# QUANTITATIVE RING CURRENT MODEL: OVERVIEW AND COMPARISON WITH OBSERVATIONS 

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#### Abstract

This paper describes a new quantitative ring current model that solves temporal evolution of the ion distribution in the magnetosphere by tracing the ion drift motion. The plasma sheet density as a boundary condition of our model depends on the solar wind density. The tracing is performed under a dipole magnetic field and a time-dependent convection electric field depending on the solar wind parameters. The ions are lost by two processes; the charge exchange with neutral hydrogen and the convection outflow due to encounter with the dayside magnetopause. Magnetic disturbance is directly derived from the calculated current density with the threedimensional Biot-Savart integral; this is a new simulation method. Using this model, we have examined the physical mechanism of the storm-time ring current responding to the interplanetary parameters. We simulated three successive storms which occurred in April 1997 as a case study. The following subjects concerned with dynamics of the ring current were examined; (1) the causes of the ring current development, (2) the electric current distribution, (3) the effects of the charge exchange loss, (4) the energy composition of the plasma pressure, (5) the response time lag of the plasma sheet density variation to the solar wind density and (6) the diamagnetic effect.


## 1. Introduction

The terrestrial ring current mainly consists of trapped energetic ions with energies 10-200 keV (Frank, 1967; Smith and Hoffman, 1973; Williams, 1981). The global intensity of the ring current can be monitored by the ground-based magnetic observation at mid- and low-latitudes. The Dst index calculated from the observed geomagnetic field gives a zeroth-order estimate of the ring current intensity. Occasionally extra particles are transported (or injected) from the magnetotail into the ring current region by a large-scale convection field, and they are adiabatically energized. Consequently, the enhanced current depresses the Dst value. The total ring current energy has been estimated from the decrease of the Dst value using the Dessler-Parker-Sckopke relation (Dessler and Parker, 1959; Sckopke, 1966). Considering total kinetic energy in the ring current region, several empirical models have been proposed to explain the depression of storm-time Dst (e.g., Burton et al., 1975; Gonzalez et al., 1989). The empirical models suggest that the Dst decrease can be expressed by a function of the southward component of interplanetary magnetic field (IMF) and the solar wind bulk
velocity.
In order to understand a physical process for the ring current development and its decay, it is useful to trace trajectories of the ions that contribute to the ring current. Many models of the ring current development have been proposed (e.g., Lee et al., 1983; Wodnicka, 1989; Takahashi et al., 1990; Fok et al., 1993, 1995, 1996; Chen et al., 1994; Jordanova et al., 1994, 1998; Bourdarie et al., 1997; Kozyra et al., 1998a, b). Recently, Jordanova et al. (1998) and Kozyra et al. (1998a, b) have pointed out that the superdense plasma sheet plays an important role in the ring current buildup during magnetic storms.

After examining the relationship between the solar wind density and the near-earth plasma sheet density, Borovsky et al. (1997, 1998) and Terasawa et al. (1997) have concluded that they are highly correlated. This means that the near-earth plasma sheet density is well responsive to the change in the solar wind density. If the plasma sheet density is positively correlated with the solar wind density, the enhanced solar wind density could resultantly cause the ring current buildup. As evidence of that, Тномsen et al. (1998) statistically examined their relationship and they have concluded that an increase in the solar wind density is positively correlated with the ring current buildup. Using our particle tracing code, the ring current development as a function of the solar wind parameters is discussed in this paper. Our ring current model solves each particle's drift motion under a dipolar magnetic field and a time-dependent convection electric field. After tracing the drift motion, we calculate the three-dimensional distribution of the differential flux, the plasma pressure and the current density in the ring current region. The magnetic disturbance at the center of the earth can be derived with the Biot-Savart law from the whole three-dimensional distribution of the calculated current density; this is a new simulation method. Therefore our model can calculate the spatial and temporal changes of the particle distribution, the pressure, the current density and the magnetic disturbance as a function of physical parameters in the interplanetary space.

Several processes have been proposed to explain the global ring current enhancement empirically and theoretically; (1) the inward transport of plasma sheet particles by the enhanced convection electric field (e.g., Williams, 1981; Wodnicka, 1989; Takahashi et al., 1990; Lui, 1993; Bourdarie et al., 1997; Wolf et al., 1997; Jordanova et al., 1998; Ebihara and Eiri, 1998), (2) the particle injection associated with a substorm (e.g., Kamide and Fukushima, 1971; Lui et al., 1987; Fok et al., 1996; Ebihara et al., 1998b), (3) the diffusive transport of energetic particles due to the magnetic and/or electric fluctuation (e.g., Lyons and Schulz, 1989; Riley and Wolf, 1992; Lui, 1993; Chen et al., 1993, 1994, 1998; Bourdarie et al., 1997), (4) the inward displacement of pre-existing trapped particles due to an enhanced electric field (e.g., Lyons and Williams, 1980), (5) the direct entry of ions from the polar region into the ring current (e.g., Shelley et al., 1972, 1976; Parks et al., 1977; Cladis and Francis, 1985; Delcourt et al., 1990; Peroomian and Ashour-Abdalla, 1996). In this paper, we examine the process (1); the development of the ring current is caused by the inward transport of plasma sheet particles. The contribution of electrons to the ring current is not discussed in this paper because the energy density of electrons is less than that of protons by a factor of 5-6 in the ring current region (e.g., Frank, 1967). This lower
energy density of electrons may be caused by the lower temperature in the plasma sheet. Satellite observations have reported that the plasma sheet temperature of electrons is less than that of ions by a factor of 5-7 (e.g., Baumjohann et al., 1989).

Unless otherwise mentioned MKS units are used throughout this paper.

## 2. Model Description

### 2.1. Source distribution function

We first examine the relationship between the solar wind density and the plasma sheet density at the geocentric distance of $9-11 \mathrm{Re}$, which is the region for the ring

Table 1. Criteria for the selection of GEOTAIL and WIND data sets.

| GEOTAIL LEP | available |
| :--- | :---: |
| MLT of GEOTAIL | $21 \mathrm{~h}-3 \mathrm{~h}$ |
| Radial distance of GEOTAIL | $9-11 \mathrm{Re}$ |
| $\mid$ GSM-Z $\mid$ of GEOTAIL | $<2 \mathrm{Re}$ |
| Radial distance of WIND | $>15 \mathrm{Re}$ |
| Ion beta $\left(=2 \mu_{0} n k T / B^{2}\right)$ | $>0.8$ |



Fig. 1. Position of GEOTAIL satisfying the criteria in 1995-1997. Top and bottom panels show the position in the GSM-X-Y plane and the GSM-Y-Z plane, respectively.
current boundary. The number density and the temperature of ions in the plasma sheet are provided by GEOTAIL LEP (Mukai et al., 1994) with the energy-per-charge analyzer (EA-ion) which gives three-dimensional velocity distributions of ions over the energy range of $32 \mathrm{eV} / \mathrm{q}-39 \mathrm{keV} / \mathrm{q}$. The criteria to select the available data sets are listed in Table 1. To exclude the data observed in the lobe, the high ion beta value is added to the criteria; the ion beta is defined as $2 \mu_{0} N_{\mathrm{ps}} k T / B^{2}$, where $\mu_{0}$ is the permeability in vacuum, $N_{\mathrm{ps}}$ the plasma sheet density, $k$ the Boltzmann constant, $T$ the temperature and $B$ the intensity of the magnetic field. The local magnetic field data are provided by GEOTAIL MGF (Kokubun et al., 1994). The data during the period of umbras and penumbras of GEOTAIL are excluded. We found 170 thirty-minute intervals that satisfied the criteria in 1995-1997. Position of the thirty-minute intervals of GEOTAIL satisfying the criteria is shown in Fig. 1. The solar wind and IMF data sets are provided by SWE (Ogilvie et al., 1995) and MFI (Lepping et al., 1995) instruments aboard the WIND satellite, respectively. A time lag from WIND to the earth is adjusted by assuming that the solar wind velocity is fixed to be $400 \mathrm{~km} / \mathrm{s}$. However, the delay of the penetration of the solar wind medium into the plasma sheet is not adjusted here.

The result of the statistical analysis is shown in Fig. 2. The plasma sheet density $N_{\mathrm{ps}}$ at the geocentric distance of $9-11 \mathrm{Re}$ is well correlated with the solar wind density $N_{\mathrm{sw}}$; the result is consistent with the previous studies that the plasma sheet densities at the geosynchronous altitude and the geocentric distance of $17.5-22.5 \operatorname{Re}$ (Borovsky et al., 1997) and the geocentric distance of $15-50 \operatorname{Re}$ (Terasawa et al., 1997) are also well correlated with the solar wind density. The data indicated in Fig. 2 can be fitted by $N_{\text {ps }}$ $=0.025 N_{\mathrm{sw}}+0.395\left(\mathrm{~cm}^{-3}\right)$ with a linear correlation coefficient of 0.75 and the fitted equation is used to estimate the plasma sheet density at the geocentric distance of 10 Re for the boundary condition of this ring current model.

The distribution function at the model boundary located at 10 Re is assumed to be isotropic and Maxwellian. The temperature at the boundary is fixed to be 5 keV and the number density $N_{\mathrm{ps}}$ is given by the fitted equation described above.


Fig. 2. The plasma sheet density at the geocentric distance of 9-11 Re ( $N_{\mathrm{ps}}$ ) as a function of the solar wind density $\left(N_{\mathrm{sw}}\right)$.

### 2.2. Convection electric field

Recently, Boyle et al. (1997) reexamined solar wind coupling functions in the literature using more than 58000 polar passes of DMSP with strict criteria to provide updated estimates of the polar cap potential. They found the function expressed as

$$
\begin{align*}
\Phi_{\mathrm{PC}}= & -4.1+0.5 \sin \left(\phi+0.056+0.015 B_{y}(\mathrm{nT})\right) \\
& \times\left(1.1 \times 10^{-4} V_{\mathrm{sw}}(\mathrm{~km} / \mathrm{s})^{2}+11.1 B_{\mathrm{IMF}}(\mathrm{nT}) \sin ^{3}\left(\theta_{\mathrm{IMF}} / 2\right)\right)(\mathrm{kV}), \tag{1}
\end{align*}
$$

where $\Phi_{\mathrm{PC}}$ is the polar cap potential, $\phi$ the magnetic local time (MLT), $B_{y}$ the GSM-Y component of IMF, $V_{\mathrm{sw}}$ the bulk velocity of the solar wind, $B_{\mathrm{IMF}}$ the magnetic intensity of IMF and $\theta_{\mathrm{IMF}}$ the polar angle of IMF. After removing the skewed angle of the potential, we applied the polar cap potential drop expressed in eq. (1) to the VollandStern type convection field model (Volland, 1973; Stern, 1975) with a shielding factor of 2 in the equatorial plane as

$$
\begin{equation*}
\Phi \approx\left[1.1 \times 10^{-4} V_{\mathrm{sw}}(\mathrm{~km} / \mathrm{s})^{2}+11.1 B_{\mathrm{IMF}}(\mathrm{nT}) \sin ^{3}\left(\theta_{\mathrm{IMF}} / 2\right)\right] \frac{\sin \phi}{2}\left(\frac{R}{R_{B}}\right)^{2}(\mathrm{kV}) \tag{2}
\end{equation*}
$$

where $\Phi, R$ and $R_{B}$ are the electric potential, the geocentric distance at a given point and the geocentric distance of the magnetopause at 0600 MLT or 1800 MLT, respectively. In a dipole magnetic field, the geocentric distance of the magnetosphere boundary $R_{\mathrm{B}}$ in the equatorial plane is given by

$$
\begin{equation*}
R_{B}=\frac{1}{\cos ^{2} \lambda_{\mathrm{PC}}}(\mathrm{Re}), \tag{3}
\end{equation*}
$$

where $\lambda_{\mathrm{PC}}$ is a latitude of the polar cap boundary. $\lambda_{\mathrm{PC}}$ is taken to be $72^{\circ}$ and then $R_{B}$ is 10.47 Re ; the magnetosphere boundary $R_{B}$ is assumed to be steady in this simulation. According to the statistics, the assumed polar cap boundary latitude of $72^{\circ}$ in the dawn and dusk meridians is consistent with the poleward boundary of the closed region for active periods (Makita et al., 1983).

### 2.3. Loss processes

The ions are lost by two processes; the charge exchange with neutral hydrogen and the convection outflow to the dayside magnetopause azimuthally located at $L=10$. The Coulomb collision loss with thermal plasmas is neglected (Ebihara et al., 1998a).

The charge exchange is one of the major loss processes of the ring current ions (e. g., Frank, 1967; Swisher and Frank, 1968; Prülss, 1973; Smith et al., 1976, 1981; Tinsley, 1976; Lyons and Evans, 1976; Smith and Bewtra, 1978; Kistler et al., 1989; Fok et al., 1993; NoËl, 1997). The change of the phase space density $f$ due to the charge exchange loss is expressed as

$$
\begin{equation*}
\frac{\partial f}{\partial t}=-\frac{f}{\tau_{\mathrm{ce}}} \tag{4}
\end{equation*}
$$

where $\tau_{\mathrm{ce}}$ is the charge exchange lifetime. We calculate $\partial f / \partial t$ along particles' bounceaveraged trajectories. The bounce-averaged charge exchange lifetime can be expressed as

$$
\begin{equation*}
\tau_{\mathrm{ce}} \simeq \frac{\cos ^{\prime} \lambda_{\mathrm{m}}}{n_{\mathrm{H}}^{\prime} \sigma_{\mathrm{H}} v}, \tag{5}
\end{equation*}
$$

where $\lambda_{\mathrm{m}}$ is the mirror latitude, $n_{\mathrm{H}}^{\prime}$ the equatorial density of the neutral hydrogen, $\sigma_{\mathrm{H}}$ the charge exchange cross section and $v$ the speed of an ion. Following Smith and Bewtra (1978), the value $j$ is given to be 3.5 . A spherically symmetric model derived by Chamberlain (1963) with its parameters given by Rairden et al. (1986) is applied to obtain the equatorial number density of the neutral hydrogen $n_{H}^{\prime}$. We use the charge exchange cross section $\sigma_{\mathrm{H}}$ from Janev and Smith (1993).

### 2.4. Overview of the scheme

The simulation scheme is schematically summarized in Fig. 3. All ions are injected through the 'injection boundary' azimuthally located at $L=10$ with 2100-0300 MLT. After tracing the ions under a dipole magnetic field, the corotation electric field


Fig. 3. A block diagram of this simulation that depends on the solar wind and IMF as input parameters. Rectangles and round rectangles indicate physical quantities and physical process, respectively. The primary output of this simulation is the directoonal differential flux in the equatorial plane.
and the Volland-Stern type convection field with its intensity depending on the solar wind velocity and IMF, the directional differential flux, the plasma pressure and the current density are calculated. The magnetic disturbance induced by the current density can be directly obtained with the Biot-Savart integral from the whole threedimensional distribution of the calculated ring current. The bounce-averaged drift velocity and the details of the derivation of the macroscopic quantities (the differential flux, the plasma pressure and the current density) are given in Appendix A.

## 3. Results

We simulated three successive magnetic storms, which occurred in April 1997, and compared the calculated results to the observed values of pressure corrected Dst. The pressure corrected Dst (hereinafter referred to as $D s t^{*}$ ) is given by


Fig. 4. The solar wind, IMF and Dst during a period the storms on April 9-25, 1997. From the top to the bottom panels, the solar wind bulk velocity $V_{\mathrm{sw}}$, the number density of the solar wind protons $N_{s w}$, the GSM-X, - $Y$ and $-Z$ components of the IMF and observed Dst are shown. Vertical lines indicate the start times of the storms.

$$
\begin{equation*}
D s t^{*}=\left(D s t-c_{1} P_{s w}^{1 / 2}+c_{2}\right) / \xi, \tag{6}
\end{equation*}
$$

where $P_{\mathrm{sw}}$ is the dynamic pressure of the solar wind, $\xi$ a coefficient for the earth's induction, and $c_{1}$ and $c_{2}$ are empirical coefficients, respectively. Typically, the coefficients, $c_{1}$ and $c_{2}$, are $0.2 \mathrm{nT} /\left(\mathrm{eV} \mathrm{cm}^{-3}\right)^{1 / 2}$ and 20 nT , respectively (e.g., Gonzalez et al., 1994). After Dessler and Parker (1959), the coefficient for the induction $\xi$ is taken to be 1.5 . The solar wind bulk velocity $\left(V_{\mathrm{sw}}\right)$, the solar wind proton number density ( $N_{\mathrm{sw}}$ ), the GSM-X, -Y and -Z components of the interplanetary magnetic field (IMF-Bx, -By and -Bz), and the observed Dst index during the storms are plotted in Fig. 4. These interplanetary parameters are obtained from SWE and MFI instruments aboard the WIND satellite. There were three major storms in April 1997; the storms began at 1300 UT on April 10, 1997 (hereinafter denoted as Storm I), at 1320 UT on April 16 (Storm II) and at 0500 UT on April 21 (Storm III), which are indicated by vertical lines in Fig. 4. The minima of Dst are $-82 \mathrm{nT},-77 \mathrm{nT}$ and -107 nT , respectively; the storms were categorized as moderate to intense ones. Especially, Storm III was caused by a passage of a huge magnetic cloud with an estimated diameter of 0.4 AU ( R . Lepping, personal communication, 1998).

### 3.1. Dst

In Fig. 5, the calculated $D s t^{*}$ is compared with the observed one during the period


Fig. 5. Comparison between calculated and observed Dst*. From the top panel, the plasma sheet number density deduced from the solar wind density, the polar cap potential drop derived by Boyle et al. (1997) and calculated Dst* (thick line) with observed Dst* (dotted line) during the period of April 9-25, 1997 are shown.
of April 9-25, 1997. We adjust the offset of the observed Dst* at the beginning of the Storm I (April 10, 1997, 1300 UT) to the calculated Dst* at this time. Because the magnetosphere is initially empty in our calculation, the curve of calculated Dst* starts with Dst* of 0 at 0000 UT on April 9, 1997. As demonstrated in Fig. 5, the calculated $D s t^{*}$ roughly tracks the observed $D s t^{*}$. The calculated Dst* tends to overshoot the observed one. The recovery of the calculated Dst* takes place earlier than the observed Dst* by 5-7 hours. The difference between calculated and observed Dst* will be discussed in Section 4.

### 3.2. Energy input rate

Number of ions passing through the 'injection boundary' depends on the polar cap potential $\Phi_{\mathrm{PC}}$ and the plasma sheet number density $N_{\mathrm{ps}}$ in this simulation. To examine both contributions to the ring current buildup as indicated by $D s t^{*}$, two cases are examined; one is a case under the steady convection with $\Phi_{\mathrm{PC}}$ of 20 kV , and the other is a case under the steady plasma sheet density with $N_{\mathrm{ps}}$ of $0.4 \mathrm{~cm}^{-3}$.

Figure 6 shows the results. The calculated Dst* keeping the plasma sheet density constant (the thick and solid line) tends to undershoot the observed one around the most disturbed periods. This may suggest that the enhancement of the plasma sheet density is needed especially for the Storm I and the Storm III. The calculated Dst* keeping the polar cap potential constant (the thick and dashed line) little agrees with the observed


Fig. 6. Plasma sheet density (top), polar cap potential drop (middle) and Dst* (bottom). Three curves in the bottom panel indicate observed Dst* (thin line), calculated Dst* with steady convection field of 20 kV (thick and dashed line) and calculated Dst* with steady plasma sheet density of 0.4 $\mathrm{cm}^{-3}$ (thick and solid line), respectively.
one.
The result can be explained by an analytical expression. The energy input rate into the ring current is analytically given by

$$
\begin{equation*}
\gamma=0.572 N_{\mathrm{ps}}\left(\mathrm{~cm}^{-3}\right) E_{0}(\mathrm{keV}) \Phi_{\mathrm{PC}}(\mathrm{kV}) \text { (gigawatt), } \tag{7}
\end{equation*}
$$

where $E_{0}$ is the plasma sheet temperature. The derivation of eq. (7) is described in Appendix B. Equation (7) indicates that the energy input rate is proportional to the number density in the plasma sheet $\left(N_{\mathrm{ps}}\right)$ times the temperature in the plasma sheet $\left(E_{0}\right)$ times the polar cap potential drop ( $\Phi_{\mathrm{PC}}$ ).

Because the plasma sheet temperature is insensitive to Dst* as examined below, the energy input rate $\gamma$ is approximately a function of the plasma sheet density $N_{\mathrm{ps}}$ and the polar cap potential $\Phi_{\mathrm{Pc}}$. If the plasma sheet temperature $E_{0}$ is fixed to be 5 keV , the energy input rate becomes simply as

$$
\begin{equation*}
\gamma=2.86 N_{\mathrm{ps}}\left(\mathrm{~cm}^{-3}\right) \Phi_{\mathrm{PC}}(\mathrm{kV}) \text { (gigawatt). } \tag{8}
\end{equation*}
$$

For a typical case in the main phase of a storm, $N_{\mathrm{ps}}$ is $1.5 \mathrm{~cm}^{-3}$ and $\Phi_{\mathrm{PC}} 120 \mathrm{kV}$. Then the energy input rate becomes 520 gigawatts.

### 3.3. Dependence on the plasma sheet temperature

As described in the previous subsection, the analytic expression of the energy input rate into the ring current eq. (7) indicates that the energy input rate is a function of $N_{\mathrm{ps}}$, $E_{0}$ and $\Phi_{\mathrm{PC}}$. As the plasma sheet temperature increases, the relative number of high energy ions increases but the relative number of low energy ions decreases. Because high energy ions tend to drift azimuthally, they hardly move towards the earth.


Fig. 7. Temperature dependence of the ring current buildup. The curve in the diagram indicates the minimum Dst* as a function of the plasma sheet temperature for Storm I occurred on April 10-11, 1997.

Therefore these ions flow away from the magnetosphere and they contribute slightly to the ring current buildup. Consequently, the ring current intensity as indicated by $D s t^{*}$ could not simply be proportional to the plasma sheet temperature.

Figure 7 shows a variation of the minimum $D s t^{*}$ of Storm I as a function of the plasma sheet temperature $E_{0}$ keeping the plasma sheet density constant. As demonstrated in Fig. 7, the ring current buildup as indicated by Dst* is insensitive to the temperature in the near-earth plasma sheet for the temperature above 3 keV . However, for the temperature below 3 keV , minimum $D s t^{*}$ is sensitive to the plasma sheet temperature.

According to the statistics of the direct satellite observations, the average temperature in the near-earth plasma sheet is $\sim 5 \mathrm{keV}$ (e.g., Baumjohann et al., 1989; Paterson et al., 1998), except during a substorm expansion phase. During the substorm expansion, Baumjohann (1996) and Birn et al. (1997) reported that the temperature in the near-earth plasma sheet increases. According to Borovsky et al. (1998), the plasma sheet temperature at $17.5-22.5 \mathrm{Re}$ is higher than 3 keV for the solar wind velocity $>350$ $\mathrm{km} / \mathrm{s}$. Therefore, we concluded that the ring current buildup during a storm is insensitive to the change of the plasma sheet temperature.

### 3.4. Electric current distribution

Figure 8 illustrates the temporal evolution of the plasma pressure perpendicular to the magnetic field ( $P_{\perp}$ ) and the current density perpendicular to the magnetic field ( $J_{\perp}$ ) in the equatorial plane for Storm I. There are two characteristics to be noted:

1) As the storm develops at 0100 UT on April 11 (labeled as $B$ ), the plasma pressure drastically increases in the dusk region; the peak is located at 5.1 Re with the perpendicular plasma pressure of 34 nPa , Both westward and eastward currents are also enhanced simultaneously. The peak of the eastward current is located at 4.4 Re and the westward current at 6.9 Re . The spatial structure of the current density is essentially asymmetric.
2) In the recovery phase at 0700 UT on April 11 (labeled as $C$ ), the previously enhanced plasma pressure and the current density decrease. The peak of the plasma pressure is still located at 5.3 Re , but the pressure decreases to 18 nPa . The spatial structure of the current density is getting more symmetric.

The asymmetric structure (during the main phase) and the symmetric structure (during the recovery phase) were observed by the NOAA-12 satellite. Figure 9 shows daily averaged count rates of protons with an energy range of $30-80 \mathrm{keV}$ observed by NOAA-12 (the daily averaged data, courtesy of Y. Miyoshi) during a period of April 9 -13, 1997. Here we regard that the daily averaged count rate of the protons with a energy range of $30-80 \mathrm{keV}$ gives the relative intensity of the core ring current. NOAA12 is the polar orbiting satellite with an altitude of 815 km and an inclination of $98^{\circ}$. The Medium Energy Proton and Electron Detector (MEPED) has solid state detector telescopes within the energy ranges of $30-80,80-250,250-800$ and $800-2500 \mathrm{keV}$. One telescope views radially outward along the earth-satellite vector. Another telescope views in a direction perpendicular to the first one. Therefore the latter telescope observes trapped particles at high latitudes. Each line in Fig. 9 indicates the count rates of the protons measured by the telescope viewing the perpendicular direction to the


Fig. 8. Snapshots of the equatorial pressure and the current density in Storm I. Top panel shows calculated Dst*. Middle and bottom panels show the pressure perpendicular to the magnetic field and the current density perpendicular to the magnetic field in the equatorial plane, respectively, at 1800 UT on April 10, 1997 (left panels; denoted as A), 0100 UT on April 11, (middle panels; B) and 0700 UT on April 11, 1997 (right panels; C). In the bottom panels (current density), the pseudo-color code indicates the strength of the azimuthal component of the current density; red as westward and blue as eastward currents. Arrows indicate the direction of the current.
earth-satellite vector. A noticeable enhancement of counts in the dusk region could be seen during the main and the early recovery phases (April 10-11, 1997). The counts in the dusk region exceeded $\sim 2000$, which were greater than the counts in the dawn region by a factor of 6-7. The dawn-dusk asymmetry suggests that most of the protons transported from the night side plasma sheet escape from the ring current region, i.e., the protons, which enhance the counts in the dusk region, hardly drift into the dawn region through the dayside region. In the recovery phase, particular protons began drifting around the earth under a weakened convection field, and then the symmetric structure of the ring current located at $L=4-5$ remained in the dawn and dusk regions. It is also noticeable in Fig. 9 that the $L$-values of the most enhanced counts in the dawn region were somewhat larger than those of the counts in the dusk region. A theoretical


Fig. 9. Daily averaged count rates of protons with an energy range of $30-80 \mathrm{keV}$. The data were obtained by the polar orbitting satellite, NOAA-12. From top to bottom, each panel indicates the count rates as a function of $L$ during a period from April 9, 1997 to April 13. Left panels show the proton count rates in the dusk region (2000 MLT) and right panels show them in the dawn region ( 0800 MLT).
calculation suggests that the closed drift trajectories of the protons are not exactly symmetric (see Fig. A1); they are swelled in the dawn region. The difference of the peaks observed by NOAA-12 in the dawn and dusk region can be explained by considering the swelled drift trajectories of the protons. We cannot quantitatively discuss the details of the observed data because they are the data observed at high latitudes. NOAA-12 cannot measure the bulk of the ring current protons which are bouncing near the equatorial region.

The calculated cross sections of the electric current and the plasma pressure in the equatorial plane at 1800 MLT are presented in Fig. 10. Four curves in the top panel in Fig. 10 indicate the total azimuthal current density $J_{\phi}$, the magnetization current density $J_{\mathrm{M}}$, the curvature drift current density $J_{\mathrm{R}}$ and the grad-B drift current density $J_{\mathrm{B}}$, respectively. Since the anisotropy of the plasma pressure is relatively small ( $P_{\|} / P_{\perp}$ ~ 1.2), the azimuthal current density $J_{\phi}$ is mainly produced by the $\nabla P_{\perp}$ term in eq. (A 20) because the second term on the right hand side of eq. (A20) can be negligible.

Next, the contribution of the eastward and the westward currents to Dst* are


Fig. 10. Cross sections of the equatorial current density (top panel) and plasma pressure (bottom panel) as a function of $L$ in the meridian at 1800 MLT at 2200 UT on April 10, 1997. In the top panel, sold, dotted, dashed and dashed-dotted lines indicate the total azimuthal current $J_{\phi}$, the magnetization current $J_{\mathrm{M}}$, the curvature drift current $\mathrm{J}_{\mathrm{R}}$ and the grad-B drift current $J_{\mathrm{B}}$, respectively. The positive value denotes the westward current. In the bottom panel, a solid line indicates the perpendicular plasma pressure $P_{\perp}$ and a dashed line the parallel plasma pressure $P_{\|}$.


Fig. 11. Calculated Dst* induced by the westward current (thin line; denoted as 'westward'), by the eastward current (thin line; denoted as 'eastward') and by the both currents (thick line; denoted as 'both') during the period of April 9-25, 1997.
examined. The total current density $J_{\phi}$ has peaks at $L=4.4$ for the eastward current $\left(12 \mathrm{nA} / \mathrm{m}^{2}\right)$ and at $L=6.9$ for the westward current $\left(17 \mathrm{nA} / \mathrm{m}^{2}\right)$. These current densities resemble each other. However, the total current flows westward because of its larger volumes than that of the eastward current, and hence, Dst* decreases. Three lines in Fig. 11 indicate $D s t^{*}$ induced by the westward current, the eastward current and both currents, respectively. The intensity of $D s t^{*}$ induced by the westward current is larger than $D s t^{*}$ induced by the eastward current by a factor of 3-4.

### 3.5. Effects of charge exchange loss

The loss effect of energetic ions in the ring current region due to the charge exchange is examined here. The charge exchange loss effect on the Dst* during the storms is clearly shown in Fig. 12. The dotted line indicates the calculated Dst* without the charge exchange loss, i.e., the convection outflow is the only loss process. The initial rapid recovery of the $D s t^{*}$ at the beginning of the recovery phase can be seen in both with the charge exchange and without the charge exchange. This means that the initial rapid recovery in the early recovery phase is mainly due to (1) the temporal change of the spatial structure and (2) the sudden decrease of the plasma sheet density. We will discuss the cause of the initial rapid recovery in future. After the initial rapid recovery, the $D s t^{*}$ decays slowly due to the charge exchange in the late recovery phase. The recovery rate of the calculated $D s t^{*}$ with the charge exchange is in good agreement with that of the observed one. On the other hand, the Dst* without the charge exchange hardly recovers in the late recovery phases; the next storm occurs before the


Fig. 12. Charge exchange loss effect on Dst*. Observed Dst* (thin line), calculated Dst* with charge exchange loss process (thick line) and calculated Dst* without charge exchange loss process (thin dotted line) are plotted for the period of April 9-25, 1997.
sufficient decay of ring currents.

### 3.6. Energy composition of the plasma pressure

The differential perpendicular pressure, which is defined as $\mathrm{d} P_{\perp} / \mathrm{d} E$ (having a dimension of the number density), is introduced here. Figure 13 shows the calculated differential pressure at different $L$-values of 4 and 5. The white lines in Fig. 13 indicate the peak energy of the differential perpendicular pressure $d P_{\perp} / d E$, i.e., ions having the


Fig. 13. The differential pressure defined as $\mathrm{d} P_{\perp}(n \mathrm{~Pa}) / \mathrm{d} E$ (keV) at (a) $L=4$ and (b) $L=5$ in cross section at 1800 MLT at 0000 UT on April 16, 1997. White lines indicate a peak of the differential pressure. Each panel shows calculated Dst* (top), the polar cap potential $\Phi_{\mathrm{PC}}$ (middle) and a time series of the differential pressure (bottom).
energies indicated by the white lines mostly contribute to the perpendicular pressure. The results suggest that the major contributor to the perpendicular plasma pressure is the ions with energies $\simeq 15-30 \mathrm{keV}$ for $L=4$ and $\simeq 30-40 \mathrm{keV}$ for $L=5-6$ during the main phase and the early recovery phase, while the energy of a major contributor is $\simeq 15$ keV in the quiet periods. The rise of the major contribution energy to the perpendicular pressure is due to the enhancement of the convection electric field.

The previous satellite observations by the AMPTE/CCE satellite (Active Magnetospheric Particle Tracer Explorer/Charge Composition Explorer) (Williams, 1985) have shown that there is another peak around 100 keV in the dusk region during the main phase of the large storm on September 5, 1984 when AMPTE/CCE was located between $L=3.7-4.7$ (Gloeckler et al., 1985). The storm-associated double peaks were also observed by the Explorer 45 satellite (Smith and Hoffman, 1973). The first peak with energies of $15-20 \mathrm{keV}$ observed by AMPTE/CCE is in good agreement with the model calculation, that is, the first peak appears due to the convective transport from the near-earth plasma sheet. However, the second peak (with energy of $\sim 100$ keV ) is hardly explained by our model. Several processes are proposed to explain the existence of the second peak: (1) Chen et al. (1994) have suggested that substormassociated temporal electric fields can produce enhancements of the ring current with energies greater than $\sim 100 \mathrm{keV}$ at $L=2.5-4$. (2) Rowland and Wygant (1998) have presented the average structure of the inner magnetospheric electric field directly observed by the CRRES satellite. They observed a noticeable development of the electric field for moderate to high $K p$ at $L=3.5-6$. The intense electric field in the inner magnetosphere is different from the shielded Volland-Stern type electric field. This electric field may push ions with high energies toward the earth (Wygant et al., 1998). (3) A diffusive process due to the electric and the magnetic field fluctuations is also proposed. For quiet periods, the classical diffusion theory, e.g., Davis and Chang (1962) and Nakada and Mead (1965), well explains the real distribution of the high energy particles. However, the diffusive transport processes for active periods are not yet well understood because of the difficulty in determining the diffusion coefficient (e.g., Boscher et al., 1998).

### 3.7. Response time of the plasma sheet density change to the solar wind

We have assumed that the plasma sheet density changes with the solar wind density without delay. Borovsky et al. (1998) have concluded that the response time from the solar wind to the near-earth plasma sheet is of the order of 4 hours. Figure 14 shows the effects of the delay time of the plasma sheet density. The open square, the full square and the full circle indicate the calculated Dst* without delay time, with delay times of 3 hours and 7 hours, respectively. It is clear from the figure that the calculated Dst* with the delay time of 7 hours is closer to the observed Dst* for three storms. It seems reasonable to conclude that the delay time causes a significant change to Dst* during the main and the early recovery phases. Further analyses and simulations are required to clarify the physical meaning of the estimated delay time.

### 3.8. Diamagnetic effect

A high plasma pressure distorts local magnetic fields; this is called the diamagnetic


Fig. 14. Effects of the time delay of the plasma sheet density responding to the solar wind density for the three storms; the top panel for Storm I, the middle panel for Storm II and the bottom panel for Storm III. Three solid lines in a panel indicate calculated Dst* with no time delay (open square), with 3 hours delay (full square) and with 7 hours delay (full circle), respectively. A dashed line indicates observed Dst*. A vertical dotted line in all three panels indicates the commencement time of a storm reported by NOAA.
effect. Such distortion has been observed near the equatorial plane; Explorer 6 (Smith et al., 1960), Explorer 26 (Cahill, 1966; Hoffman and Cahill, 1968), Explorer 45 (e.g., Cahill, 1973), AMPTE/CCE (e.g., Potemra et al., 1985), CRRES (Wygant et al., 1998), ETS-6 (Terada et al., 1998) and POLAR (Tsyganenko et al., 1999). It can also be calculated in the present model as shown in Fig. 15.

Figure 15 shows a time series of contour plots of the distorted equatorial magnetic field induced by the ring current during the main and the early recovery phases of the storm on April 10-11, 1997. The distorted field is derived from the Biot-Savart integral


Fig. 15. Distorted equatorial magnetic field due to the ring current. Top panel indicates calculated Dst*. Each panel labeled as (a)-(f) shows a contour map of constant equatorial magnetic fields during the main and the early recovery phases of the storm on April 10-11, 1997 at (a) 1300 UT, April 10, 1997 (b) 1600 UT, (c) 1900 UT, (d) 2200 UT, (e) 0100 UT, April 11 and (f) 0400 UT, April 11. Numerical figures written in the contour are the intensity of the magnetic field in nanotesla.
over the whole three-dimensional distribution of the calculated current; this is the same method as $D s t^{*}$ is calculated in this study. Note that the ring currents are calculated under the condition of the geomagnetic field to be a given dipole field. This means that our simulation is not self-consistent.

The contour shown in Fig. 15 is a line equivalent to the $\nabla B$ drift trajectory. We classified the $\nabla B$ drift trajectories into four patterns;

1) an ion drifting westward around the earth (Type 1),
2) an ion drifting westward around the earth but partly drifting eastward in the reversed 'S' shape structure (Type 2),
3) an ion locally drifting anticlockwise around the magnetic depression, (Type 3),
4) an ion locally drifting clockwise around the magnetic hill (Type 4).


Fig. 16. Schematic ions' $\nabla B$ drift trajectories predicted from the calculation. The trajectories are categorized into four patterns (see text).

The trajectories for electrons are opposite to the cases of ions. Type 3 and Type 4 are the trajectories that a particle is locally trapped, never drifting around the earth. These patterns are schematically summarized in Fig. 16.

## 4. Discussion

The differences between calculated and observed $D s t^{*}$ are probably attributed to the following reasons.

1) There is an ambiguity of the estimated polar cap potential for given solar wind and IMF conditions. Furthermore, the solar wind and the IMF observed by WIND (located at $\sim 200$ Re in April 1997) do not always correspond to those in the vicinity of the earth's magnetosphere. According to the direct observations by CRRES, the storm-time electric field in the inner magnetosphere is different from the Volland-Stern type electric field (Rowland and Wygant, 1998; Wygant et al., 1998). In future, we will examine the particle motion under the realistic electric field.
2) There is an ambiguity of the estimated plasma sheet density. Discussing this ambiguity is beyond the scope of this paper because the penetration process of the solar wind ions into the magnetosphere is still controversial.
3) High energy ions with energies greater than $\sim 100 \mathrm{keV}$ also contribute to the ring current (e.g., Smith and Hoffman, 1973; Lui et al., 1987; Hamilton et al., 1988; Lyons and Schulz, 1989; Sheldon and Hamilton, 1993). Such high energy ions are steadily trapped by the earth's magnetic field but their trajectories may be changed by the electric and the magnetic field disturbances during a storm. In future, the contribution of such ions to the ring current will be studied to understand the dynamics of the ring current as a whole.
4) This simulation considers no other ion species, e.g., $\mathrm{O}^{+}$. Energetic $\mathrm{O}^{+}$ions become dominate in the inner magnetosphere with the development of magnetic storms. Hamilton et al. (1988), Roeder et al. (1996) and Daglis (1997) reported that the energy density of $\mathrm{O}^{+}$ions in the ring current often dominates that of $\mathrm{H}^{+}$ions in the ring current region during a strong magnetic storm. Daglis (1997) has indicated that the energy density of $\mathrm{O}^{+}$exceeds that of $\mathrm{H}^{+}$during intense magnetic storms with its minimum Dst of less than -200 nT , while the energy density of $\mathrm{O}^{+}$contributes only to $30 \%$ during a moderate magnetic storm (January 2, 1991). According to his analysis, the energy density of $\mathrm{O}^{+}$does not always exceed that of $\mathrm{H}^{+}$during storms, and the relative amount of $\mathrm{O}^{+}$energy density strongly depends on the intensity of Dst. The intensity of the minimum Dst of the moderate storm reported by Daglis (1997) is similar to those of the storms we examined. If the relative amount of $\mathrm{O}^{+}$energy density depends on the intensity of Dst for all storms, the energy density of $\mathrm{O}^{+}$could be a minor contributor to the storm-time energy density in the ring current region for these storms we examined.
5) The induction electric field due to the dipolarization event during the expansion phase of a substorm may change the energy input rate into the ring current. Although the substorm effect to the ring current buildup hardly explains the whole negative variation of storm time Dst* (e.g., Ebihara et al., 1998b), minor variations with short time scales of Dst* could be the result of the substorm effect.

## 5. Conclusions

1) The major variation of $D s t^{*}$ is mainly due to the convection electric field and the plasma sheet density. The effectiveness of the solar wind density to the plasma sheet density differs among the storms.
2) The ring current buildup is insensitive to the near-earth plasma sheet temperature for the temperature above 3 keV .
3) $D s t^{*}$ induced by the westward current is larger than $D s t^{*}$ induced by the eastward current by a factor of 3-4.
4) The ions with energies of $\sim 15-30 \mathrm{keV}$ at $L=4$, and $\sim 30-40 \mathrm{keV}$ at $L=5-6$ in the dusk region contribute mostly to the perpendicular pressure in the ring current.
5) The previous satellite observations have shown that there is typically another peak with the energy of around 100 keV . This second peak is hardly described by our model. We concluded that further physical processes, an additional electric field or diffusion, are necessary to explain the second peak.
6) The time lag of the plasma sheet density behind the solar wind density causes a significant change to $D s t^{*}$ during the main and the early recovery phases.
7) The diamagnetic effect by the ring current is greatly enhanced in the equatorial dusk region. The diamagnetic effect will play an important role in both the storm-time ring current buildup and the storm-time redistribution of relativistic particles.

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## Appendix A: Model Calculation

## A1. Particles' motion

The motion of energetic charged particles in the dipolar magnetic field is a superposition of three periodic motions; gyration about the local magnetic field line, bouncing between the northern and southern hemispheres along the field line and drifting perpendicular to the field line. Many computer resources are, in general, required to solve the Lorentz equation that governs the complete particle motion.

If the temporal variation of the electromagnetic fields for a gyroperiod is negligible, motion of a particle can be approximated by motion of its guiding center (Northrop, 1963). Averaging over the bouncing motion between mirror points, the guiding center motion can be projected onto an equatorial plane if the configuration of the magnetic field is a dipole or particular cases (e.g., Roederer, 1970; Shukhtina, 1993). The bounce-averaged drift velocity $\left\langle V_{s}\right\rangle$ under the dipolar magnetic field is given by

$$
\begin{equation*}
\left.<V_{s}\right\rangle=\frac{E \times B}{B^{2}}+\frac{W G\left(\alpha_{0}\right)}{q B^{3}} B \times \nabla B, \tag{A1}
\end{equation*}
$$

where $B$ is the magnetic field, $\mathbb{E}$ the electric field, $W$ the kinetic energy, $\alpha_{0}$ the equatorial pitch angle and $q$ the charge. The function of $G\left(\alpha_{0}\right)$ is given by EJiRI (1978).

The bounce-averaged drift trajectories under the dipole magnetic field and the Volland-Stern type convection field can be obtained by solving the following equations (Ejiri, 1978) as

$$
\begin{align*}
& \frac{\mathrm{d} X}{\mathrm{~d} t}=-\frac{\omega}{\gamma} X^{\gamma+2} \cos \phi,  \tag{A2}\\
& \frac{\mathrm{~d} \phi}{\mathrm{~d} t}=\omega X^{\gamma+1} \sin \phi+\omega-\frac{3 \mu G\left(y_{0}\right)}{q} \overline{R_{\mathrm{s}}^{2} X^{2}},  \tag{A3}\\
& \frac{\mathrm{~d} y_{0}}{\mathrm{~d} t}=-\frac{y_{0} I\left(y_{0}\right)}{4 f\left(y_{0}\right)} \frac{1}{X} \frac{\mathrm{~d} X}{\mathrm{~d} t},  \tag{A4}\\
& X \equiv \frac{R}{R_{\mathrm{s}}}, \tag{A5}
\end{align*}
$$

where $\omega$ is the angular velocity of the earth's rotation, $\phi$ the local time, $\gamma$ the shielding factor of the convection field, $y_{0}$ the sine of the equatorial pitch angle, $I\left(y_{0}\right)$ a function related to the second invariant, $\mu$ the first adiabatic invariant (magnetic moment) and $R_{\text {s }}$ is the geocentric distance of a stagnation point at 1800 MLT for a zero energy particle. The geocentric distance of a stagnation point ( $R_{\mathrm{s}}$ ) for a zero energy particle is

$$
\begin{equation*}
R_{\mathrm{s}}=\left(\frac{\omega R e^{3} B_{0}}{A \gamma}\right)^{\frac{1}{\gamma+1}} . \tag{A6}
\end{equation*}
$$

The function $I\left(y_{0}\right)$ is given by

$$
\begin{equation*}
I=\frac{1}{R} \int_{s_{1}}^{s_{2}}\left[1-\frac{B(s)}{B_{\mathrm{m}}}\right]^{1 / 2} \mathrm{~d} s \tag{A7}
\end{equation*}
$$

where $B_{\mathrm{m}}$ is the intensity of the magnetic field at the mirror points $s_{1}$ and $s_{2}$.
Solving the equations, eq. (A2), eq. (A3) and eq. (A4), one can trace the bounce-averaged drift trajectory for a given initial condition. Several proton and electron bounce-averaged trajectories are illustrated in Figs. A1 and A2.


Fig. A 1. Magnetic moment dependence of bounce-averaged trajectories for protons (top panels) and electrons (bottom panels) with their pitch angles of $90^{\circ}$. From left to right panels, each panel indicates the trajectories for the initial kinetic energies of $0.1,1$, and 10 keV , respectively. They correspond to the magnetic moments of $3.23 \mathrm{eV} / n T, 32.3 \mathrm{eV} / \mathrm{nT}$ and $323 \mathrm{eV} / n T$, respectively. The trajectories are traced under the dipole magnetic field and the Volland-Stern type convection field with its intensity for $K p=4$ case by the Maynard and CHEN (1975) model. All particles start at $L=10$. Particle positions are represented by dots at 10-minute steps.

## A2. Differential fux and macroscopic quantities

Following Cladis and Francis (1985), we derived a method to calculate the absolute differential flux of the trapped particles. A basic concept for the derivation is that a packet particle carries a number of real particles in a small phase space bin; this is conceptually corresponding to the Liouville theorem.

A directional differential flux $j$ is defined as


Fig. A 2. Same as previous figure except that the intensity of the convectoon field dependence of the bounce-averaged trajectories for protons (top panels) and electrons (bottom panels) with their pitch angles of $30^{\circ}$ and the initial kinetic energy of $10 \mathrm{keV}(323 \mathrm{eV} / \mathrm{nT})$. Left panels indicate the trajectories for $K p=1$ and bottom panels for $K p=4$.

$$
\begin{equation*}
j=\frac{\mathrm{d} N}{\mathrm{~d} S_{\mathrm{l}} \mathrm{~d} t \mathrm{~d} \Omega \mathrm{~d} W}, \tag{A8}
\end{equation*}
$$

where $\mathrm{d} S_{\perp}$ and $\Omega$ are the area of a (virtual) detector and the solid angle, respectively. $\mathrm{d} N$ is a number of particles that flow through the detector per unit perpendicular area. The directional differential flux whose detector is placed in the equatorial plane gives a three-dimensional distribution of the differential flux because all particles pass through the equatorial plane except for particles with their pitch angles within a loss cone. The differential flux in the equatorial plane $j_{0}$ is given by

$$
\begin{equation*}
j_{0}=\frac{\mathrm{d} N}{2 \pi \mathrm{~d} S \tau_{\mathrm{b}}\left(y_{0}\right) y_{0} \mathrm{~d} y_{0} \mathrm{~d} W}, \tag{A9}
\end{equation*}
$$

where $\tau_{\mathrm{b}}$ is the bounce period. Here the sine of the equatorial pitch angle $\left(y_{0}\right)$ is introduced instead of the solid angle ( $\Omega$ ).

Assuming that the magnetic flux is conserved and that the total kinetic energy and
the magnetic moment of particles are conserved, the off-equatorial flux $j$ at a latitude of $\lambda$ can be derived from the Liouville theorem (Roederer, 1970) as

$$
\begin{equation*}
j(y)=j_{0}\left(y_{0}\right), \tag{A10}
\end{equation*}
$$

with

$$
\begin{equation*}
y_{0}=y \frac{\cos ^{3} \lambda}{\left(1+3 \sin ^{2} \lambda\right)^{1 / 4}} . \tag{Al1}
\end{equation*}
$$

The perpendicular pressure $P_{\perp}$ and the parallel plasma pressure $P_{\|}$are given by

$$
\begin{align*}
& P_{\|}=\int m v^{2} F(v) \cos ^{2} \alpha \mathrm{~d} v,  \tag{A12}\\
& P_{\perp}=\int \frac{1}{2} m v^{2} F(v) \sin ^{2} \alpha \mathrm{~d} v, \tag{A13}
\end{align*}
$$

where $F$ is the velocity distribution function and $m$ the particle's mass. The pressure can be expressed by using the directional differential flux $j$ instead of the velocity distribution function $F$, as

$$
\begin{align*}
& P_{\perp}=\pi \sqrt{2 m} \int_{J} \int_{W} j \sqrt{W} \sin ^{3} \alpha \mathrm{~d} \alpha \mathrm{~d} W,  \tag{A14}\\
& \boldsymbol{P}_{\|}=2 \pi \sqrt{2 m} \int_{\alpha} \int_{W} j \sqrt{W} \cos ^{2} \alpha \sin \alpha \mathrm{~d} \alpha \mathrm{~d} W . \tag{A15}
\end{align*}
$$

The current density perpendicular to the magnetic field in the ring current has been considered as a combination of the three currents (PARKER, 1957); the magnetization current $J_{\mathrm{M}}$, the curvature drift current $J_{\mathrm{R}}$ and the grad-B drift current $J_{\mathrm{B}}$. The polarization current and the gravitational current are neglected here. The magnetization current is due to particles' gyration and is expressed as

$$
\begin{equation*}
\boldsymbol{J}_{\mathbf{M}}=\nabla \times \boldsymbol{M}, \tag{A16}
\end{equation*}
$$

with

$$
\begin{equation*}
\boldsymbol{M}=-\frac{P_{\perp} \boldsymbol{B}}{B^{2}} . \tag{A17}
\end{equation*}
$$

The drift currents, $J_{\mathrm{R}}$ and $\boldsymbol{J}_{\mathrm{B}}$, are

$$
\begin{equation*}
\boldsymbol{J}_{\mathrm{R}}=\frac{\boldsymbol{P}_{\|}}{\boldsymbol{B}^{4}} \boldsymbol{B} \times(\boldsymbol{B} \cdot \nabla) \boldsymbol{B}, \tag{A18}
\end{equation*}
$$

and

$$
\begin{equation*}
J_{\mathrm{B}}=\frac{P_{\perp}}{B^{3}} B \times \nabla B, \tag{A19}
\end{equation*}
$$

respectively. Thus the total azimuthal current density $J_{\perp}$ is

$$
\begin{align*}
J_{\perp} & =\mathbb{J}_{\mathrm{M}}+\mathbb{J}_{\mathrm{R}}+J_{\mathrm{B}} \\
& =\frac{\mathbb{B}}{B^{2}} \times\left[\nabla P_{\perp}+\left(P_{\|}-P_{\perp}\right) \frac{(B \cdot \nabla) B}{B^{2}}\right] . \tag{A20}
\end{align*}
$$

In the dipolar magnetic field, three components of the current density, the radial component $\left(J_{\mathrm{r}}\right)$, the azimuthal component positive eastward $\left(J_{\phi}\right)$ and the latitudinal component ( $J_{\lambda}$ ), become

$$
\begin{align*}
J_{\mathrm{r}}(r, \phi, \lambda)= & -\frac{B_{\lambda}}{B^{2} r \cos \lambda} \frac{\partial P_{\perp}}{\partial \phi} \\
= & -\frac{r^{2}}{M\left(1+3 \sin ^{2} \lambda\right)} \frac{\partial P_{\perp}}{\partial \phi},  \tag{A21}\\
J_{\phi}(r, \phi, \lambda)= & \frac{1}{B^{2}}\left(\frac{B_{\mathrm{r}}}{r} \frac{\partial P_{\perp}}{\partial \lambda}-B_{\lambda} \frac{\partial P_{\perp}}{\partial \mathrm{r}}\right) \\
& +\frac{1}{B^{3}}\left(P_{\|}-P_{\perp}\right)\left(\frac{B_{\mathrm{r}}}{r} \frac{\partial B}{\partial \lambda}-B_{\lambda} \frac{\partial B}{\partial r}\right)  \tag{A22}\\
= & \frac{r^{3}}{M\left(1+3 \sin ^{2} \lambda\right)} \\
& {\left[-\frac{2}{r} \sin \lambda \frac{\partial P_{\perp}}{\partial \lambda}-\cos \lambda \frac{\partial P_{\perp}}{\partial r}\right.} \\
& \left.+\frac{P_{\|}-P_{\perp}}{r}\left(-\frac{6 \sin ^{2} \lambda \cos \lambda}{1+3 \sin ^{2} \lambda}+3 \cos \lambda\right)\right],  \tag{A23}\\
J_{\lambda}(r, \phi, \lambda)= & \frac{B_{\mathrm{r}}}{B^{2} r \cos \lambda} \frac{\partial P_{\perp}}{\partial \phi} \\
= & -\frac{r^{2} \sin \lambda}{M\left(1+3 \sin ^{2} \lambda\right) \cos \lambda} \frac{\partial P_{\perp}}{\partial \phi}, \tag{A24}
\end{align*}
$$

where $B_{\mathrm{r}}$ is the radial component of the magnetic field, $B_{>}$the latitudinal component of the magnetic field, $r$ the radial distance from the center of the earth and $M$ the magnetic moment of the earth.

By integrating the three-dimensional distribution of the current density, the magnetic disturbance at the geocentric distance of $r$ can be derived from the Biot-Savart law as

$$
\begin{equation*}
\Delta B(\boldsymbol{r})=\frac{\mu_{0} \int J_{\perp}\left(\boldsymbol{r}^{\prime}\right) \times\left(\boldsymbol{r}-\boldsymbol{r}^{\prime}\right)}{4 \pi}\left|\boldsymbol{r}-\boldsymbol{r}^{\prime}\right|^{3} \mathrm{~d}^{3} \boldsymbol{r}^{\prime}, \tag{A25}
\end{equation*}
$$

where $\mu_{0}$ is the permeability in vacuum. Especially, the magnetic disturbance parallel
to the earth's dipole at the center of the earth $\Delta B_{\mathrm{C}}$ is given by

$$
\begin{equation*}
\Delta B_{\mathrm{C}}=\frac{\mu_{0}}{4 \pi}-\iint_{\tau} \int_{\lambda} \int_{\phi} \cos \lambda J_{\phi}(r, \lambda, \phi) \mathrm{d} r \mathrm{~d} \lambda \mathrm{~d} \phi . \tag{A26}
\end{equation*}
$$

## Appendix B: Derivation of the Energy Input Rate into the Ring Current

The total energy $\Gamma$ injected from the near-earth plasma sheet through the 'injection boundary' is defined as

$$
\begin{align*}
\Gamma & \equiv \int \frac{1}{2} m v^{2} F(v) \mathrm{d} x \mathrm{~d} v  \tag{B1}\\
& =\int \frac{1}{2} m v^{2} F(v) \mathrm{d} S v \mathrm{~d} t \cos \alpha v^{2} \mathrm{~d} \Omega \mathrm{~d} v \\
& =2 \pi m \int \mathrm{~d} S \int F(v) v^{4} S_{\mathrm{b}}(y) y \mathrm{~d} y \mathrm{~d} v, \tag{B2}
\end{align*}
$$

where $F(v)$ is the velocity distribution function, $m$ the mass, $S$ the area, $\alpha$ the pitch angle, $y$ the sine of a pitch angle, $\tau_{\mathrm{b}}$ the bounce period and $S_{\mathrm{b}}(y)$ is

$$
\begin{align*}
\tau_{\mathrm{b}} & =\frac{2}{v} \int\left(1-\frac{B_{\mathrm{m}}}{B}\right)^{-1 / 2} \mathrm{~d} s  \tag{B3}\\
& \equiv \frac{2}{v} S_{\mathrm{b}}(y) \tag{B4}
\end{align*}
$$

where $B_{\mathrm{m}}$ and ds are the magnetic field at a mirror point, the line element along with a field line, respectively.

Since the velocity distribution function at the 'injection boundary' is assumed to be isotropic and Maxwellian in this simulation, the velocity distribution function $F$ becomes

$$
\begin{equation*}
F=\frac{N_{\mathrm{Ps}}}{\left(\pi v_{0}^{2}\right)^{3 / 2}} \exp \left(-\frac{v^{2}}{v_{0}^{2}}\right), \tag{B5}
\end{equation*}
$$

where $N_{\mathrm{ps}}$ and $\nu_{0}$ are the number density in the plasma sheet and the most probable velocity, respectively.

In polar coordinates, the area $d S$ in the equatorial plane is given by

$$
\begin{align*}
\mathrm{d} S & =R_{0} \mathrm{~d} \phi \mathrm{~d} R \\
& =R_{0} \mathrm{~d} \phi \mathrm{~d} t \frac{\mathrm{~d} R}{\mathrm{~d} t} \\
& =R_{0} \mathrm{~d} \phi \mathrm{~d} t \frac{E_{\phi}}{B} \tag{B6}
\end{align*}
$$

where $R_{0}, \phi$ and $E_{\phi}$ are the geocentric distance of the 'injection boundary', the local time and the azimuthal component of the electric field, respectively. If the convection field is expressed by the Volland-Stern type, the azimuthal component of the electric field is
easily obtained as

$$
\begin{align*}
E_{\phi} & =-\frac{1}{R} \frac{\partial \Phi}{\partial \phi}  \tag{B7}\\
& =-A R \cos -\phi,  \tag{B8}\\
A & =\frac{\Phi_{\mathrm{PC}}}{2{R_{\mathrm{B}}{ }^{2}}^{2}}, \tag{B9}
\end{align*}
$$

where $\Phi, R_{\mathrm{B}}$ and $\Phi_{\mathrm{PC}}$ are the electric potential, the geocentric distance of the magnetopause and the polar cap potential, respectively. Then the area $\mathrm{d} S$ becomes

$$
\begin{equation*}
\mathrm{d} S=\frac{\Phi_{\mathrm{PC}}}{2 B}\left(\frac{R_{0}}{R_{\mathrm{B}}}\right)^{2} \mathrm{~d} t \cos \phi \mathrm{~d} \phi . \tag{B10}
\end{equation*}
$$

After substituting eq. (B5), eq. (B6) and eq. (B10) into eq. (B2), the total energy $\Gamma$ is given by

$$
\Gamma=\frac{3}{2} N_{\mathrm{ps}} E_{0} \frac{\Phi_{\mathrm{PC}}}{2 B}\left(\frac{R_{0}}{R_{\mathrm{B}}}\right)^{2} \int \mathrm{~d} t \int \cos \phi \mathrm{~d} \phi \int S_{\mathrm{b}} y \mathrm{~d} y,
$$

where $E_{0}$ is the temperature corresponding to $m v_{0}{ }^{2} / 2$. Now, the energy input rate $\gamma$ is introduced as

$$
\begin{align*}
& \gamma=\Gamma / \int \mathrm{d} t \\
&=\frac{3 N_{\mathrm{px}} E_{0}}{4 B} \Phi \Phi_{\mathrm{PC}}  \tag{B11}\\
& Y\left(\frac{R_{0}}{R_{\mathrm{B}}}\right)^{2} Y(R) \int \cos \phi \mathrm{d} \phi(\text { watt }),  \tag{B12}\\
& Y\left(R_{0}\right) \equiv \int S_{\mathrm{b}}(y) y \mathrm{~d} y .
\end{align*}
$$

After substituting $R_{0}$ of $10 \mathrm{Re}, \phi$ of $21 \mathrm{~h}-3 \mathrm{~h}(27 \mathrm{~h})$ and $R_{\mathrm{B}}$ of 10.47 Re , which are used in this simulation, into eq. (B11), the energy input rate $\gamma$ becomes

$$
\begin{equation*}
\gamma=0.572 N_{\mathrm{ps}}\left(\mathrm{~cm}^{-3}\right) E_{0}(\mathrm{keV}) \Phi_{\mathrm{PC}}(\mathrm{kV}) \text { (gigawatt). } \tag{B13}
\end{equation*}
$$

## Appendix $\mathbb{C}$ : Bounce $\mathbb{P e r i o d s ~ a n d ~} \mathbb{D r i f t} \mathbb{P}$ eriods

A bounce period of a magnetically trapped particle in a dipole magnetic field is given by

$$
\begin{equation*}
\tau_{\mathrm{b}}=\frac{2}{v} \int\left(1-\frac{B_{\mathrm{m}}}{B}\right)^{-1 / 2} \mathrm{~d} s \tag{C1}
\end{equation*}
$$



Fig. A 3. Bounce periods of trapped protons (top) and electrons (bottom) as a function of kinetic energy. Six lines indicate the periods at $L=1,2,3$, 4, 6 and 8, respectively.
where $B$ is the magnetic field intensity, $B_{\mathrm{m}}$ the magnetic field intensity at a mirror point, ds the line element along a field line, $v$ the particle's speed. The bounce periods of particles with their pitch angles of $90^{\circ}$ as a function of a kinetic energy and $L$ are shown in Fig. A3.

In a dipole magnetic field, a grad-B drift velocity of a trapped particle in the equatorial plane is given by

$$
\begin{equation*}
V_{\nabla B}=\frac{m v^{2}}{2 q B^{2}}\left|\nabla_{\perp} \boldsymbol{B}\right|, \tag{C2}
\end{equation*}
$$

where $m$ and $q$ are the particle's mass and the charge, respectively. Thus the drift period of a particle with its equatorial pitch angle of $90^{\circ}$ becomes


Fig. A 4. Drift periods of trapped protons (top) and electrons (bottom) as a function of kinetic energy due to the grad-B drift. The $\boldsymbol{E} \times \boldsymbol{B}$ drift is excluded. Six lines indicate the periods at $L=1,2,3,4,6$ and 8, respectively.

$$
\begin{equation*}
\tau_{\nabla B}=\frac{2 \pi r}{V_{\nabla B}}, \tag{C3}
\end{equation*}
$$

where $r$ is the geocentric distance. The drift periods of the particles are shown in Fig. A4.

The bounce periods and the drift periods at $L=3,4,5,6,7$ and 8 are listed in Table A1 with gyroperiods and gyroradii.

Table A 1. Gyroperiod, bounce period, drift period and gyroradius of a trapped particle with a pitch angle of $90^{\circ}$ at $L=3,4,5,6,7$ and 8 for kinetic energies (E) of $1,10,100$ and 1000 keV in the equatorial plane.

| $L=3, \alpha_{0}=90^{\circ}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\underset{(\mathrm{keV})}{\mathrm{E}}$ | gyroperiod <br> (s) | bouncing period (s) | drift period (hour) | gy roradius (km) |
| Protons | 1.0 | $5.713 \times 10^{-2}$ | 129.3 | 244.0 | 3.98 |
|  | 10.0 | $5.713 \times 10^{-2}$ | 40.87 | 24.4 | 12.59 |
|  | 100.0 | $5.714 \times 10^{-2}$ | 12.93 | 2.44 | 39.8 |
|  | 1000.0 | $5.719 \times 10^{-2}$ | 4.091 | 0.2441 | 125.9 |
| Electrons | 1.0 | $3.118 \times 10^{-5}$ | 3.021 | 244.3 | 0.09292 |
|  | 10.0 | $3.172 \times 10^{-5}$ | 0.9678 | 24.64 | 0.2951 |
|  | 100.0 | $3.720 \times 10^{-5}$ | 0.3442 | 2.658 | 0.9731 |
|  | 1000.0 | $9.200 \times 10^{-5}$ | 0.2005 | 0.3647 | 4.131 |
| $L=4, \alpha_{0}=90^{\circ}$ |  |  |  |  |  |
|  | $\begin{gathered} \mathrm{E} \\ (\mathrm{keV}) \end{gathered}$ | gyroperiod <br> (s) | bouncing period <br> (s) | drift period (hour) | gy roradius (km) |
| Protons | 1.0 | 0.1354 | 172.3 | 183.0 | 9.434 |
|  | 10.0 | 0.1354 | 54.50 | 18.3 | 29.83 |
|  | 100.0 | 0.1354 | 17.24 | 1.83 | 94.34 |
|  | 1000.0 | 0.1356 | 5.454 | 0.1831 | 298.4 |
| Electrons | 1.0 | $7.389 \times 10^{-5}$ | 4.028 | 183.2 | 0.2203 |
|  | 10.0 | $7.520 \times 10^{-5}$ | 1.29 | 18.48 | 0.6996 |
|  | 100.0 | $8.819 \times 10^{-5}$ | 0.459 | 1.993 | 2.307 |
|  | 1000.0 | $2.181 \times 10^{-4}$ | 0.2674 | 0.2735 | 9.792 |
| $L=5, \alpha_{0}=90^{\circ}$ |  |  |  |  |  |
|  | $\begin{gathered} \mathrm{E} \\ (\mathrm{keV}) \end{gathered}$ | $\begin{gathered} \text { gyroperiod } \\ \text { (s) } \\ \hline \end{gathered}$ | bounce period (s) | drift period (hour) | $\begin{gathered} \text { gy roradius } \\ (\mathrm{km}) \end{gathered}$ |
| Protons | 1.0 | 0.2645 | 215.4 | 146.4 | 18.42 |
|  | 10.0 | 0.2645 | 68.12 | 14.64 | 58.27 |
|  | 100.0 | 0.2645 | 21.54 | 1.464 | 184.3 |
|  | 1000.0 | 0.2648 | 6.818 | 0.1465 | 582.8 |
| Electrons | 1.0 | $1.443 \times 10^{-4}$ | 5.035 | 146.6 | 0.4302 |
|  | 10.0 | $1.469 \times 10^{-4}$ | 1.613 | 14.78 | 1.366 |
|  | 100.0 | $1.722 \times 10^{-4}$ | 0.5737 | 1.595 | 4.505 |
|  | 1000.0 | $4.259 \times 10^{-4}$ | 0.3342 | 0.2188 | 19.13 |

$L=6, \alpha_{0}=90^{\circ}$

|  | E <br> $(\mathrm{keV})$ | gyroperiod <br> $(\mathrm{s})$ | bounce period <br> $(\mathrm{s})$ | drift period <br> $($ hour $)$ | gy roradius <br> $(\mathrm{km})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Protons | 1.0 | 0.457 | 258.5 | 122.0 | 31.84 |
|  | 10.0 | 0.4571 | 81.75 | 12.2 | 100.7 |
|  | 100.0 | 0.4571 | 25.85 | 1.22 | 318.4 |
|  | 1000.0 | 0.4575 | 8.181 | 0.1221 | 1007.0 |
| Electrons | 1.0 | $2.494 \times 10^{-4}$ | 6.042 | 122.1 | 0.7434 |
|  | 10.0 | $2.538 \times 10^{-4}$ | 1.936 | 12.32 | 2.361 |
|  | 100.0 | $2.976 \times 10^{-4}$ | 0.6885 | 1.329 | 7.785 |
|  | 1000.0 | $7.360 \times 10^{-4}$ | 0.4011 | 0.1823 | 33.05 |

Table A 1. (continued).

| $L=7, \alpha_{0}=90^{\circ}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\underset{(\mathrm{keV})}{\mathrm{E}}$ | gyroperiod <br> (s) | bounce period (s) | drift period (hour) | $\underset{(\mathrm{km})}{\text { gyroradius }}$ |
| Protons | 1.0 | 0.7258 | 301.6 | 104.6 | 50.56 |
|  | 10.0 | 0.7258 | 95.37 | 10.46 | 159.9 |
|  | 100.0 | 0.7258 | 30.16 | 1.046 | 505.6 |
|  | 1000.0 | 0.7265 | 9.545 | 0.1046 | 1599.0 |
| Electrons | 1.0 | $3.959 \times 10^{-4}$ | 7.049 | 104.7 | 1.18 |
|  | 10.0 | $4.030 \times 10^{-4}$ | 2.258 | 10.56 | 3.749 |
|  | 100.0 | $4.726 \times 10^{-4}$ | 0.8032 | 1.139 | 12.36 |
|  | 1000.0 | $1.168 \times 10^{-3}$ | 0.4679 | 0.1563 | 52.48 |
| $L=8, \alpha_{0}=90^{\circ}$ |  |  |  |  |  |
|  | $\begin{gathered} \mathrm{E} \\ (\mathrm{keV}) \end{gathered}$ | gyroperiod <br> (s) | bounce period <br> (s) | drift period (hour) | $\begin{aligned} & \text { gyroradius } \\ & (\mathrm{km}) \end{aligned}$ |
| Protons | 1.0 | 1.083 | 344.7 | 91.5 | 75.47 |
|  | 10.0 | 1.083 | 109.0 | 9.151 | 238.7 |
|  | 100.0 | 1.083 | 34.47 | 0.9151 | 754.7 |
|  | 1000.0 | 1.085 | 10.91 | $9.155 \times 10^{-2}$ | 2387.0 |
| Electrons | 1.0 | $5.911 \times 10^{-4}$ | 8.056 | 91.59 | 1.762 |
|  | 10.0 | $6.016 \times 10^{-4}$ | 2.581 | 9.239 | 5.597 |
|  | 100.0 | $7.055 \times 10^{-4}$ | 0.9179 | 0.9966 | 18.45 |
|  | 1000.0 | $1.745 \times 10^{-3}$ | 0.5347 | 0.1368 | 78.34 |

