Displacement Method in the Shear Field Theory of the Reinforced Panels and Experiments on Panels Used in the Buildings at Syowa Station

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変形法による棒パネル構造理論と南極基地建物パネルの実験

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要旨

本研究は，1）不静定棒平板結合構造の剪断場理論に対する，我々が変形法と名付けた解法を理論的に確立すること，および 2）国立科学博物館の御好意によって提供をうけた，第一次南極基地建物のパネルについて実験し，剪断場理論と比較して，設計上本解法が使えるかどうか調べるという 2 つの目的を持っている．

本解法は，従来のエネルギー法またはマトリックス法系の剪断場理論とその本質において，まったく等価であるが，変位をすべて未知数にとることによって，従来のものより解法上の方程式作成，即ち，機械的作表がきわめて容易となる．従って，マトリックス法への応用も容易となる．その他弾性方程式の一般的表現，記号法や近似解法など工夫した点が多々ある．提供を受けた南極基地パネルについての実験結果は，二次応力の影響や挫屈破壊を除いて，ほぼその剪断場理論値と一致した．従って，設計上この解法が利用出来るという結論を得た．

Introduction

The present paper has two objects, namely, 1) consideration on the displacement method of the statically indeterminate reinforced panels, in the shear field theory, 2) examination of whether or not the test values correspond to this theory, in designing the unit panels for the use of the Japanese Antarctic Expedition. The shear field theory had been established by the energy method or matrix method[1,5]. However, we find it inconvenient to make the mechanical tabulation of simultaneous equations owing to the complicated stiffness matrix in their theories. Then the paper considers all the components of displacement at joints for unknown factors of these equations. The basic tool used in the convenient mechanical tablation for stiffness matrix is that of two simple equiliblium equations for

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internal axial force at joint and stiffness member in shear field.

Methods are then developed to give reasonable approximate deflection and highly simplified panel structure with several windows. The test was to apply load to the simply supported panels used in the buildings at Syowa Station established in 1957[2-4], using Amsler bending testing machine for comparison. The theoretical value corresponded with the result of test, except for the secondary stress and local buckling at panels or stiffeners. Then, we concluded that the shear field theory may be adopted in the design of panels of prefabricated buildings at Syowa Station.

§ 1. Assumption, Notation and Rule of Signs

This method is based on the same assumption as that of the shear field theory in the past[1,5]. In other words, the rectangular panels around the straight stiffeners with hinged joints that formed the rectanguler wall of grid state have produced shear flow only. It is assumed as follows:

1) The material obeys HOOKE's law until the stress reaches the yield value.

2) External force was applied to the hinged joints only in a plane, then the members have produced the axial force or shear flow in a straight stiffener or panels. In Fig. 1, we consider that the unit panel supported by the internal force and shear flow in a vertical plane generally produced the panel deformation.

The Cartesian coordinate X, Y indicate k, i the notation of a point, *i. e.*, the

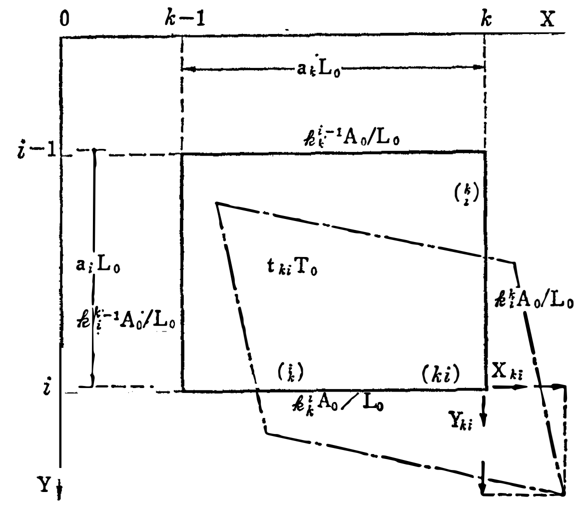

Fig. 1. External forces, deformation and structural constants.

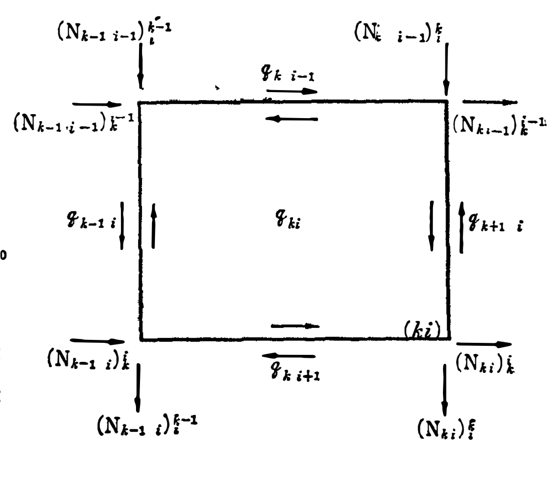

Fig. 2. Internal axial forces and shear flows.

general number of a joint is (ki), the stiffness members in X, Y direction are $\binom{i}{k}$ $\binom{k}{i}$ respectively, the X, Y components of displacement and external force vector are u_{ki}, v_{ki}, X_{ki} and Y_{ki} respectively. On the other hand, the notation of structural constant in analysis is as follows; a_k, a_i is length ratio of general stiffeners per unit length, L_0, t_{ki} is width ratio of general panel per unit width T_0, k_k^i, k_i^k is axial stiffness of general stiffeners against the axial deformation per unit sectional area A_0.

The rule of signs is as follows: The free-body diagram, in which the stiffener has been isolated from its boundary panel and all axial forces acting upon it, both active and reactive, except that shear flow acts upon panel side only, is indicated by positive vector as shown in Fig. 2. For instance, the axial forces $(N_{ki})_k^i$, $(N_{ki})_i^k$ in X, Y direction, respectively acting upon the joint (ki), are positive, if these vectors moved in the positive direction of the system coordinate X, Y and the shear flow q_{ki} is positive if the force acting upon the panel from members faces the direction indicated in Fig. 2 or produces the deformation in Fig. 1.

As stated before, the exchange of suffixes k and i in the internal forces causes the transition of the vector components.

§ 2. Equilibrium Equations

Equilibrium equations on stiffness member $\binom{i}{k}$ and the joint (ki) about the direction of X-axis are obtained as follows, referring to Fig. 2.

$$(N_{ki})_k^i+(N_{k-1i})_k^i=(q_{ki}-q_{ki+1})a_kL_0 \tag{1}$$

$$(N_{ki})_k^i+(N_{ki})_{k+1}^i=X_{ki} \tag{2}$$

§ 3. Compatibility Equation

On condition that elongation of the member is plus, its axial deformation δL_k^i is expressed as,

$$\begin{aligned}\delta L_k^i&=u_{ki}-u_{k-1i}\\ \delta L_i^k&=v_{ki}-v_{ki-1}\end{aligned} \tag{3}$$

On the other hands, its angle of the rotation of member R_k^i is expressed as follows, provided that the clockwise direction is plus,

$$R_k^i=(v_{ki}-v_{k-1i})/a_kL_0$$
$$R_i^k=-(u_{ki}-u_{ki-1})/a_iL_0 \tag{4}$$

§ 4. Equation of Elasticity

The states of general equilibrium condition of the panel ki (Fig. 2) are expressed as in Figs. 3 (a)-(d).

Summing up Fig. 3 by the principle of superposition, shear flow q_{ki} remains independent as in Fig. 3 (c).

The shearing strain of the panel γ_{ki} can be expressed with modules of rigidity G as,

$$\gamma_{ki}=\frac{1}{GT_0}\ \frac{q_{ki}}{t_{ki}} \tag{5}$$

Besides, in Fig. 3 (c)

$$N_c=\frac{a_kL_0}{2}, \qquad N_{c'}=\frac{a_iL_0}{2}$$

which results in $q_{ki}=1$.

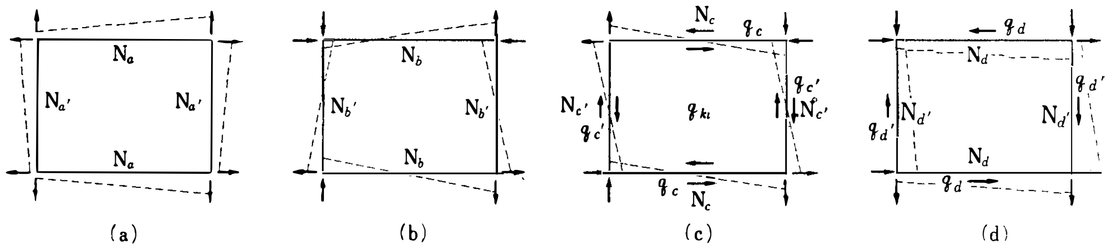

Fig. 3. Equilibrium condition of the panel.

N_a, q_c, etc.: Absolute values of axil forces and shear flow.

Provided that the above is the equilibrium condition, general deformation of Fig. 1 is virtually displacement. Principle of the virtual work, where the external work is W_0, is expressed as follows:

$$W_0=\int\frac{N_{(C)}N}{EA}dx+\gamma_{ki}\,a_k\,a_iL_0^2 \tag{6}$$

But,

$$W_0=[a_k(u_{ki}-u_{ki-1}+u_{k-1i}-u_{k-1\,i-1})$$
$$+a_i(v_{ki}-v_{k-1i}+v_{ki-1}-v_{k-1\,i-1})]L_0/2 \tag{7}$$

$$\int\frac{N_{(C)}N}{EA}dx=\frac{L_0^2}{2EA_0}\left\{\frac{a_k}{k_k^i}\Big[(N_{ki})_k^i+(N_{k-1i})_k^i\Big]-\frac{a_k}{k_k^{i-1}}\Big[(N_{ki-1})_k^{i-1}+(N_{k-1\,i-1})_k^{i-1}\Big]\right.$$

$$+\frac{a_i}{k_i^k}\left[(N_{ki})_i^k+(N_{ki-1})_i^k\right]-\frac{a_i}{k_i^{k-1}}\left[(N_{k-1i})_i^{k-1}+(N_{k-1i-1})_i^{k-1}\right]\Bigg\} \quad (8)$$

Since the elasticity condition of the member is,

$$\delta L_k^i=\frac{L_0}{2EA_0}\left[(N_{ki})_k^i-(N_{k-1\,i})_k^i\right] \quad (9)$$

then, considering the equilibrium equation (1), the basic equation of the axial force is obtained as,

$$\frac{(N_{ki})_k^i}{L_0}=+\frac{EA_0}{L_0{}^2}k_k^i(u_{ki}-u_{k-1i})+\frac{a_k}{2}(q_{ki}-q_{ki+1}) \quad (10)$$

$$\frac{(N_{k-1\,i})_k^i}{L_0}=-\frac{EA_0}{L_0{}^2}k_k^i(u_{ki}-u_{k-1i})+\frac{a_k}{2}(q_{ki}-q_{ki+1}) \quad (10')$$

On the other hand, substituting Eq. (10) in Eq. (8), the equation of the shear flow is expressed as follows, referring to Eqs. (5) and (6):

$$\begin{aligned}&-D_{ki}\,q_{ki}+\kappa_k^i q_{ki+1}+\kappa_k^{i-1}q_{ki-1}+\kappa_i^k q_{k+1i}+\kappa_i^{k-1}q_{k-1i}\\&+\frac{EA_0}{L_0{}^2}\left\{\frac{a_k}{2}(u_{ki}-u_{ki-1}+u_{k-1i}-u_{k-1\,i-1})\right.\\&\left.+\frac{a_i}{2}(v_{ki}-v_{k-1i}+v_{ki-1}-v_{k-1\,i-1})\right\}=0\end{aligned} \quad (11)$$

where $D_{ki}=\frac{EA_0}{GT_0L_0}\,\frac{a_k\,a_i}{t_{ki}}+\kappa_k^i+\kappa_k^{i-1}+\kappa_i^k+\kappa_i^{k-1}$

$\kappa_k^i=a_k/2k_k^i, \qquad \kappa_i^k=a_i/2k_i^k$

Substituting Eq. (10) into Eq. (2), joint equation is expressed as,

$$\begin{aligned}&\frac{EA_0}{L_0{}^2}\left\{-k_k^i u_{k-1\,i}+\left(k_k^i+k_{k+1}^i\right)u_{ki}-k_{k+1}^i u_{k+1i}\right\}\\&\quad+\frac{a_k}{2}(q_{ki}-q_{ki+1})+\frac{a_i}{2}(q_{k+1i}-q_{k+1\,i+1})=\frac{X_{ki}}{L_0}\end{aligned} \quad (12)$$

If we substitute Y for X, i for k and v for u respectively in Eq. (12), joint equation for the direction of Y-axis will be obtained.

§ 5. Basis of the Method

Considering the boundary condition, the solution can be obtained in the simultaneous equation of joint equation (12) and equation of shear flow (11) with the unknown u, v and q. Mechanical tabulation and the condition of symmetry and asymmetry on the stiffness matrix are exemplified in § 6.

Neglecting the axial deformation of member, approximate solution is solved in the equation of shear flow only where independent rotations of stiffness members are the unknown factor. Equation of shear flow is expressed as follows, referring to Eq. (4).

$$q_{ki}=\frac{GT_0}{2}t_{ki}\left[\left(R_k^i-R_i^k\right)+\left(R_k^{i-1}-R_i^{k-1}\right)\right] \tag{13}$$

But, in case the panel contains more than 2 members with different rotations, the above R_k^i should be the mean value of them.

§ 6. Example of Panel Structure with Opening

In Figs. 5 (a) and (b), the panels (the oblique lined parts) are symmetrical to the $A-A$ and $B-B$ axes, and the loads, V, V', H and H', which are asymmetrical to the axes, acting upon the joints ① and ② of the panels.

Considering that the above condition are symmetric to a point, the u, v and q

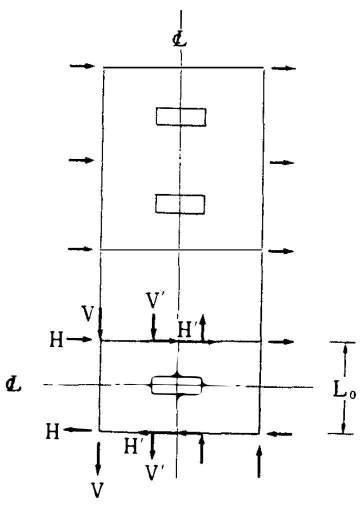

Fig. 4. Free body diagram of the n-th storied wall with opening.

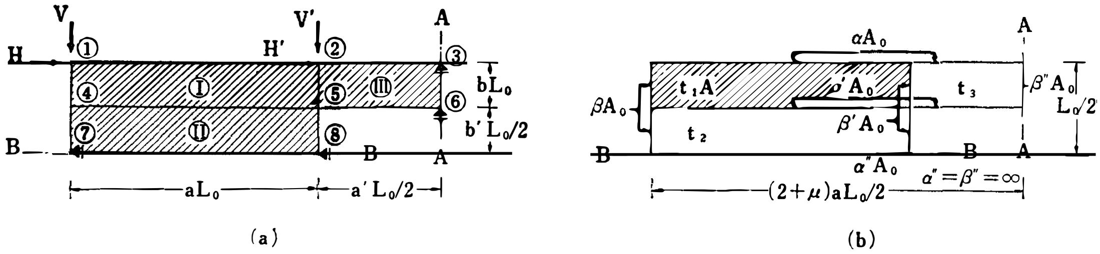

Fig. 5. External forces and structural constants in the 1st quadrant of a wall.

are able to shown in the first quadrant only. Therefore, assuming that infinitive cross section is put on the axes, u or v is restricted in content with condition of symmetry to a point.

Then, the horizontal force $2(H+H')$ is denoted as the shearing force at n-th story from the upper of the n-th storied wall and m is denoted these story moment.

In the mechanical tabulation Table 1 u, v and q are arranged horizontally and the equations, X_1, X_2,...... Y_1, Y_2,...... I, II,...... vertically.

Considering the equilibrium forces of Fig. 4 and solving joint equation for q_1, q_2 and q_3 we find,

$$2\{V(2a+a')+V'a\}=m/L_0,$$

$$V(2a+a')+V'a=(2b+b')(H+H') \tag{14}$$

assuming,

$$\frac{V'}{V}=c, \qquad \frac{a'}{a}=\mu, \qquad \frac{b'}{b}=\nu, \qquad A=abL_0{}^2$$

then,

$$\begin{aligned} \bar{q}_1&=\frac{2A}{m}q_1=-(2+\mu)\\ \bar{q}_2&=\frac{2A}{m}q_2=-\frac{(2+\nu)(2+\mu)}{2+\mu/(1+c)}\\ \bar{q}_3&=\frac{2A}{m}q_3=-\left\{1-\frac{\mu}{2}\,\frac{(2+\nu)}{2+\mu/(1+c)}\right\} \end{aligned} \tag{15}$$

Table 1. Simultaneous equations of panel structure with opening.

	u'_1	u'_2	u'_3	u'_4	u'_5	u'_6	v'_1	v'_4	v'_7	v'_2	v'_5	v'_8	q_1	q_2	q_3	Load terms or left side of Eq
X_1	k_1	$-k_1$											$-a/2$			H/L_0
X_2	$-k_1$	k_1+2k_2	$-2k_2$										$-a/2$		$-a'/4$	H'/L_0
X_3		$-2k_2$	$2k_2$												$-a'/4$	0
X_4				k'_1	$-k'_1$								$a/2$	$-a/2$		0
X_5				$-k'_1$	$k'_1+2k'_2$	$-2k'_2$							$a/2$	$-a/2$	$a'/4$	0
X_6					$-2k'_2$	$2k'_2$									$a'/4$	0
Y_1							k_3	$-k_3$					$-b/2$			V/L_0
Y_4							$-k_3$	k_3+2k_4	$-2k_4$				$-b/2$	$-b'/4$		0
Y_7								$-2k_4$	$2k_4$					$-b'/4$		0
Y_2										k'_3	$-k'_3$		$b/2$		$-b/2$	V'/L_0
Y_5										$-k'_3$	$k'_3+2k'_4$	$-2k'_4$	$b/2$	$b'/4$	$-b/2$	0
Y_8											$-2k'_4$	$2k'_4$		$b'/4$		0
I	$-a/2$	$-a/2$		$a/2$	$a/2$		$-b/2$	$-b/2$		$b/2$	$b/2$		$-D_1$	$a^2/2\alpha'$	$b^2/2\beta'$	0
II				$-a/2$	$-a/2$			$-b'/4$	$-b'/4$		$b'/4$	$b'/4$	$a^2/2\alpha'$	$-D_2$		0
III		$-a/4$	$-a/4$		$a'/4$	$a'/4$				$-b/2$	$-b/2$		$b^2/2\beta'$		$-D_3$	0

Notes:

$u'=\frac{EA_0}{L_0^2}u \quad v'=\frac{EA_0}{L_0^2}v$

$k_1=\alpha/a \quad k'_1=\alpha'/a \quad k_3=\beta/b \quad k'_3=\beta'/b$

$k_2=\alpha/a' \quad k'_2=\alpha'/a' \quad k_4=\beta/b' \quad k'_4=\beta'/b'$

$D_1=\frac{EA_0}{GT_0L_0}\,\frac{ab}{t_1}+\frac{a^2}{2\alpha}+\frac{b^2}{2\beta}+\frac{a^2}{2\alpha'}+\frac{b^2}{2\beta'}$

$D_2=\frac{EA_0}{GT_0L_0}\,\frac{ab'}{2t_2}+\frac{a^2}{2\alpha'}+\frac{bb'}{4\beta}+\frac{bb'}{4\beta'}$

$D_3=\frac{EA_0}{GT_0L_0}\,\frac{a'b}{2t_3}+\frac{a'a}{4\alpha}+\frac{a'a}{4\alpha'}+\frac{b_2}{2\beta}$

But, the standard length L_0 is taken as in Fig. 5 (b). Therefore, A is equivalent to the area hatched in Fig. 5 (b).

Then, assuming $\mu=1$, $\nu=2$ and $c\fallingdotseq 1$, we get,

$$\bar{q}_1=-0.05, \qquad \bar{q}_2=-0.25, \qquad \bar{q}_3=-0.4.$$

We can expect to find crack on the panel Ⓘⓘⓘ, since the absolute value of the Ⓘⓘⓘ is the greatest.

§ 7. Structure of Actual Panels

Five panels; two standard wall panels, two standard roof panels and one roof panel with mouthpiece of chimney, were experimented with.

Measurements of half size of the panels are shown in Figs. 6 and 7. Errors of the measurements are less than ± 0.5 m/m.

All the members of the panels were not placed symmetrically, but some reinforcing members of wall seat and of beam seat were placed in one side as shown in Figs. 6 and 7.

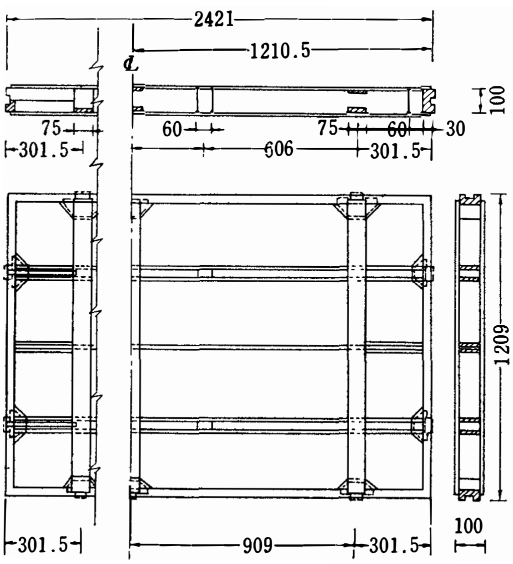

Fig. 6. Detail of wall panel.

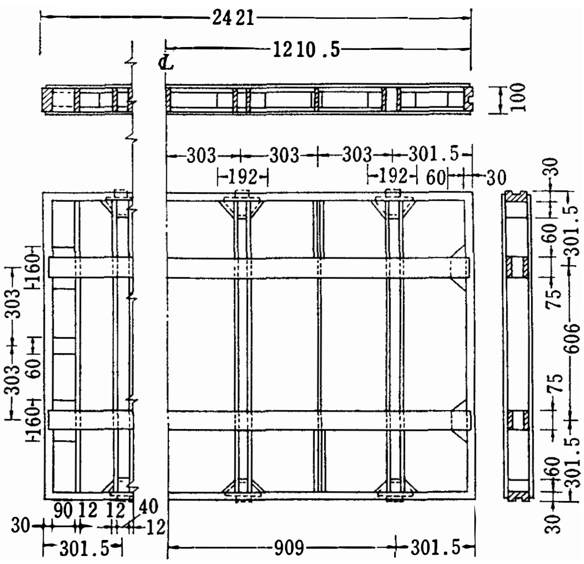

Fig. 7. Detail of roof panel.

The mouthpiece of chimney was also reinforced as shown in Photo 1.

The panels were made of the members of the selected Hinoki, Japanese cypress and of the sixply veneer of rolling birch of 6 m/m.

'Resolutinol', formaldehide resin, was used as the adhesive binding agent, and polystyrol bubble was put for insulation.

The surfaces of the panels were finished with paint except the rubber packing parts.

The veneer was soaked into phenol resin for moisture proof.

§ 8. Method of Experiment

The panels were examined by means of Amsler testing machine of 100 tons (limited bending load, 30 tons).

Static loads were added in the direction of arrows as shown in Figs. 8–11.

It was assumed that wedged parts of connector, JIS 3101 SF–34, were loaded with forces, and FB-100×9 and □-50×50 were used to distribute the forces uniformly as in Photo 2.

The wall panels were loaded on one side repeatedly, and the roof panels except No. 5 were loaded increasingly.

In the parts and in the direction as shown in Figs. 8–11, displacement was measured by means of dial gauges (1/100 m/m) and strain by paper gauges (S. B. P. Q. S 21N).

For examining YOUNG's modulus of the cypress and of the veneer of the panels, 16 double assemblies of the cypress (using vynyl acetate resin) of $40\times40\times100$ m/m

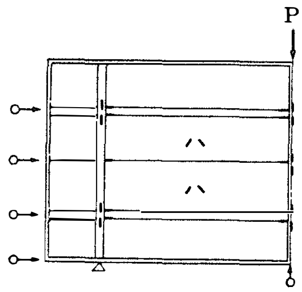

Fig. 8. Panel Nos. 1 and 2.

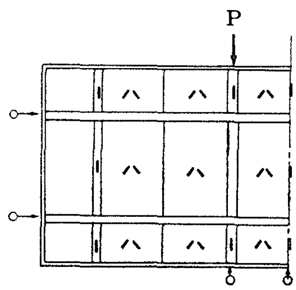

Fig. 9. Panel No. 3.

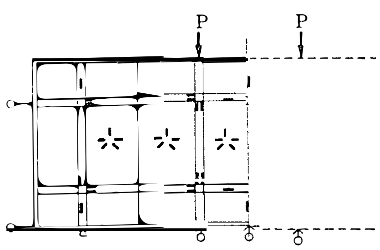

Fig. 10. Panel No. 4.

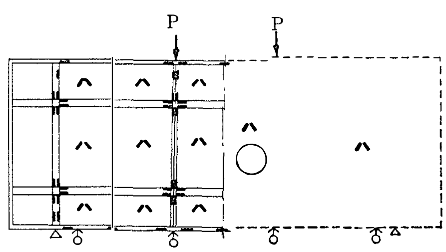

Fig. 11. Panel Nos. 5 and 6.

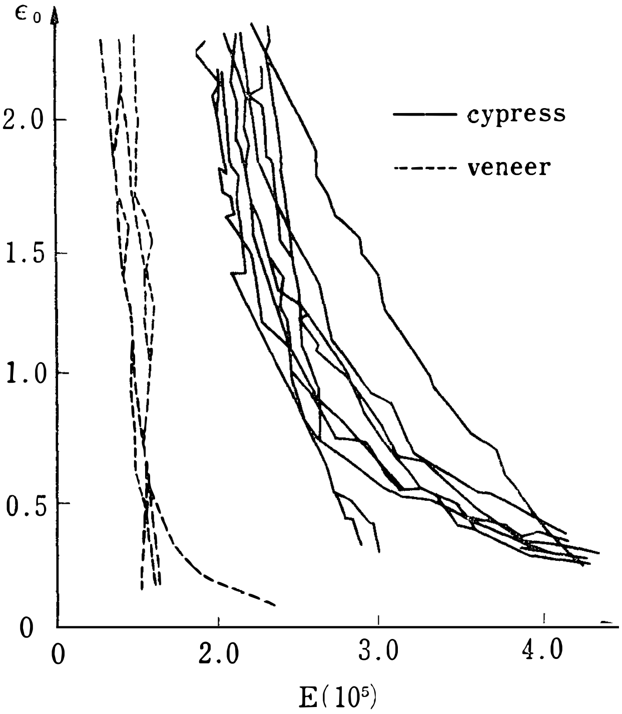

Fig. 12. Variatons of YOUNG'*s modulus.*

and 4 sixhold veneer of 36×40×100 m/m were prepared from the test panel, for approximate way.

After the moisture content of the test assemblies was measured, paper gauges were applied to both sides of the principal axis of their cross section, and YOUNG's modulus of the cypress and the veneer were measured through the compression test by Amsler testing machine.

The specific gravity of the cypress was 0.38–0.4 and that of the veneer was

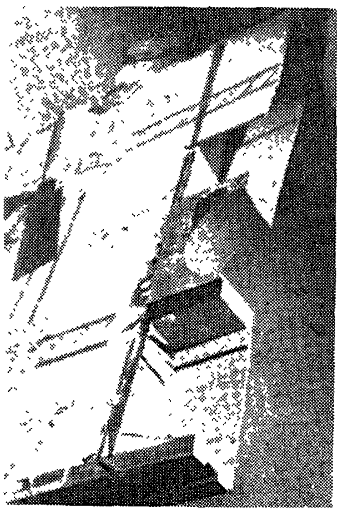

Photo 1.

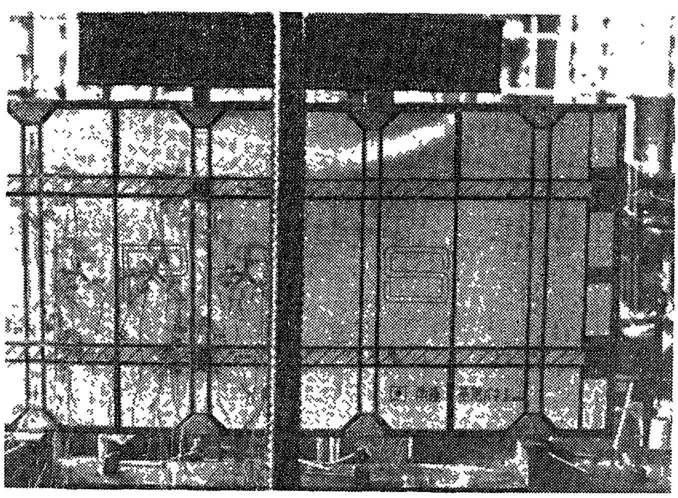

Photo 2.

0.74−0.76. The average moisture content of the cypress was 11.7 %.

The variations of YOUNG's modulus of the cypress and of the veneer are shown in Fig. 12.

§ 9. Theoretical Solutions by Displacement Method

Assuming that the panels in Figs. 6 and 7 were symmetric, we get the theoretical solution as shown in Figs. 13 and 14 by using Eqs. (11) and (12) described in § 4. They were the solutions on the u', v' and q obtained by means of the digital computer NEAC 2230.

For the wall panel, we resolved the simultaneous equation of the 32nd dimension as in Fig. 13, and for the roof panel, we get the result in Fig. 14 after combining the solutions of the simultaneous equations of the 31st, 22nd dimensions provided that all the members were placed symmetrically and asymmetrically.

In these cases, the constant of elasticity E was

$$E = 1.6 \times 10^5 \text{ kg/cm}^2,$$

then,

$$G = 8.9 \times 10^3 \text{ kg/cm}^2 \text{ (assumption)}.$$

The theoretical values for other G vary as indicated by dotted lines in Figs. 13 and 14.

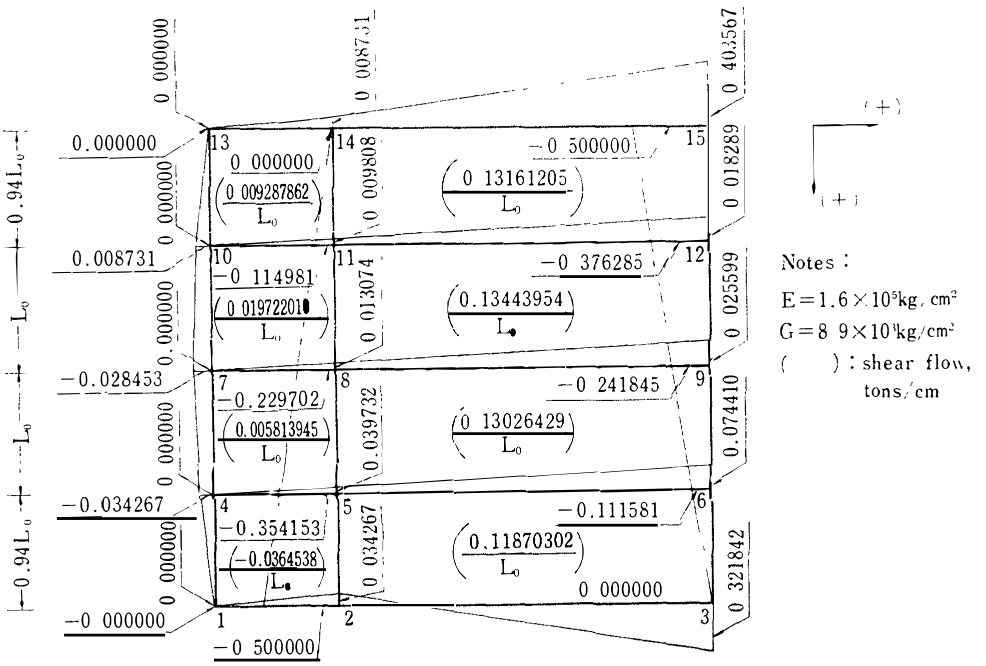

Fig. 13. Diagram of axial stress on wall panel at p tons (theoretical values).

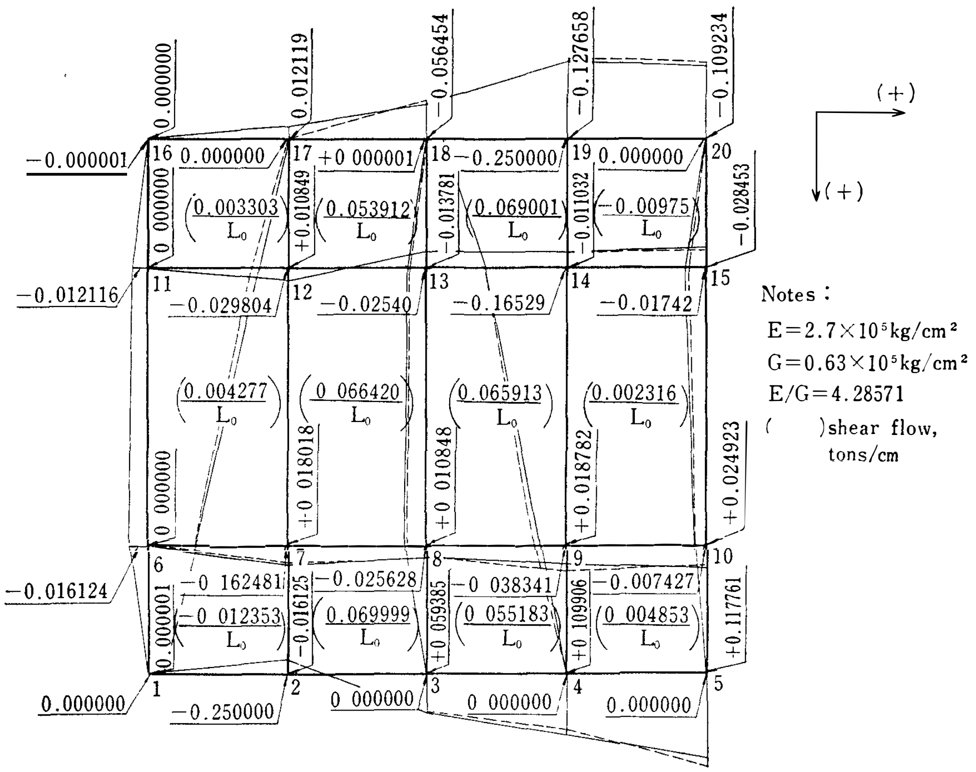

Fig. 14. Diagram of axial stress on roof panel at p/2 tons (theoretical values).

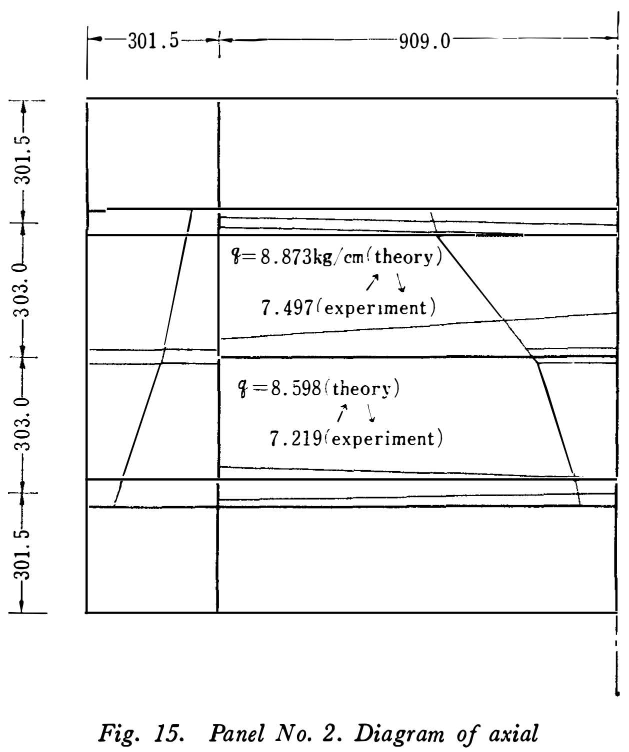

Fig. 15. Panel No. 2. Diagram of axial stress on wall panel at 2 tons.

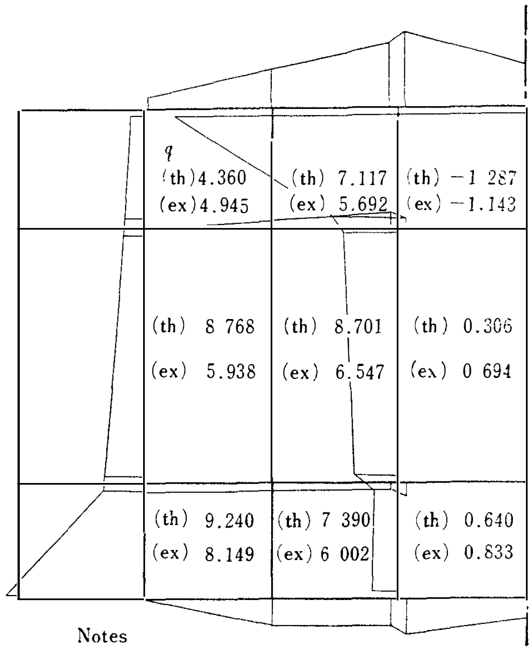

Fig. 16. Panel No. 5. Diagram of axial stress on roof panel at 2 tons.

In consideration of the above, it may be assumed that the distribution of internal forces is independent of the axial deformation of the member. However, the experimental values are considerably dependent of the disposition of stiffness members. We suspected that the secondary stress is generated on the panel.

Therefore, in designing the panel it is desirable to consider the normal stress in the shear field.

§ 10. Investigation

From the values measured by paper gauges (in Fig. 12), we obtain,

$$E=1.6\times10^5 \text{ kg/cm}^2$$

then,

$$G=\frac{E}{2(1+\nu)}=\frac{E}{2.54}=0.63\times10^5 \text{ kg/cm}^2$$

but,

$$\nu \fallingdotseq 0.27$$

and the states of the internal forces are described as in Figs. 15 and 16, provided that e_{45° means the strain in 45° direction and that the shear flow q_{ki} is given as,

$$q_{ki}=G(e_{45^\circ}-e_{135^\circ})t_{ki}$$

Almost the same values were obtained when the experiment was made for the load by other forces and on other panels.

Force-Max deformation curves between the loads and the base of the panel are shown in Figs. 17 and 18.

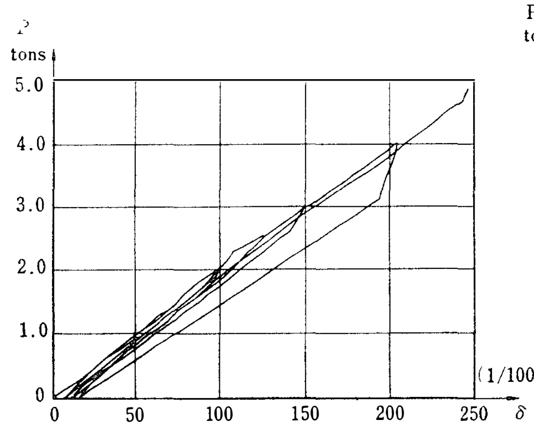

Fig. 17. Panel No. 2.

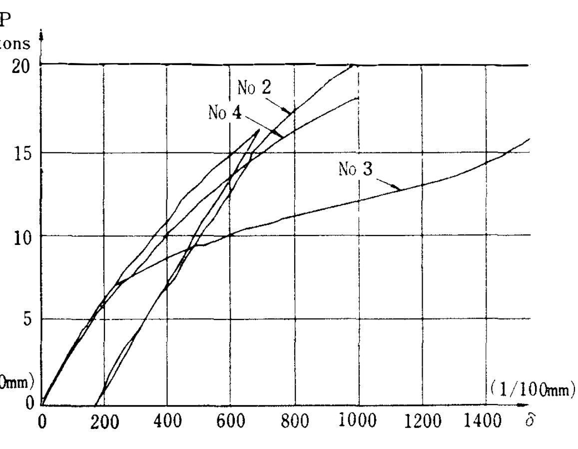

Fig. 18. Panel Nos. 2, 3 and 4.

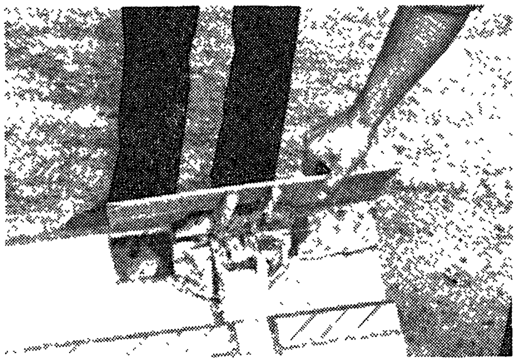

Photo 3.

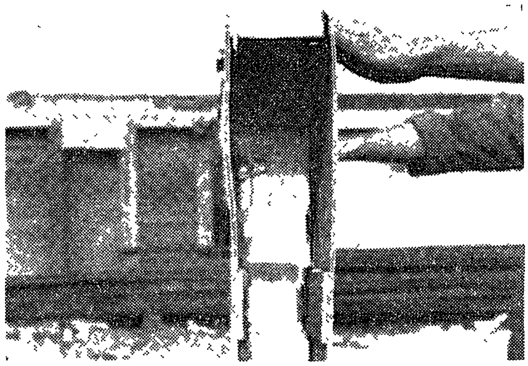

Photo 4.

§ 11. Rupture and Strength

As shown in Figs. 17 and 18, the panels are almost rigid and elastic until rupture.

Two dimentional strength of this Antarctic panels are determined by the buckling collapse at plates or stiffeners of the loading points and the local buckling as shown in Photos 3 and 4.

Moreover, the adhesive binding parts are disadvantageous when the loads are repeated.

§ 12. Comment on Design of Panel

We could not make a complete pursuit of the two problems mentioned at first, as they are limited in the phenomenal examination. In this experiment, however, we could get the following conclusion on the distribution of members of panel for prefabricated frame structure.

1) Compression zone of panel should be reinforced as to the beam seat of the panel.

2) Connector of the panel has a good durability.

3) When the larger side of sectional area of stiffness members is bound to the plane of the veneer as shown in Figs. 6 and 7, the member takes effect as stiffener and heat-resisting. But, it decreases in shearing rigidity and local buckling strength in their panels. Therefore, the members of the panel should be arranged in square grid system uniformly.

4) Crack or flow of veneer grows mold and decreases its durability and strength.

In general, the safety factor of wooden panel is required to be 2–2.5.

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References

1) EBNER, H. und H. KÖLLER: Jb. dt. LuftForsch., **15** (10–11), 527–542, 1958.
ROBINSON, J.: Structural Matrix Analysis for the Engineer, 123, John Wiley & Sons, 1966.
2) ARCHITECTURAL INSTITUTE OF JAPAN: An extra issue of Antarctic logistics. Mon. Rec. Archit., 1957.
3) NATIONAL ACADEMY OF SCIENCES: Symposium on Antarctic Logistics, 778p, 1963.
4) HIRAYAMA, Z.: The present condition of the building at Syowa Station. Antarctic Rec., **11**, 980–990, 1961.
5) SATO, T.: A prefabricated wooden house built by use of panels made by pasting together plywood boads with an adhesive agent. Bldg Engng (建築技術), **162**, 1965.

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