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ESTIMATION OF TOTAL ELECTRON CONTENT USING VERY LONG BASELINE INTERFEROMETER

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Abstract: Both X (8GHz) and S (2GHz) band delays between two stations out of two or more stations are measured in very long baseline interferometer (VLBI) observations for geodetic use in order to calibrate excess delays depending on the ionospheric total electron content (TEC). Using a number of observed delay differences between X and S bands, TEC at each station can be estimated by a least squares method. TEC estimated from VLBI data was compared with that obtained by the Faraday rotation measurements using a geostationary satellite beacon, and a satisfactory agreement was obtained.

1. Introduction

A very long baseline interferometry (VLBI) technique has been applied to various fields, such as radio astrometry (*e.g.*, SCHILIZZI, 1986), earth rotation monitoring (*e.g.*, DICKEY and EUBANKS, 1985), time synchronization (*e.g.*, HAMA *et al.* 1989), and geodesy (*e.g.*, CLARK *et al.*, 1985). Among these applications it is essential for geodesy to calibrate the propagation medium (*i.e.*, the atmosphere and ionosphere) delays. The atmospheric delays are corrected by means of a posteriori parameter fitting of model atmosphere (MA, 1978). On the other hand the ionospheric delays proportional to the inverse square of the wave frequency used are directly corrected through receiving two frequency bands, S-band (2GHz) and X-band (8 GHz).

In the VLBI observation, the difference between the arrival time of a radio signal at one end of the interferometer and its arrival at the other end is measured by means of a group delay technique and is referred to as an "observed delay". The difference between the observed delays measured at the S and the X bands, which is referred to as a "differential delay" here, is employed to calibrate the excess delays caused by the charged particles existing along the ray paths from a radio source to each antenna. This differential delay arises mainly from the difference in the earth's ionospheric total electron content (TEC) along each ray path to antenna; a differential delay due to the differential ray path between S and X bands is negligible and that due to interplanetary and interstellar plasma is also negligible. Up to the present time the differential delays have been used merely for the calibration of observed delay in the analyses. However as described above, the information about TEC at each station is included in the differential delays. We have developed an estimation method for TEC from the differential delays.

The purpose of this paper is to describe a TEC estimation method from VLBI data and is also to demonstrate some results.

2. Ionospheric Excess Delay Calibration in VLBI Experiment

Radio signals propagated through the magnetoionic media suffer excess delays depending on the frequency. Therefore the observed delay at a frequency f is expressed as

$$\tau_{\mathrm{obs}}(f)=\tau_g+\tau_{\mathrm{ion1}}(f)-\tau_{\mathrm{ion2}}(f) \qquad (1)$$

where τ_g is a geometrical delay which is an important parameter for VLBI, and τ_{ion1} and τ_{ion2} are ionospheric excess delays at station 1 and 2, respectively (see Fig. 1). To obtain τ_g, $\tau_{\mathrm{ion1}}(f)-\tau_{\mathrm{ion2}}(f)$ must be subtracted from τ_{obs}.

In the VLBI experiments for a geodetic purpose, receiving frequencies at each antenna are 8 GHz (X band) and 2 GHz (S band), and they are sufficiently higher than the plasma frequency and the cyclotron frequency in the earth's ionosphere, which are of the order of 10 MHz and 1 MHz, respectively. In this case, using the refractive index of the medium with a quasilongitudinal approximation (RATCLIFFE, 1959), the excess group delay in the ionosphere can be expressed as

$$\tau_{\mathrm{ion}}(f)=1.34\times10^{-7}N_S f^{-2} \quad \mathrm{(s)} \qquad (2)$$

where N_S denotes TEC per unit area along the integrated line of sight (electron/m^2). We can, therefore, get the differential ionospheric excess delay at X band, $\tau_{\mathrm{diff}}(f_X)$, by combining eqs. (1) and (2) as follows,

$$\tau_{\mathrm{diff}}(f_X)=\tau_{\mathrm{ion1}}(f_X)-\tau_{\mathrm{ion2}}(f_X)$$

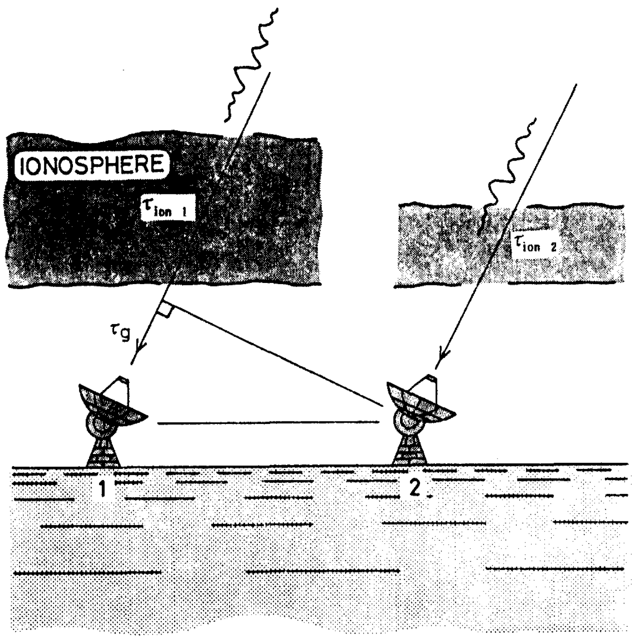

Fig. 1. Delay observed in VLBI experiment. Radio waves arriving at stations 1 and 2 suffer ionospheric excess delays τ_{ion1} and τ_{ion2}. τ_g is a geometrical delay.

$$=(\tau_{obs}(f_X)-\tau_{obs}(f_S))f_S^2/(f_S^2-f_X^2) \tag{3}$$

where f_X, f_S, $\tau_{obs}(f_X)$, and $\tau_{obs}(f_S)$ are X band frequency, S band frequency, observed delay at X band, and that at S band, respectively. In the VLBI database $\tau_{obs}(f_X)$ and $\tau_{diff}(f_X)$ are stored with other observables such as correlation amplitude, fringe phase, *etc.*, for a further analysis. For a geodetic analysis, a geometrical delay calculated by

$$\tau_g=\tau_{obs}(f_X)-\tau_{diff}(f_X) \tag{4}$$

is employed.

Here $\tau_{diff}(f_X)$ is used for estimating TEC. From eqs. (2) and (3), a difference between TEC at the station 1 and that at the station 2, ΔN_t, can be expressed as

$$\begin{aligned}\Delta N_t&=N_{t1}-N_{t2}\\&=7.46\times 10^6 f_X^2\tau_{diff}(f_X)\end{aligned} \tag{5}$$

where N_{t1} and N_{t2} are TECs along the wave paths to two stations. Thus TEC directly obtained by VLBI observation is not that at each station but a differential TEC between two stations. In other words, it is impossible to get TEC at each station independently from one VLBI observation. The geodetic VLBI experiment usually lasts for 24 h or more with changing sources one after another, where each observation length requires several hundred seconds depending on the expected correlated flux of the sources. Hence total number of observations during one experiment exceeds one hundred (typically about 150). In the present study TEC at each station is derived from a number of τ_{diff} observed in an experiment by means of the least squares estimation.

3. Least Squares Estimation of TEC

3.1. Linearized least squares estimation

In the least squares estimation N observable values are modeled as

$$\boldsymbol{Y}=\boldsymbol{F}(\boldsymbol{X})+\boldsymbol{e} \tag{6}$$

where $\boldsymbol{Y}$, $\boldsymbol{X}$, $\boldsymbol{F}$, and $\boldsymbol{e}$ denote a measured observable vector (dimension N), a parameter vector (dimension $M<N$), a mathematical model for the effect of $\boldsymbol{X}$ on $\boldsymbol{Y}$, and an observation error vector (dimension N), respectively. $\boldsymbol{F}(\boldsymbol{X})$ is linearized by expanding as follows,

$$\boldsymbol{F}(\boldsymbol{X})=\boldsymbol{F}(\boldsymbol{X}_0)+A\boldsymbol{X} \tag{7}$$

where $\boldsymbol{X}_0$ is a nominal parameter value vector (dimension M) which is assumed to be close to the true value vector and A is a partial derivative (Jacobian) matrix (size $N\times M$), where an element of A is defined as

$$A_{nm}=\partial \boldsymbol{F}_n(\boldsymbol{X})/\partial \boldsymbol{X}_m \quad (n=1\sim N,\ m=1\sim M)\ . \tag{8}$$

Then the observed values minus calculated values, $\Delta\boldsymbol{Y}$ (dimension N), are expressed by the N linearized equations as

$$\begin{aligned}\Delta\boldsymbol{Y}&=\boldsymbol{Y}-\boldsymbol{F}(\boldsymbol{X}_0)\\&=A\boldsymbol{X}+\boldsymbol{e}\end{aligned} \tag{9}$$

The least squares solution $\hat{X}$ which minimizes the mean square observation error is computed by

$$\hat{X}=(\tilde{A}A)^{-1}\tilde{A}\Delta Y \tag{10}$$

where $\tilde{A}$ is a transposed matrix of A.

3.2. *TEC estimation*

In the case of TEC estimation used here an observable is $\tau_{\mathrm{diff}}(f_X)$, and this differential ionospheric excess delay at X band is to be expressed by a mathematical function in which TEC at each station is included. For TEC variation with time, periodic variations are supposed. An example of frequency spectrum of actual TEC variations observed at Kokubunji (35.7°N, 139.5°E), Tokyo by means of Faraday rotation measurement is shown in Fig. 2. It is clearly demonstrated in the figure that frequency with a period of one day and its harmonics up to the 4th order (*i.e.*, 6 h period) are dominant components of the spectrum. Hence we use a following mathematical model to describe the variation of TEC at a zenith at station i,

$$N_{ti}(t)=a_{i0}+\sum_{k=1}^{4}\left(a_{ik}\cos\left(\frac{kt\pi}{12}\right)+b_{ik}\sin\left(\frac{kt\pi}{12}\right)\right) \tag{11}$$

where t is UT hour. Therefore TEC variation at a station is here modeled by nine parameters (a_{i0}, a_{i1}, $\cdots$, a_{i4}, and b_{i1}, b_{i2}, $\cdots$, b_{i4}). A τ_{diff} observed by VLBI is the differential delay along the line of sight at each station, so that using a function $S(E_i)$, which describes the elevation angle dependence of the path length in the ionosphere at station i, the ionospheric excess delay at X band, $\tau_{\mathrm{ion}\,i}$, can be modeled as

$$\tau_{\mathrm{ion}\,i}(t)=1.34\times 10^{-7} f_X^{-2} N_{ti}(t) S(E_i) \tag{12}$$

The function $S(E_i)$ is given as

$$S(E_i)=1/\cos\{\sin^{-1}[R\cos E_i/(R+h)]\} \tag{13}$$

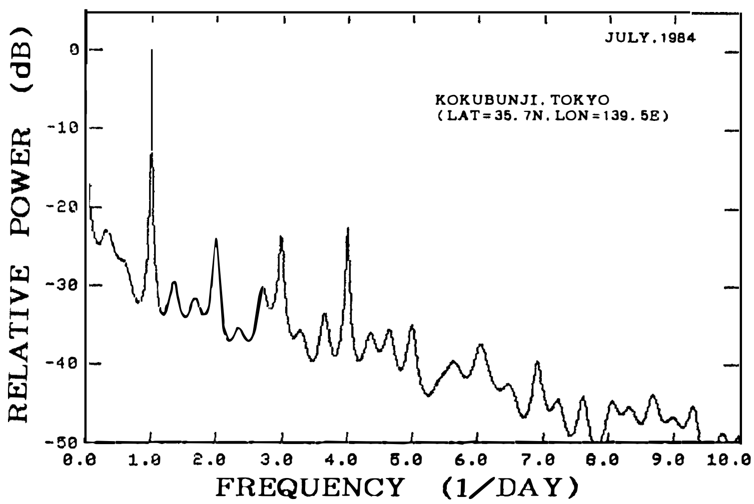

Fig. 2. Frequency spectrum of TEC variation observed in July 1984 at Kokubunji, Tokyo. Note that one day component and its harmonics up to the 4th order are dominant components.

where E_i is a source elevation angle at station i, R is the earth's radius (6371.2 km), and h is a mean altitude of the ionosphere. In eq. (13) a uniform thickness of the ionosphere over the station concerned is assumed and h is taken to be 300 km. Consequently the difference of X band excess delays between station 1 and 2, τ_{model}, which corresponds to the observable $\tau_{\mathrm{diff}}(f_X)$, can be modeled as follows,

$$\tau_{\mathrm{model}}(t)=\tau_{\mathrm{ion1}}(t)-\tau_{\mathrm{ion2}}(t)+\tau_{\mathrm{offset1}}-\tau_{\mathrm{offset2}} \tag{14}$$

where τ_{offset1} and τ_{offset2} denote the instrumental delay offsets of station 1 and 2 and are introduced into the model so as to reflect an actual observation condition; they are also estimated from the data. After all, ten parameters ($a_{i0}, a_{i1}, \cdots, a_{i4}, b_{i1}, b_{i2}, \cdots b_{i4}$, and $\tau_{\mathrm{offset}\,i}$) for a station can be determined by the estimation. For an example, in case of one baseline experiment total 20 parameters should be adjusted because two stations attend the experiment: however in case of the estimation using one (single) baseline data, it is impossible to get the instrumental delay offset at each station independently because of a lack of the independence between these instrumental delay offset parameters.

The partial derivatives of τ_{model} with respect to parameters, which are necessary for computing the least squares solution, are given by

$$\left.\begin{aligned} &\frac{\partial\tau_{\mathrm{model}}}{\partial\tau_{\mathrm{offset}}}=\pm 1 \\ &\frac{\partial\tau_{\mathrm{model}}}{\partial a_0}=\pm D \\ &\frac{\partial\tau_{\mathrm{model}}}{\partial a_k}=\pm D\cos\left(\frac{kt\pi}{12}\right) \quad (k=1,2,3,4) \\ &\frac{\partial\tau_{\mathrm{model}}}{\partial b_k}=\pm D\sin\left(\frac{kt\pi}{12}\right) \quad (k=1,2,3,4) \end{aligned}\right\} \tag{15}$$

with

$$D=1.34\times 10^{-7} f_X^{-2} S(E_i)$$

where positive and negative signs are taken for remote and reference stations, respectively. Then we can get the least squares solution of each parameter by computing eq. (10). Substituting obtained parameters into eq. (11), TEC at time t can be calculated.

The most simple estimation of TEC is to use single baseline data. In this case, a size of Jacobian matrix is $N\times 20$ where N is a number of observations, and TECs at two stations, which are both ends of baseline, can be obtained simultaneously by means of the least squares estimation. By using other baseline data we can obtain TECs at other stations. As described previously, as long as single baseline data are used for the estimation τ_{offset} of at least one station must be fixed.

It is of course possible to compute the least squares solution of multi baseline data at a time. In this case the number of observable values becomes $N\times L$ where L is the number of baselines used, and the number of parameters becomes $10\times I$ where I is the number of stations relevant to these baselines. So that a partial derivative matrix consists of $N\times L\times 10\times I$ elements. Each partial derivative relating to a baseline of which τ_{diff} is an observable is calculated by eq. (15). Other partial derivatives not relevant to the baseline are set to zero.

4. Results

The data obtained by VLBI experiment conducted on July 29, 1984 have been used for an estimation of TEC. Five stations participated in this experiment (Fig. 3), so that there are ten baselines included in the station configuration. TEC of each station was estimated by using various combination of baselines. Figure 4 represents the results obtained for Kashima (35.9°N, 140.7°E). In the figure four thin broken lines (S1–S4) and four solid lines (M1–M4) indicate TECs estimated from single baseline data and those from multi baseline data, respectively, where single baseline means that only one baseline VLBI data are used for the estimation, while multi baseline estimation uses the data obtained from baselines greater than or equal to two. In Fig. 4 TEC obtained from the Faraday rotation measurements of geostationary satellite beacon at Kokubunji (about 100 km west from Kashima), TEC_F, is also shown by a thick broken line for the sake of comparison. We can see a rough agreement between them; *i.e.*, they take their minima around 18 UT and two maxima around 7 UT and 23 UT. TECs derived from multi baseline data are much closer to TEC_F. They coincide with each other within an error of less than 5×10^{16} electron/m^2. A systematic discrepancy however remains between them; TECs estimated from VLBI data are lower than TEC_F at 11–20 UT.

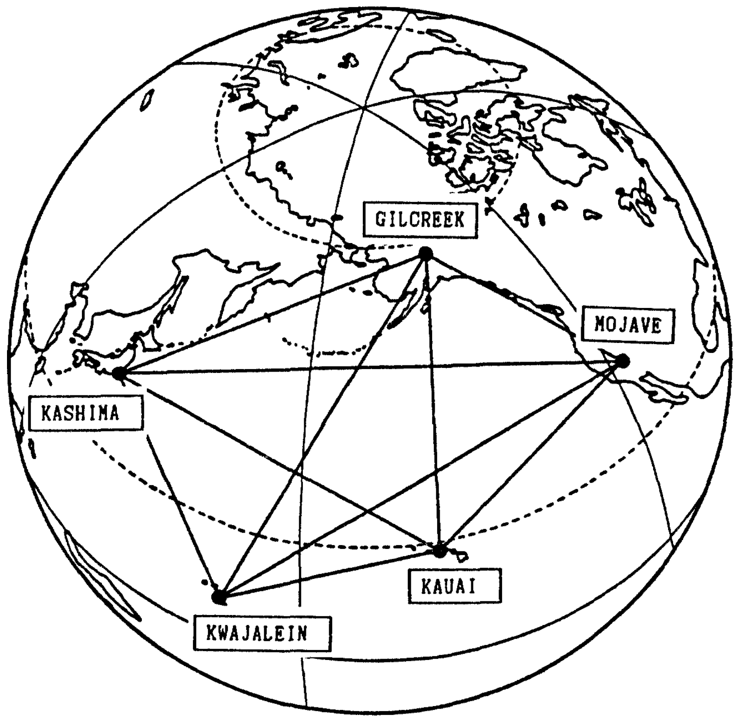

Fig. 3. Station configuration of VLBI experiment conducted on July 29, 1984.

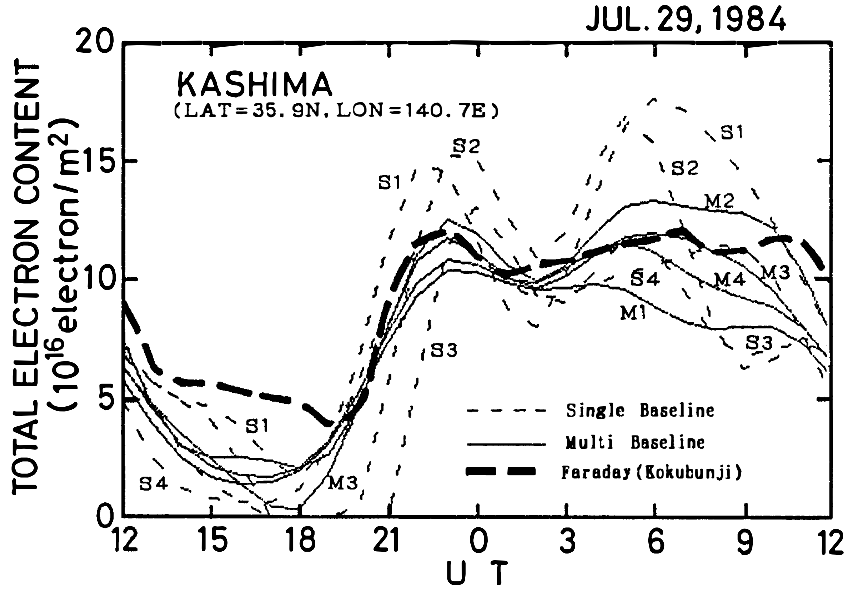

Fig. 4. *TEC at Kashima estimated from VLBI data and that observed at Kokubunji, Tokyo by means of the Faraday measurement of the geostationary satellite beacon. Four thin broken lines represent TEC estimated from single baseline data; S1, S2, S3, and S4 denote Kahima (KAS)-Mojave (MOJ), KAS-Kwajalein (KWA), KAS-Kauai (KAU), and KAS-Gilcreek (GIL) baselines, respectively. Four solid lines represent TEC from multi baseline data; M1 is estimated from three baseline data (KAS-KAU, KAS-GIL, and KAU-GIL), M2 is also from three baseline data (KAS-GIL, KAS-MOJ, and MOJ-GIL), M3 is from four baseline data (KAS-GIL, KAS-KAU, KAS-MOJ, and KAS-KWA), and M4 is from six baseline data (KAS-KAU, KAS-KWA, KAS-MOJ, KAU-KWA, KAU-MOJ, and KWA-MOJ). A thick broken line means TEC obtained by the Faraday measurement.*

5. Discussion

It can be concluded from the results described above that the TEC estimation method presented here gives the following reasonable results; *i.e.*, TEC estimated from the VLBI data is roughly consistent with TEC obtained by the conventional method such as the Faraday rotation measurement of the satellite beacon. There are, however, some discrepancies between them as shown in Fig. 4. Some possibilities have been considered to explain the discrepancies.

First the wave path in the ionosphere is usually different between the VLBI and the Faraday measurements. Radio waves from various directions are observed at a station in the VLBI experiment, while the direction is fixed in the Faraday measurement because the beacon is radiated from the geostationary satellite. Figure 5 shows the geographic locations of subionospheric points of the ray paths from the wave sources to Kashima during the VLBI experiment conducted on July 29, 1984, where subionospheric point is defined as the point at which the path from the source to the observation station intersects an ionospheric altitude of 300 km. Each point with capital letters "A", "B", $\cdots$, and "K" corresponds to each observation in the VLBI experiment. Their mean coordinates, which are indicated by a symbol "$*$", are 36.6°N and 143.5°E. A subionospheric point of the geostationary satellite ETS-II for the Faraday rotation measurement at Kokubunji, Tokyo, is also shown in the figure by a

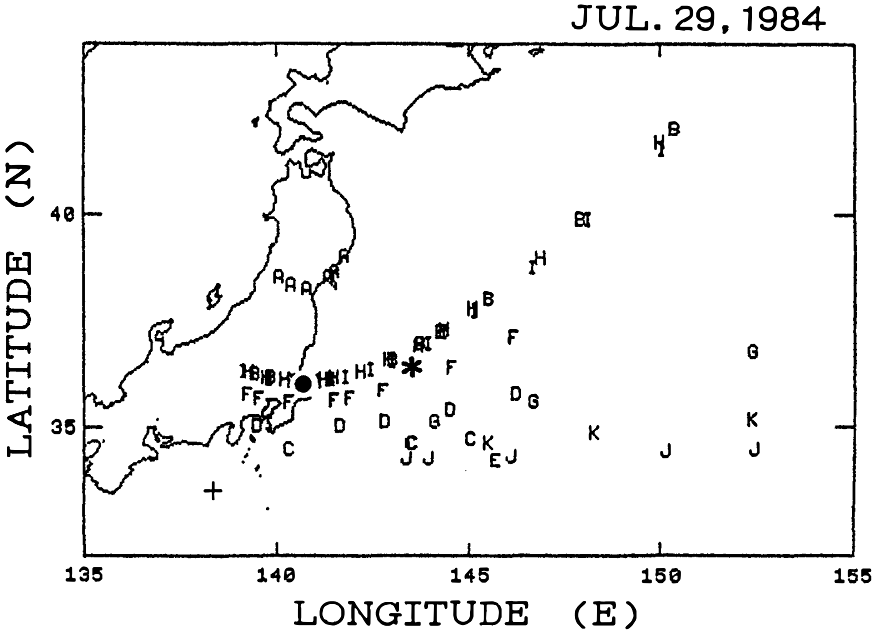

Fig. 5. Subionospheric points of radio sources observed from Kashima at VLBI experiment conducted on July 29, 1984. A mean ionospheric altitude of 300 km is assumed. Capital letter symbols "A", "B", ..., and "L" correspond to each radio source. An averaged position over all observations is displayed by "". The position of Kashima is shown by "●". The symbol "+" denotes the subionospheric point of the ray path from the geostationary satellite, ETS-II to Kokubunji, Tokyo.*

symbol "+" and its coordinates are 33.5°N and 138.4°E. As shown in Fig. 5 TECs obtained by VLBI data are averaged over the region whose latitudinal and longitudinal extents are about 6° (700 km) and 12° (1100 km), respectively. In other words, horizontal structure less than 700–1100 km of ionospheric plasma distribution is smeared out. On the other hand TEC_F reflects the change of the fixed ionospheric position. Therefore it is supposed that one reason of the discrepancies seen in Fig. 4 is due to the difference of ionospheric areas observed by the two techniques.

Next, a time resolution of TEC obtained by VLBI data is lower than that obtained by Faraday measurement. In the present estimation, fluctuations with period shorter than 6 h are not estimated from the VLBI data. On the other hand the time resolution of TEC_F displayed in the figure is 15 min. This means that TEC obtained by VLBI is averaged not only over the spatial extent but also in a time domain. To express a rapid increase around 20 UT seen in TEC derived from the Faraday measurement (see Fig. 4) by the Fourier series, which is used as the model in this study, it should be required to use higher order terms that have a period shorter than 6 h. This might reduce the discrepancy seen at 12–20 UT. But it is difficult to append more parameters to the current TEC variation model, because this will cause degradation of independence among parameters and will increase a difficulty to obtain a unique solution in the estimation.

As described above, TEC estimated from VLBI data is averaged over spatial extent of several hundred kilometers, and fluctuations with the period shorter than 6 h are smeared out. These are thought to be main causes of the discrepancies between TEC obtained by VLBI and TEC_F.

6. Conclusion

We have described the estimation method of ionospheric total electron content from the VLBI data, and have demonstrated the results. By comparing the TEC obtained from VLBI data and that derived by the conventional TEC measurement which observe the Faraday rotation of geostationary satellite beacon, it is confirmed that the TEC estimation method presented here gives reasonable results, particularly when the multi baseline data are used for the estimation. It is however true that there are some limitations due to the VLBI experiment itself and the TEC variation model; *i.e.*, TEC obtained from VLBI data is that averaged over spatial extent of several hundred kilometers and its time resolution is at most 6 h. From these reasons it is not suitable for research work that require both high spatial resolution and high time resolution, but we can obtain the TEC of all stations participating in the VLBI experiment simultaneously with the same accuracy for every station. Therefore TEC obtained from VLBI data is thought to be applicable to the investigation of the global ionosphere.

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