are calculated at every 1 km altitude and the results are illustrated by ray curves and by wave positions at every 100 seconds on the curves. Further, attenuation of the wave of (1/40) Hz frequency is calculated along the ray path; the attenuation is very little when the wave propagates under 150 km, it becomes not negligible when the wave goes above 200 km.

1. Introduction

A natural infrasonic wave which has relations with a magnetic substorm was first recognized in the observation at Washington D.C., U.S.A. (CHRZANOWSKY *et al.*, 1961). The cause of this wave is discussed by several authors (MAEDA and WATANABE, 1964; PIDDINGTON, 1964; KATO *et al.*, 1977). In Alaska, the relations between the generation of specific infrasonic signals and the supersonic motion of the aurora (WILSON

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and NICHPARENKO, 1967) were studied and a shock wave model (WILSON, 1967) was proposed to explain the amplifying mechanism of the signal in the directions of the auroral motion. Further, auroral infrasonic waves were shown to be infrasonic bow waves generated by supersonic equatorward drift motions of the electrojet arc. The Lorentz-force is the coupling mechanism working between the electrojet current carriers and the neutral particles (WILSON, 1972).

The present author (SUZUKI, 1973) studied the auroral-infrasonic wave data observed at Syowa Station, Antarctica, from 1969 to 1972, and concluded that the infrasonic wave was generated by supersonic drift motion of a locally existing jet current with narrow width (SUZUKI, 1979a). An auroral infrasonic wave is seen when the disturbed area travels at a supersonic speed, and at that time syncronized with the zenith crossing of the westward current, a rotation of horizontal vectors of the geomagnetic field is recognized on the ground (SUZUKI, 1979b).

As described above, generation of an auroral infrasonic wave has been discussed by many authors, while propagation of this wave has scarcely been discussed. There still remain many problems unsolved about the propagation of this wave. For example, (1) we can see the wave being delayed several minutes to the equatorward zenith crossing of the luminous aurora. By calculating the wave path and the speed, it should be determined from this delay time whether it is reasonable to regard this luminous aurora as a real wave source. (2) A reflected wave is occasionally seen 10 or 12 minutes after the direct wave. In spite of such long propagation, the reflected wave is attenuated little; its intensity is usually stronger than 70% of that of the direct. (3) However, though the reflected wave is attenuated little by such long propagation, no auroral infrasonic waves are observed at lower latitudes. The present paper is a preliminary study in order to discuss the propagation of the auroral infrasonic wave and to give answers to such problems described above.

In the present study, the propagation of an infrasonic wave will be studied on a wave path and on a wave attenuation. The energy flow by sound wave will be illustrated by ray tracings. Though the shock wave model has been proposed (WILSON, 1967), the stationary sound source is assumed for simplicity in the present paper. On the basis of the present study, a moving source will be treated in the next paper, where a ray path and an attenuation of a reflected wave, curves to deduce the altitude of a current of the wave source, and impossibility of observing the wave in low latitudes will be discussed (SUZUKI, 1980a).

2. Sound Propagation in the Earth's Atmosphere

Sound waves are capable to travel up to altitudes of several thousand kilometers in the earth's atmosphere. Variations of the atmospheric density, of the temperature, and of the composition with altitudes restrict the acoustic frequencies and intensities. The lowest frequency that can travel at any altitudes is of the order of 0.002 Hz (BARRY, 1963); however, the energy of the wave, undergoing dissipation, drops sharply with increasing frequency and altitude.

The lower limit is imposed because of the variation of atmospheric density with altitude. The atmosphere has the appearance of an exponentially tapered transmission line to a vertically travelling sound wave. The acoustic impedance Z is,

$$Z=a\rho, \tag{1}$$

where a is the speed of sound and ρ is the atmospheric density (RAYLEIGH, 1945). The density varies rapidly with h, typically,

$$\rho=\rho_0 \exp(-h/H), \tag{2}$$

where ρ_0 is the density at $h=0$ and H is the scale height. The lowest cutoff wave length λ_L for this exponentially tapered transmission line is $\lambda_L=4\pi H$ or,

$$f_L=a/\lambda_L=a/4\pi H. \tag{3}$$

The lowest cutoff frequency f_L takes the value $0.0013 \sim 0.0039$ Hz for $h=0 \sim 400$ km (BARRY, 1963).

The higher acoustic frequency that can travel is limited by the finite collision rate. The energy of an acoustic wave is transferred from one molecule to another molecule at the time of collision. The upper cutoff frequency f_U is expressed as,

$$f_U \propto \sqrt{\nu a}, \tag{4}$$

where ν is the mean collision frequency. At the altitude of 300 km, the absorption rate for the wave of 0.004 Hz is equal to 1 db/km, and at 200 km the wave of 0.1 Hz is absorbed by the rate of 1 db/km (BARRY, 1963).

The definition of an acoustic wave velocity requires the assumption of linearity. The definition is appropriate only when the sound pressure is much less than the ambient atmospheric pressure. The power carried by a linear, plane, harmonic wave per unit area is,

$$P=\rho a^3 (\Delta p/p)^2 2\gamma^2, \tag{5}$$

where ρ, Δp, p and γ are the ambient density, peak sound pressure, ambient pressure and ratio of specific heats, respectively (BARRY, 1963). According to our observation, Δp of the auroral infrasonic wave is about 4 μb (SUZUKI, 1979a), the power P carried by this wave is $P=1.59\times10^{-4}$ W/m^2. If the linearity is supposed to be held when a peak sound pressure is weaker than 10% of the ambient pressure, *i.e.* $\Delta p/p \leqq 0.1$, the acoustic power P satisfies this condition that $\Delta p/p$ is not greater than 0.1 under the altitude of about 230 km; according to the above assumption the linearity is not held above there.

3. Ray Calculation

3. 1. Model atmosphere

On the basis of the model atmosphere after the CIRA 1972 (COSPAR, 1972)

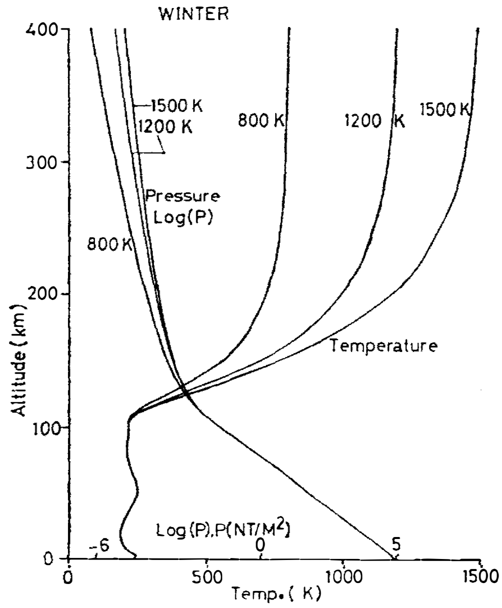

Fig. 1. Temperature and pressure changes at 70° south latitude in winter given by CIRA 1972 model atmosphere. 800 K, 1200 K and 1500 K are the exospheric temperatures. Discontinuities at 110 km of CIRA model are smoothed (see Fig. 2).

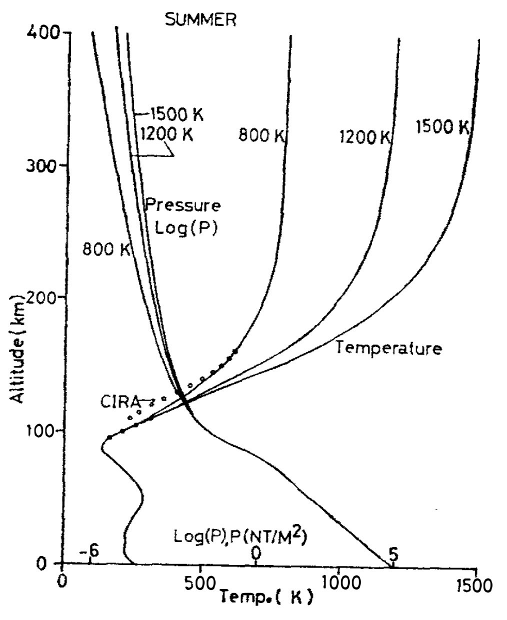

Fig. 2. Temperature and pressure changes at 70° south latitude in summer. The discontinuity around 110 km height in CIRA model (shown by circles) is smoothed.

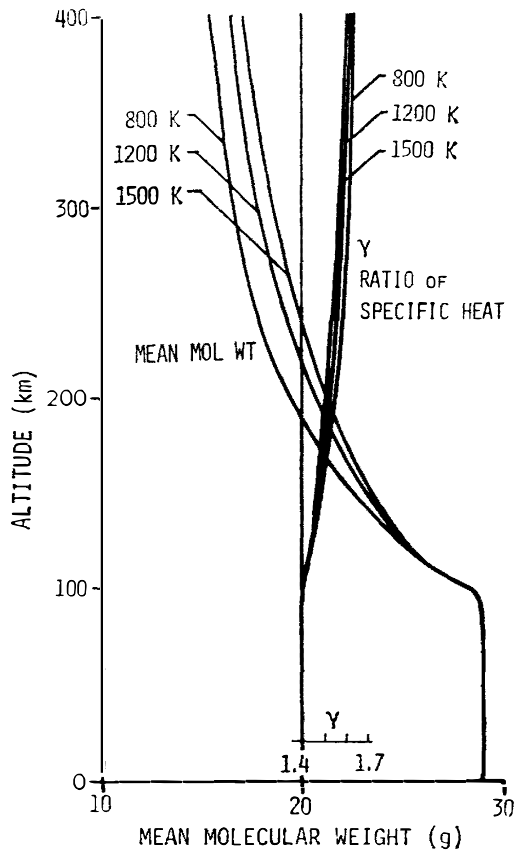

Fig. 3. The altitude changes of molecular weight and ratios of specific heats of the model atmospheres for various exospheric temperatures at 70°S latitude.

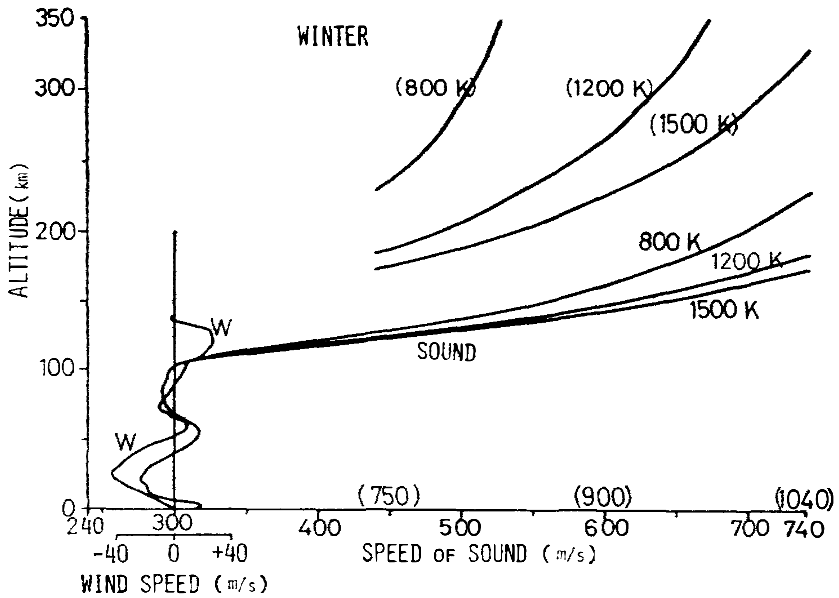

Fig. 4. Wind velocity (equatorward positive) in winter in the geomagnetic meridian is given by W-curve. Sound velocity is calculated using models given in Figs. 1 and 3.

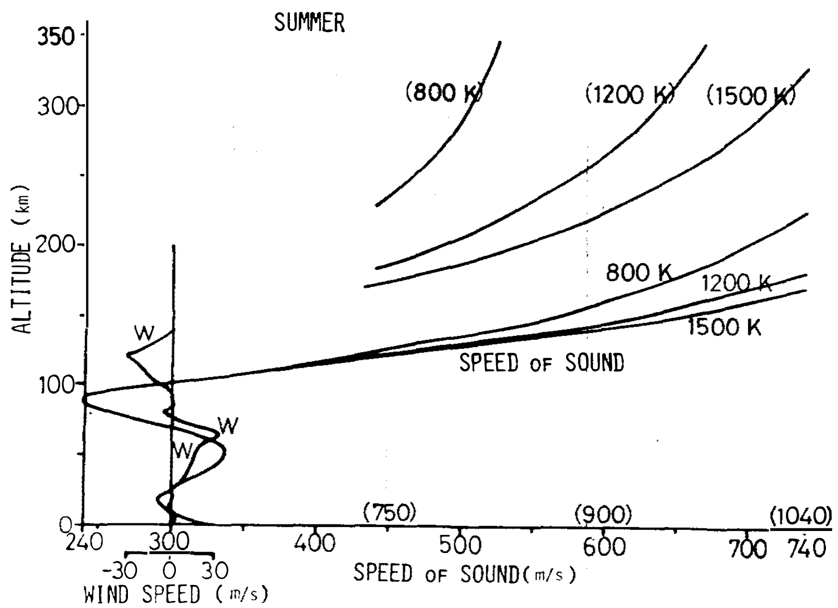

Fig. 5. Wind and sound velocities in summer.

at 70° south latitude, curves showing temperature and pressure are given in Figs. 1 and 2, and those showing ratio of specific heat and weight of a molecule are also given in Fig. 3. Further, wind speed is given in Figs. 4 and 5, where the equatorward component is shown. The atmospheric models derived from these figures are used for ray calculation. As the *CIRA* model has discontinuous changes around 110 km altitude as shown in Fig. 2, the smoothed curves are used for the ray calculation. For the altitudes 0–25 km, the data obtained by a meteorological sonde at Syowa Station (69.00°S, 39.35°E) in Antarctica are used.

The ratio of specific heats is calculated by assuming that the model gas is composed of monoatomic and diatomic gases (SUZUKI, 1980b). A monoatomic gas has three degrees of freedom, and a diatomic gas has five degrees of freedom. When the number density of monoatomic gas is q, the ratio of specific heats γ is given by,

$$\gamma = 1 + [0.4/(1 - 0.4q)]. \tag{6}$$

3. 2. Ray calculation

Ray paths for the acoustic waves through the horizontally stratified atmosphere are calculated. The ray equation is known as the Snell's law of sound,

$$(a/\sin i) + W = \text{const.}, \tag{7}$$

where a, i and W are speed of sound, an angle between the wave normal and the vertical, and speed of horizontal wind, respectively. The velocity of sound in an ideal gas is written as,

$$a = \sqrt{\gamma RT/M}, \tag{8}$$

where R, T and M are the universal gas constant, *i.e.*, $R=8.317 \times 10^7$ erg/deg, the temperature and the molecular weight, respectively.

In eqs. (7) and (8), all but the universal gas constant are functions of an altitude. On the basis of the model atmosphere described in the previous section, the curves showing the velocities of sound and wind are given in Figs. 4 and 5. As no wind speed is given for the altitudes above 140 km in *CIRA* 1972, the wind speed is supposed to be zero above this height.

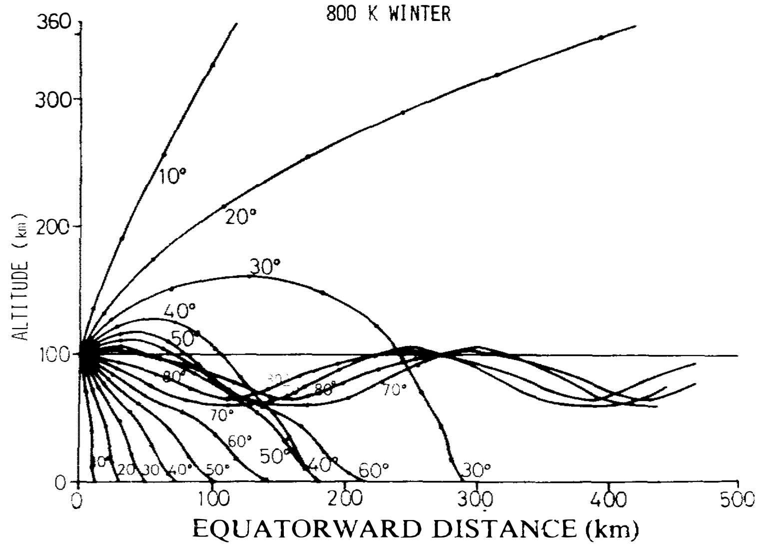

Fig. 6. Ray paths calculated every 10° of incident angles for the model atmosphere of 800 K exospheric temperature T_{ex} in winter. Nodes on the ray show the wave position at every 100 s.

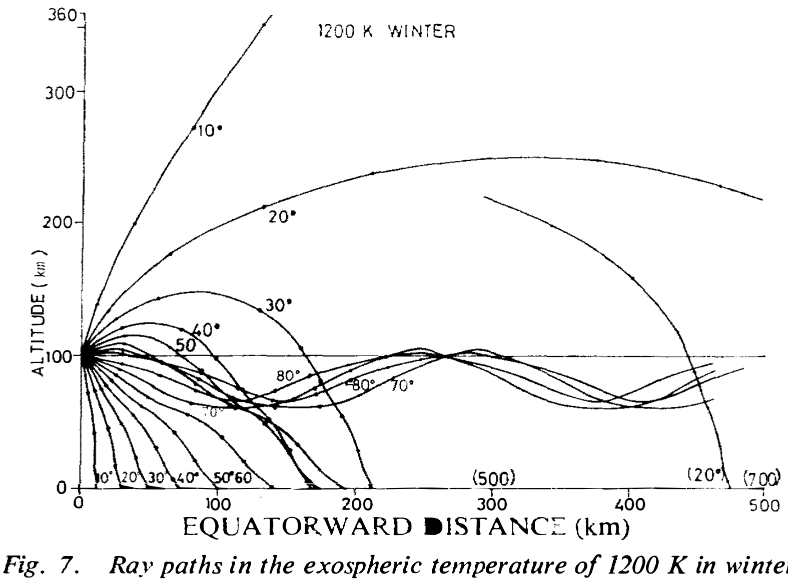

Fig. 7. Ray paths in the exospheric temperature of 1200 K in winter.

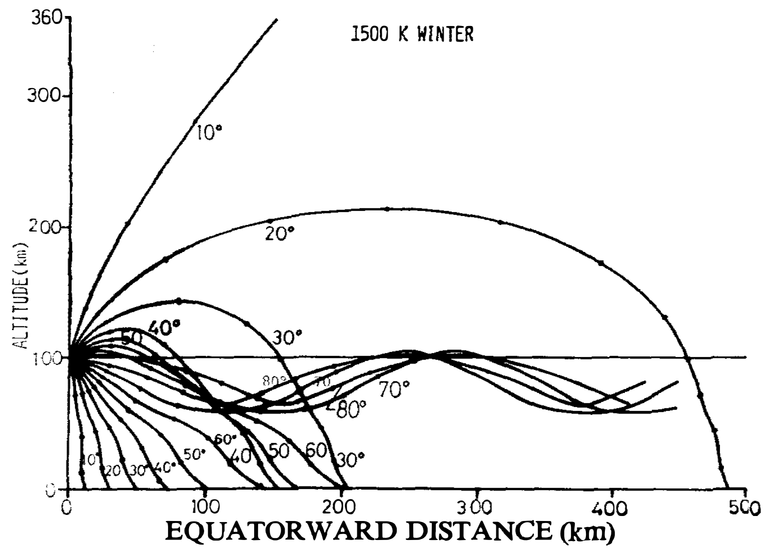

Fig. 8. Ray paths in the exospheric temperature of 1500 K in winter.

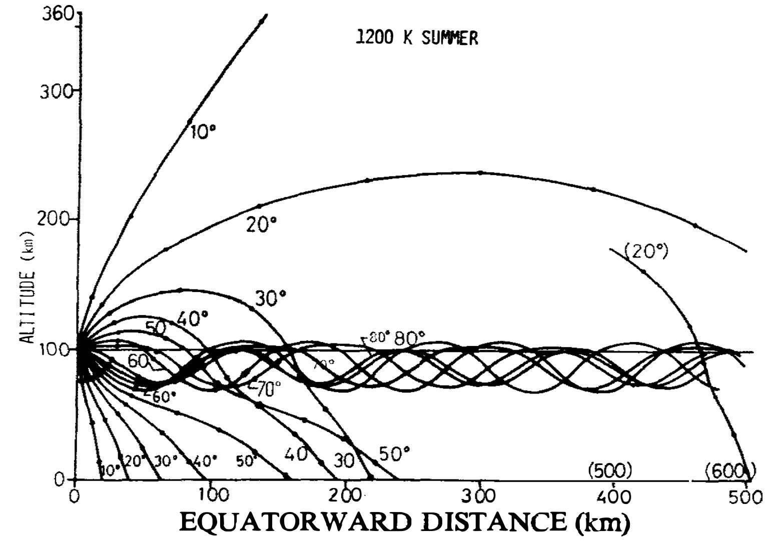

Fig. 9. Ray paths in the exospheric temperature of 1200 K in summer.

The eq. (7) shows that sin i increases with increase in the sound speed and/or in the wind speed, and finally the sound ray going upwards is refracted horizontally or is reflected downwards. In summer, for example, sound speed shows minimum at 89 km altitude (Fig. 5), and thus the upward and the downward sound starting at this altitude may be reflected; in this way the well-known sound ducts are formed.

Ray paths emitted from the source at the altitude of 100 km are calculated at every 1 km altitude using eq. (7) for six model atmospheres with the exospheric temperatures of 800 K, 1200 K and 1500 K both in winter and summer. The curves in Figs. 6, 7, 8 and 9 show the results, where the wave is supposed to propagate in a geomagnetic meridian. The nodes of the ray curves show the locations of wave front every 100 seconds.

As it is supposed from the curves of sound speed and wind speed (Figs. 4 and 5), the wave ducts are formed at about 60–106 km in winter and 70–107 km in summer. Since the minimum value of the sound speed at the mesopause is small in summer, the wave duct in summer is narrower than that in winter.

Differences of exospheric temperature mainly affect the ray paths reflected between 150–250 km altitudes. Such differences are seen between waves emitted upwards at the angle of 20° with the exospheric temperatures of 1200 K (Fig. 7) and 1500 K (Fig. 8), respectively. Ray paths of waves emitted upwards with angles of 20° and 30° in $T_{ex}=800$ K are also quite different from those in $T_{ex}=1200$ K. Seasonal differences are not clear for the upward waves. In winter the wave emitted downwards with the angle of 60° is able to reach the ground, but in summer it propagates only within the duct at the mesopause.

3. 3. Wave attenuation

The attenuation of acoustic wave intensity is given by an 'attenuation coefficient' α per unit of distance traveled. A sound wave of intensity I_0 at the origin attenuates to intensity $I(R)=I_0\exp[-\int_0^R \alpha(r)dr]$ at the point apart R from the origin, $I(R)/I_0=\exp[-\int_0^R \alpha(r)dr]$ is called 'attenuation factor'. According to MAEDA and WATANABE (1964) and the earlier paper by RAYLEIGH (1945), the acoustic wave in the atmosphere attenuates due to viscosity and thermal conductivity, and the attenuation coefficient $\alpha(f)$ in (cm^{-1}) for the wave of frequency f is given approximately by,

$$\alpha(f)=4\pi^2\frac{f^2}{a^3}\left(\frac{4}{3}b+\frac{\gamma-1}{\gamma}c\right), \tag{9}$$

where the coefficient of kinematic viscosity b is given approximately by,

$$b=\frac{1.7\times10^{-4}}{\rho(h)} \quad \text{in} \quad \text{cm}^2\,\text{s}^{-2},$$

and the coefficient of thermal conductivity c is given by

$$c=\frac{2.1\times10^{-5}}{\rho(h)} \quad \text{in} \quad \text{cm}^2\,\text{s}^{-2}.$$

Several different expressions of α were given by other authors, and these will be discussed later.

The coefficient α of the wave of (1/40) Hz frequency in $T_{ex}=1200$ K model atmosphere is calculated by using eq. (9) and the results are shown in Fig. 10; the frequency of (1/40) Hz lies almost at the center of the frequency range of the auroral

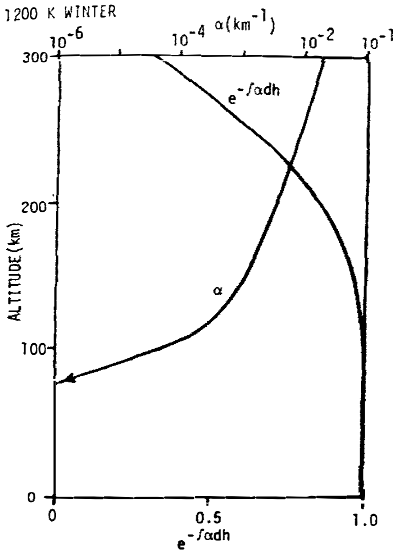

Fig. 10. Attenuation coefficient α and attenuation factor exp(−∫αdh) for the (1/40) Hz wave. In the calculation, vertical propagation of an wave at every altitude is assumed.

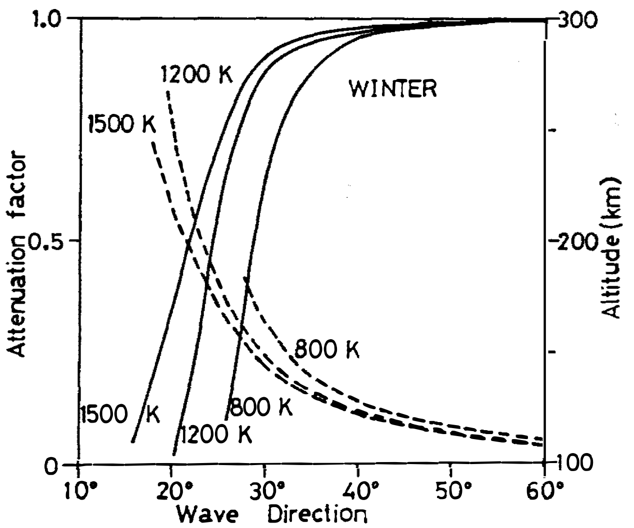

Fig. 11. Solid lines show attenuation of the wave propagating from the source to the ground as the function of an incident angle at the source. The attenuation factors for all waves emitted downwards are greater than 0.999. The altitudes where the waves emitted upwards are reflected are also shown by dotted lines.

infrasonic wave. Further, an attenuation factor of vertically travelling wave from the ground is also calculated by assuming that the wave travels vertically at every altitude. A change in the curvature of the α-curve at about 120 km is related to the change of ρ at this altitude (see pressure curve in Fig. 1). As the coefficient α is smaller than 10^{-3} km^{-1} under 140 km altitude, attenuation of the wave which travels under this altitude is very little; the attenuation does not exceed 10% even when the wave travels horizontally as long as 100 km through 140 km altitude.

Attenuation factors of the wave travelling from the source to the ground were calculated for various paths given in Figs. 6, 7, 8 and 9. The results of the calculation for winter are shown in Fig. 11, in which the altitudes where the rays are reflected are also shown. The attenuation in summer is not much different from that in winter. Energy dissipation due to ray divergence is not considered in the results, as the rays do not diverge when the sound source moves with the supersonic speed (SUZUKI, 1980a). The waves emitted downwards are attenuated little; the attenuation factors of these waves are larger than 0.999. According to eq. (9), the attenuation of the wave strongly depends on the density of the atmosphere in which the wave travels. As the atmospheric density decreases sharply with altitude, the attenuation of waves emitted upwards depends on the altitude where the wave is reflected. The attenuation of the wave is almost negligible unless the wave travels below 150 km, while the attenuation of the wave which travels above 200 km is rapidly increased. The wave emitted with an acute angle arrives at upper altitude. The angles about 30° and 20° are the critical angles whether the waves go above 150 km and 200 km, respectively, hence the waves emitted with former angle (30°) are considerably and those with latter angle (20°) are very strongly attenuated.

4. Discussion

The acoustic wave behavior is analogous to the behavior of radio waves in the ionosphere. In both cases wave propagation is studied in a stratified medium through which the phase velocity increases, though not monotonically, with height. The resultant refraction of a sound wave shows focusing, skip and critical-angle effects as commonly seen in the ionospheric radio-wave transmission. The use of ray optics as an aid in visualizing energy transmission is, of course, properly restricted to cases where the wave length is insignificantly small compared with dimensions of the system and the phase velocity is not a function of frequency. At the frequencies near the lower cutoff, f_L, neither of these restrictions is satisfied. Nevertheless, the only reason why we used it is that there is no other simple method that can give the qualitative behavior of acoustic energy flow.

The auroral infrasonic wave is emitted by moving sources. Therefore, the ray path obtained in the present study does not show the real propagation of the actual auroral infrasonic wave. Propagation of the actual auroral infrasonic wave will be discussed and be described in another paper by using the present results (SUZUKI, 1980a). However, the present study is very useful to know the energy flow of the waves emitted from the source at 100 km altitude.

In Figs. 7, 8 and 9, the waves emitted upwards at an initial angle of 20° travel a long horizontal distance at about the altitude of 250 km, 215 km and 238 km, respectively. However, such long-distance propagations are seen only on calculation, as such ray paths are calculated by using ideal model atmosphere which has no fluctuations. Nevertheless, there are no atmosphere without fluctuations. In the thermosphere, the rapid increase of temperature with respect to altitude always forces the wave to be refracted downwards. If, by a fluctuation, the wave is given a downward direction, it will be further refracted downwards due to the temperature gradient described above. Therefore, the wave does not travel for a long horizontal length usually.

Though the waves emitted upwards with the initial angle of 20° reach the ground in the ray curves, these are, being strongly dissipated owing to the attenuation in the thermosphere, scarcely recognized as auroral infrasonic waves. In winter, the wave emitted upwards by an initial angle of 30°～60° is seen on the ground after being reflected at the thermosphere, while in summer only the wave of 30°～50° is seen because the wave of 60° will be trapped in a wave duct at the mesosphere. Such a wave that arrived on the ground after being reflected at the thermosphere was found at Syowa Station in Antarctica (SUZUKI, 1980a).

The other expressions from eq. (9) of attenuation coefficient are discussed in the paper by REED (1972) where the coefficient α is given as,

$$\alpha = K(f^2/p), \tag{10}$$

where p is the ambient pressure and K is almost constant under the thermosphere bottom. The value of K is calculated by eq. (9) as,

$$K = 4\pi^2 \frac{1}{\gamma}\left(\frac{4}{3}b' + \frac{\gamma-1}{\gamma}c'\right)\frac{1}{a},$$

where $b' = \rho b$ and $c' = \rho c$. The K is given under the thermosphere bottom as,

$$K \fallingdotseq 2.2 \times 10^{-7} \pm 20\% \quad (\mathrm{g\,cm^{-2}}).$$

The 20% deviation is caused by a change of the sound speed (see Figs. 4 and 5). According to COX (1958), and attributed to SCHRÖDINGER (1917), the value K is given approximately as,

$$K \fallingdotseq 1.362 \times 10^{-7} \quad (\mathrm{g\,cm^{-2}}),$$

under the thermosphere bottom. GOLITSYN (1961) gives the value of K by observation as,

$$K = 1.869 \times 10^{-7} \quad (\mathrm{g\,cm^{-2}}).$$

There are some differences between these values of K. The attenuation factor calculated according to GOLITSYN agrees well with the observation of an intensity difference between a direct and a reflected auroral infrasonic waves (SUZUKI, 1980a). However, according to the above paper, the differences of K-value are not particularly important in atmospheric propagation.

5. Conclusion

Propagation of the actual auroral infrasonic wave is seen by applying the results of ray tracings in the present study to the moving source (SUZUKI, 1980a). However, the sound propagation from the source at 100 km altitude can be seen by this study.

The waves emitted downwards reach the ground with little attenuation unless they are trapped in the duct at mesosphere. The waves emitted with angles $60^\circ \sim 90^\circ$ in summer and $70^\circ \sim 90^\circ$ in winter are trapped in the duct at the mesosphere.

The upwards-emitted waves with angles $40^\circ \sim 90^\circ$ are reflected under 130 km and some ($40^\circ \sim 50^\circ$ rays in summer and $40^\circ \sim 60^\circ$ in winter) arrive at the ground with little attenuation and the other are trapped in the duct at the mesosphere. The dissipation of the wave energy depends strongly on the altitude where the wave is reflected. If the wave travels above 150 km, the dissipation is no more negligible, and if it travels above 200 km, the wave is strongly attenuated; such altitudes of reflection are seen when the waves are emitted with initial angles of 30° and 20° respectively.

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