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MHD VORTICES IN THE MAGNETOSPHERIC LOW LATITUDE BOUNDARY LAYER

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Abstract: A set of equations for MHD vortices is derived, in which the resistivity, the viscosity, the velocity and magnetic field shears, and the density inhomogeneity in the magnetospheric boundary layer are included self-consistently. When only the density inhomogeneity and magnetic shear are taken into account, this set of equations is reduced to an equation similar to the famous Hasegawa-Mima equation. In this case, a dipole vortex solution is obtained, with its parameters determined by the magnetospheric boundary layer conditions. This shows that a stable irregularity with two density perturbations $w>0$ and $w<0$ distributed closely may exist in the boundary layer. It is called a dipole vortex irregularity here, some characteristics of it are analytically determined.

1. Introduction

In the study of magnetospheric boundary layer, there are mainly two kind of models: the closed magnetosphere model and the magnetic reconnection model (or the open magnetosphere model). In both of them, however, in contrast to the case of MHD waves, the importance of MHD vortices has not been adequately evaluated. To maintain the convection in a closed magnetosphere model, the necessary driven power is evaluated to be about 10^{13} cm^2/s. Before the introduction of Kelvin-Helmholtz instability (PU and KIVELSON, 1983; MIURA, 1984), which may cause vortex structure in the boundary layer; and for which, the anomalous viscosity obtained through numerical simulation is of the same order of magnitude, many researchers have tried anomalous diffusions or anomalous viscosity caused by wave-particle interactions or by many kinds of instabilities. However, neither the anomalous viscosity induced by the wave-particle interaction, nor the anomalous diffusion caused by low hybrid instability induced by density gradient perpendicular to the magnetic field, or caused by kinetic Alfvên waves (HASEGAWA and MIMA, 1978) could appropriately account for it. On the other hand, fluid vortices induced by Kelvin-Helmholtz instability also play an important role in magnetic reconnection. The vortex induced magnetic reconnection mechanism proposed by LIU and HU (1988) has given a very good explanation for some characteristics of FTEs.

In study of vortices, because the equations are nonlinear, they usually cannot be analytically solved, and so far, research has been mainly carried out by numerical simulation. However, because of the obvious appeal of the analytical solution, efforts to find special solutions have continued. In the present paper, first, the MHD equations, which are used to describe the motions in the magnetospheric boundary layer, are reduced to a set of scalar equations appropriate for describing vortices. These equations include self-consistently the resistivity, viscosity, the velocity and magnetic field shears, and the density inhomogeneity in the magnetospheric boundary layer, and can be taken as starting equations in general. Then in an example, in which only the density gradient and magnetic shear are taken into account, this set of equations is reduced to an equation similar to the famous Hasegawa-Mima equation. Lastly, a dipole vortex solution is obtained with its parameters determined here by the boundary layer conditions. Some features of density perturbations are analytically derived.

2. The Derivation of MHD Vortex Equations

The plasma in magnetospheric boundary layer can be described by MHD equations:

$$\frac{\partial \vec{V}}{\partial t}+\vec{V}\cdot\nabla\vec{V}=-\frac{1}{\rho}\nabla p+\frac{1}{C}\frac{1}{\rho}\vec{J}\times\vec{B}+\nu\nabla^2\vec{V}, \tag{1}$$

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho\vec{V})=0, \tag{2}$$

$$\frac{\partial \vec{B}}{\partial t}=-C\nabla\times\vec{E}, \tag{3}$$

$$\nabla\times\vec{B}=\frac{4\pi}{C}\vec{J}, \tag{4}$$

$$\vec{E}+\frac{1}{C}\vec{V}\times\vec{B}=\eta\vec{J}, \tag{5}$$

$$\nabla\cdot\vec{B}=0, \tag{6}$$

where Gauss units are used, and $\vec{E}$, $\vec{B}$, $\vec{V}$, $\vec{J}$, ρ and η have the meanings in common use. Suppose that the plasma is an incompressible fluid:

$$\nabla\cdot\vec{V}=0, \tag{7}$$

and that its state equation is,

$$p=p(\rho). \tag{8}$$

Divide each physical quantity into two parts, that is, the background component and the perturbation component:

$$\vec{B}=\vec{B}_0+\vec{B}' \tag{9}$$

$$\vec{V}=\vec{V}_0+\vec{V}', \tag{10}$$

$$\vec{E}=\vec{E}_0+\vec{E}', \tag{11}$$

$$\rho=\rho_0+\rho'. \tag{12}$$

Suppose that the background components have the following forms:

$$\vec{B}_0=B_0(x)\hat{y}, \tag{13}$$

$$\vec{V}_0=V_0(x)\hat{z}, \tag{14}$$

$$\rho_0=\rho_0(x), \tag{15}$$

$$\vec{E}_0=0, \tag{16}$$

and that perturbation components vary in two dimensions. Therefore, the stream function, Ψ, and magnetic potential, A, may as well be introduced as,

$$\vec{V}'=\hat{y}\times\nabla\Psi, \tag{17}$$

$$\vec{B}'=\nabla A\times\hat{y}. \tag{18}$$

From eq. (3), it is easy to obtain:

$$\vec{E}'=-\frac{1}{C}\frac{\partial A}{\partial t}\hat{y}-\nabla\varphi, \tag{19}$$

where φ is the static electric potential with arbitrary value. Define:

$$[a,b]=\frac{\partial a}{\partial x}\frac{\partial b}{\partial z}-\frac{\partial a}{\partial z}\frac{\partial b}{\partial x}, \tag{20}$$

$$\nu_m=\eta c^2/4\pi, \tag{21}$$

$$\frac{\mathrm{d}f}{\mathrm{d}t}=\frac{\partial f}{\partial t}+[f,\Psi], \tag{22}$$

and take:

$$\rho'/\rho_0=W. \tag{23}$$

After some algebraic calculations and taking the following approximations:

$$\frac{1}{\rho}B_0(x)\frac{\partial\rho'}{\partial z}\approx\frac{1}{\rho_0}B_0(x)\frac{\partial\rho'}{\partial z}=B_0\frac{\partial W}{\partial z}, \tag{24}$$

$$\ln(\rho/\rho_0)=\ln(1+W)\simeq W. \tag{25}$$

The above MHD equations can be rewritten in following forms:

$$\frac{\mathrm{d}A}{\mathrm{d}t}+V_0\frac{\partial A}{\partial z}=\nu_m\nabla^2A, \tag{26}$$

$$\frac{\partial W}{\partial t}+V_0\frac{\partial W}{\partial z}+\frac{\partial}{\partial x}\ln\rho_0(x)\frac{\partial\Psi}{\partial z}=0, \tag{27}$$

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\nabla^2\Psi+V_0\frac{\partial}{\partial z}\nabla^2\Psi-\frac{\partial\Psi}{\partial z}\frac{\partial^2V_0}{\partial x^2}\\&=\frac{1}{4\pi\rho}\left\{\nabla^2A\left([A,W]-2\frac{\partial A}{\partial z}\frac{\partial}{\partial x}\ln\rho_0(x)\right)+B_0\frac{\partial B_0}{\partial x}\frac{\partial W}{\partial z}+[A,\nabla^2A]\right\}\\&+\nu\nabla^4\Psi-\frac{\partial^3V_0}{\partial x^3},\end{aligned} \tag{28}$$

Suppose that the characteristic values of the magnetospheric layer thickness, magnetic field, and characteristic density are l_0, B_{00}, and ρ_{00}. Then $V_{00}=B_{00}^2/4\pi\rho_{00}$, and $t_0=l_0/V_{00}$ are characteristic Alfvén speed and the characteristic transit time of Alfvén waves across the boundary layer. Taking t_0, l_0, B_{00}, ρ_{00}, V_{00}, $V_{00}l_0$ *and* $B_{00}l_0$ respectively as the units of time, space, magnetic field, density, velocity, stream function, and magnetic potential, eqs. (26), (27), and (28) are normalized:

$$\frac{\mathrm{d}A}{\mathrm{d}t}+V_0\frac{\partial A}{\partial z}=\nu_m{}^*\nabla^2A, \tag{29a}$$

$$\frac{\mathrm{d}W}{\mathrm{d}t}+V_0\frac{\partial W}{\partial z}+\frac{\partial}{\partial x}\ln\rho_0(x)\frac{\partial\Psi}{\partial z}=0, \tag{29b}$$

$$\frac{d}{dt}\nabla^2\Psi + V_0\frac{\partial}{\partial z}(\nabla^2\Psi) - \frac{\partial\Psi}{\partial z} - \frac{\partial^2 V_0}{\partial x^2}$$
$$= \frac{1}{\rho_0}\left\{\nabla^2 A\left([A, W] - 2\frac{\partial A}{\partial z}\frac{\partial}{\partial x}\ln\rho_0(x)\right) + B_0\frac{\partial B_0}{\partial x}\frac{\partial W}{\partial z} + [A, \nabla^2 A]\right\} \tag{29c}$$
$$+ \nu^*\left(\nabla^4\Psi - \frac{\partial^3 V_0}{\partial x^3}\right),$$

where $\nu^* = \nu t_0/l_0^2$ and $\nu_m^* = \nu_m t_0/l_0^2$ are normalized viscosity and resistivity. Apparently eqs. (29a–c) include not only the terms of viscosity, velocity shear, and density gradient as in a usual fluid, but also the terms of resistivity and magnetic field shear, which always exist in the magnetospheric boundary layer. So they are appropriate equations for the study of MHD vortices in the magnetospheric low latitude boundary layer.

3. The Vortex Solutions in the Boundary Layer

Equations (29a–c), include the terms of kinetic viscosity, velocity shear, density inhomogeneity, magnetic viscosity and magnetic field shear simultaneously. Here as an example, taking only the density inhomogeneity and magnetic shear into account, that is,

$$V_0 = \text{Constant},\ \rho_0 = \rho_0(x),\ B_0 = B_0(x), \tag{30}$$

and also assuming $\nu^* = 0$ for simplicity, eqs. (29a–c) can be reduced to:

$$\frac{dA}{dt} + V_0\frac{\partial A}{\partial z} = \nu_m{}^*\nabla^2 A, \tag{31a}$$

$$\frac{dW}{dt} + V_0\frac{\partial W}{\partial x} + \frac{\partial}{\partial x}\ln\rho_0(x)\frac{\partial W}{\partial z} = 0, \tag{31b}$$

$$\frac{d}{dt}\nabla^2\Psi + V_0\frac{\partial}{\partial z}\nabla^2\Psi$$
$$= \frac{1}{\rho_0}\left\{\nabla A^2\left([A, W] - 2\frac{\partial A}{\partial z}\frac{\partial}{\partial x}\ln\rho_0(x)\right) + [A, \nabla A^2] + B_0\frac{\partial B_0}{\partial x}\frac{\partial W}{\partial z}\right\}. \tag{31c}$$

This is a drift wave equation when the density gradient and magnetic shear in the magnetospheric boundary layer are taken into account.

When the disturbance is supposed to be electrostatic, that is, $A=0$, then eqs. (31a–c) may be further reduced to:

$$\frac{dW}{dt} + V_0\frac{\partial W}{\partial z} + \frac{\partial}{\partial x}\ln\rho_0(x)\frac{\partial\Psi}{\partial z} = 0, \tag{32a}$$

$$\frac{d}{dt}\nabla^2\Psi + V_0\frac{\partial}{\partial z}\nabla^2\Psi = \frac{B_0}{\rho_0}\frac{\partial B_0}{\partial x}\frac{\partial W}{\partial z}. \tag{32b}$$

In the above equations, $\frac{df}{dt} = \frac{\partial f}{\partial t} + [f, \Psi]$, if we re-define it as,

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + [f, \Psi] + V_0\frac{\partial f}{\partial z}, \tag{33}$$

then it can be rewritten as,

$$\frac{dW}{dt} + \frac{\partial}{\partial x}\ln\rho_0(x)\frac{\partial\Psi}{\partial z} = 0, \tag{34a}$$

$$\frac{d}{dt}\nabla^2\Psi-\frac{B_0}{\rho_0}\frac{\partial B_0}{\partial x}\frac{\partial W}{\partial z}=0. \tag{34b}$$

Here, there is no special limitation put on the relation between W and Ψ, so we may as well take (WU *et al*., 1991),

$$W=\Psi. \tag{35a}$$

And for simplicity, let

$$\frac{B_0(x)}{\rho_0(x)}\frac{\partial B_0}{\partial x}=\frac{\partial}{\partial x}\ln b(x). \tag{35b}$$

Subtracting eq. (34a) from eqs. (34b), (34b)-(34a), easily leads to:

$$\frac{d}{dt}(\nabla^2\Psi-\Psi)-\frac{\partial}{\partial x}\ln[\rho_0(x)b(x)]\frac{\partial\Psi}{\partial z}=0. \tag{36}$$

Because

$$\frac{d}{dt}\ln[\rho_0(x)b(x)]=\frac{\partial}{\partial t}\ln[\rho_0(x)b(x)]+[\ln[\rho_0(x)b(x)],\ \Psi]+V_0\frac{\partial}{\partial z}\ln[\rho_0(x)b(x)]$$
$$=\frac{\partial}{\partial x}\ln[\rho_0(x)b(x)]\frac{\partial\Psi}{\partial z},$$

eq. (36) can be written as:

$$\frac{d}{dt}(\nabla^2\Psi-\Psi-\ln[\rho_0(x)b(x)])=0. \tag{37}$$

When the magnetic shear is ommitted, that is, $b(x)=1$, eq. (37) becomes:

$$\frac{d}{dt}(\nabla^2\Psi-\Psi-\ln\rho_0(x))=0.$$

This is the famous Hasegawa-Mima equation (TANIUTI and HASEGAWA, 1982).

Therefore, eq. (37) is a generalized Hasegawa-Mima equation. Instead of the density gradient term, eq. (37) includes both the density gradient and the magnetic shear contributions in its third term. In the derivation of eq. (37), assumption eq. (35a) has been used, so the second term here in fact comes from W, that is, from the density disturbance, different from that in the original Hasegawa-Mima equation, where it comes from the electron inertia. It should be pointed out here, however, the introduction of eq. (35a) limited our study thereafter to cases in which Ψ and W are on the same order.

To concentrate on vortices, it is necessary to remove the wave term. This can be done as usual by making the following coordinate transformations:

$$x=x,\ \xi=z-u_0t, \tag{38}$$

where u_0 is the traveling wave speed of a static solution. So eq. (37) becomes:

$$[\nabla^2\Psi-\Psi-\ln[\rho_0(x)b(x)],\ \Psi-(V_0-u_0)x]=0. \tag{39}$$

If the density in the boundary layer has a simple form such as:

$$\ln[\rho_0(x)b(x)]=\beta x+C,\ C=\text{Constant}. \tag{40}$$

β and C are determined by $B_0(x)$ and $\rho_0(x)$. Equation (39) easily leads to:

$$\nabla^2\Psi-\Psi-\beta x=f(\Psi-(V_0-u_0)x), \tag{41}$$

where $f(A)$ is an arbitrary function of argument A. Introduce polar coordinates,

$$x = r\cos\theta,\ \xi = r\sin\theta,$$

and suppose that a is the characteristic size of a vortex; then it is obviously satisfied that

$$\nabla^2 \Psi = -k^2(\Psi - (V_0 - u_0)x) + \Psi + \beta x,\ r \le a, \tag{42a}$$

$$\nabla^2 \Psi = l^2(\Psi - (V_0 - u_0)x) + \Psi + \beta x,\ r \ge a. \tag{42b}$$

Using the boundary condition, $\Psi \to 0$, when $r \to \infty$, from eq. (42b), we obtain:

$$l^2 = \beta/(V_0 - u_0). \tag{43}$$

So eq. (42) can be written as:

$$\nabla^2 \Psi = -(t^2/a^2)\Psi - Q[(s^2 + t^2)/a^2]x,\ r \le a, \tag{44a}$$

$$\nabla^2 \Psi = (s^2/a^2)\Psi,\ r \ge a, \tag{44b}$$

where

$$s^2/a^2 = l^2 + 1, \tag{45a}$$

$$t^2/a^2 = k^2 - 1, \tag{45b}$$

$$Q = -1/(V_0 - u_0). \tag{45c}$$

For eqs. (45a–c), by use of its conditions of continuity of Ψ and $\frac{\partial \Psi}{\partial r}$ at $r = a$, many authors (PAVLENKO and WEILAND, 1980; TANIUTI and HASEGAWA, 1982) have described the localized solution:

$$\Psi(r,\theta) = \begin{cases} AK_1(sr/a)\cos\theta,\ r \ge a, \\ B(r/a)\cos\theta + CJ_1(tr/a)\cos\theta,\ r \le a, \end{cases} \tag{46}$$

where $r = (\xi^2 + x^2)^{\frac{1}{2}}$, $\theta = arctg(\xi/x)$, J_1 and K_1 are the first order Bessel function and the first order second kind modified Bessel function respectively, and

$$A = -aQ/K_1(s),\ B = -aQ[1 + s^2/t^2],\ C = aQ[(s^2/t^2)J_1(t)]. \tag{47}$$

In addition, constants s and t satisfy as well the following equation:

$$K_2(s)/[sK_1(s)] = -J_2(t)/[tJ_1(t)]. \tag{48}$$

Equation (46) with eqs. (47) and (48) describes a typical dipole vortex. Its five parameters a, u, Q, s, and t represent the size, speed, and intensity of a dipole vortex, and the varying rates inside and outside the vortex respectively. They are controlled by the four eqs. (45a–c) and (48), so one of them can take an arbitrary value. Therefore, eqs. (46) and (47) have a series of vortex solutions.

Now let us discuss the features of such dipole vortices. To demonstrate them in a straightforward way, numerical calculations are also carried out to accompany the analysis.

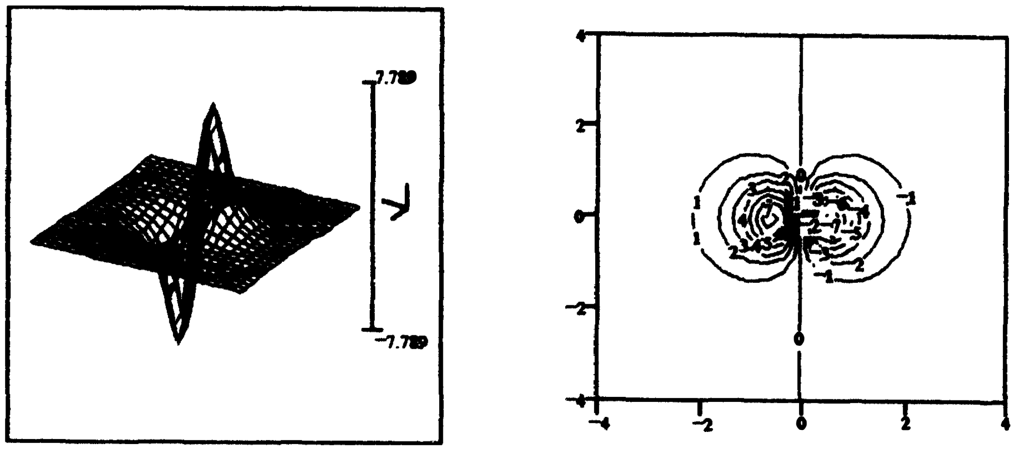
1) The intensity Q

From eq. (45c), $|Q| = 1/|u_0 - V_0|$, that is, the greater the difference between flow speed and vortex traveling speed (or wave traveling speed) is, the weaker the vortex appears. When $\beta = -0.01$, $u_0 - V_0$ varies from 0.2 to 5.2; it is found that the shape of vortex varies very little and the maximum of Q varies from 7.789 to 0.298 (see Fig. 1).

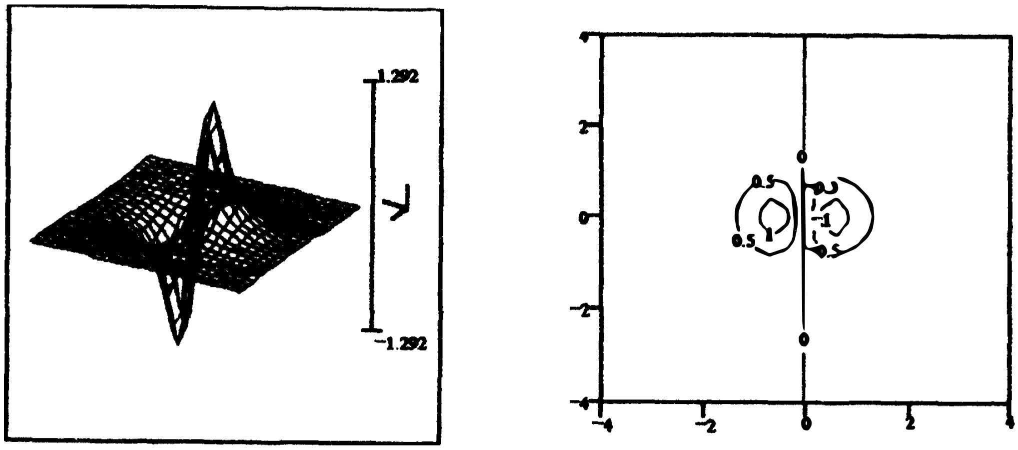
2) Parameter t

The parameter t determines the varying rate inside a vortex. From eq. (48), for any

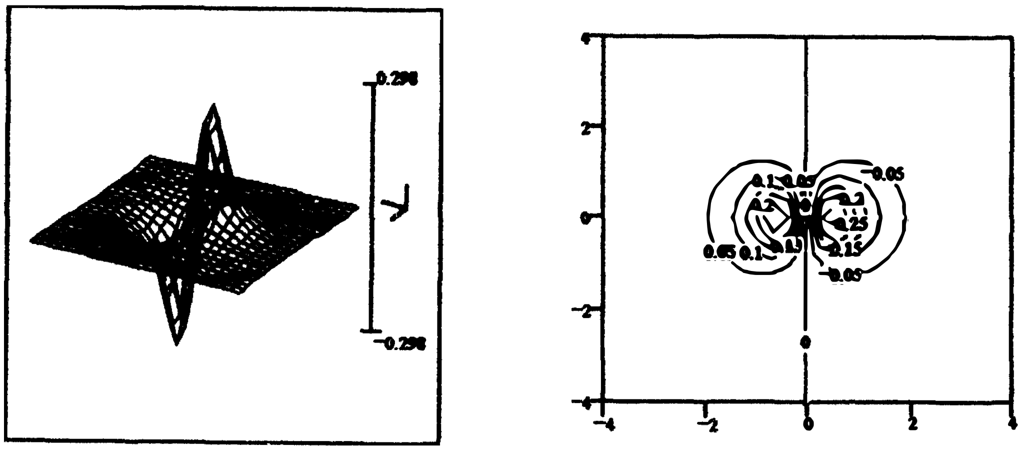
(A) (B)



(a) $u_0 - V_0 = 0.2$



(b) $u_0 - V_0 = 1.2$



(c) $u_0 - V_0 = 5.2$

Fig. 1. *Surface (A) and contour (B) plots of density disturbance varying with (a)* $u_0 - V_0 = 0.2$*, (b)* $u_0 - V_0 = 1.2$*, (c)* $u_0 - V_0 = 5.2$ *when* $\beta = -0.01$*,* $t = 3.923$*.*

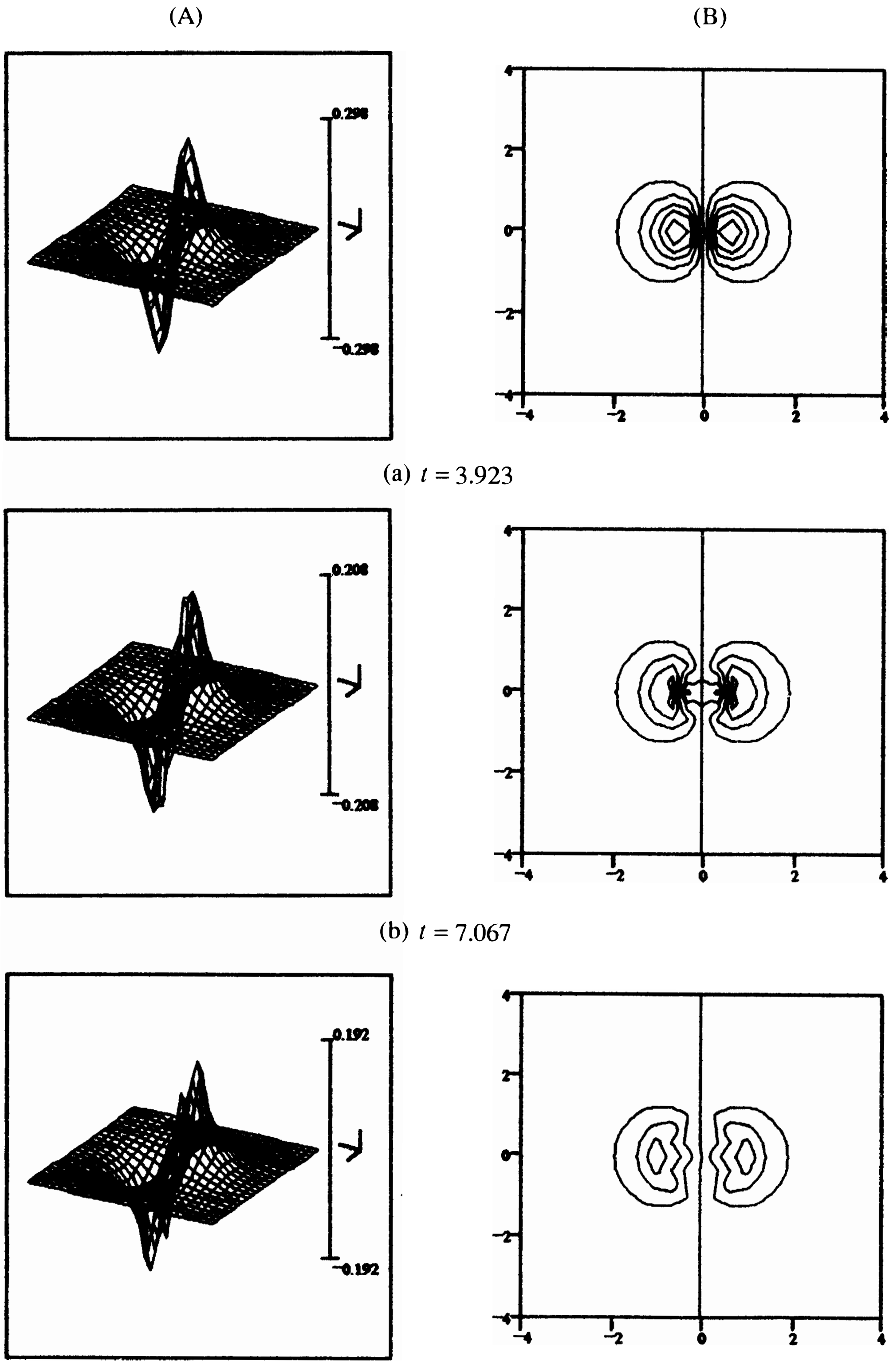


Fig. 2. *Surface (A) and contour (B) plots of density disturbance varying with (a) $t = 3.923$, (b) $t = 7.067$, (c) $t = 19.635$ when $\beta = -0.02$, $u_0 - V_0 = 5.2$.*

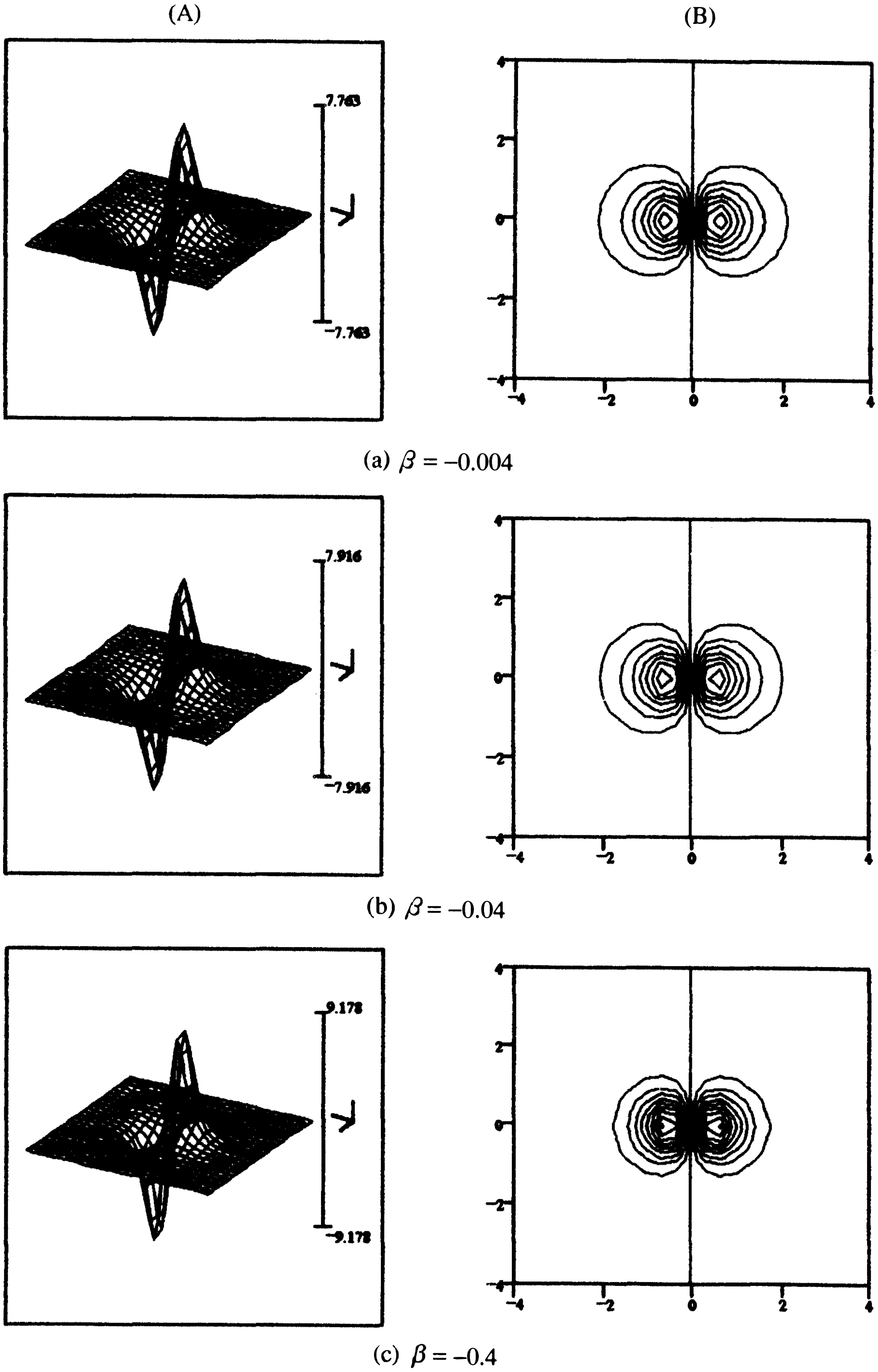


Fig. 3. *Surface (A) and contour (B) plots of density disturbance varying with (a) $\beta = -0.004$, (b) $\beta = -0.04$, (c) $\beta = -0.4$ when $u_0 - V_0 = 0.2$, $t = 4.033$.*

given s, there are innumerable values of t which satisfy the conditions. When taking $\beta = -0.02$, $u_0 - V_0 = 5.2$; the first, second and sixth roots of t in eq. (48) are used for calculations; the results in Fig. 2 show that the greater the value of t, the more complicated the structure inside the vortex. This means that, even with the same boundary layer conditions, there may simultaneously exist many vortices with very different inner structure.

3) Parameter s

This measures the variation rate outside a vortex. From eq. (43), parameter s is determined by β and $(u_0 - V_0)$, that is, determined by boundary layer state conditions. The greater s is, the faster the disturbance decays outside a vortex. Numerical calculations in Fig. 3 demonstrate such a trend, and effects of parameter s to the intensity and inner structure of a vortex.

Therefore, there may exist vortex structures (dipole vortex irregulatrities) caused by the density gradient and magnetic shear in the magnetospheric low latitude boundary layer, when the relation between $\rho_0(x)$ and $B_0(x)$ satisfies eq. (40).

4. Conclusions

Under the assumptions $\vec{B}_0 = B_0(x)\hat{y}$, $\vec{V}_0 = V_0(x)\hat{z}$, $\rho_0 = \rho_0(x)$, and $\vec{E}_0 = 0$, a vortex equation set, (29), was derived from the MHD equations by using the perturbation method. They can be used as starting equations of small scale vortices in the magnetospheric low latitude boundary layer.

It was proved that when only the density gradient and magnetic shear are taken into account, that is

$$\rho_0 = \rho_0(x), \ B_0 = (x),$$
$$V_0 = \text{Constant}, \ \gamma^* = 0,$$

and when destiny and stream disturbances (W, Ψ) are assumed as eq. (35a), its electrostatic vortices can be described by a generalized Hasegawa-Mima eq. (37). Following the standard procedure, a dipole vortex solution was given to the generalized Hasegawa-Mima eq. (37). It shows that when above conditions are satisfied, solitaty vortices may exist in the magnetospheric low latitude boundary layer.

It should be pointed out here, however, that velocity shear always exists and is an important factor causing vortices in the magnetospheric boundary layer. It is expected that velocity shears modulate above dipole vortices. The modulation depth should be made clear in the future by numerical calculations.

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