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ON THE STATISTICAL PROPERTIES OF MAGNETOSPHERIC ELF/VLF HISS

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Abstract: In order to have a clue to the study of a link between the hiss and chorus, including the mechanism of the hiss-triggered chorus, statistical properties of magnetospheric VLF/ELF hiss including the stationarity, ergodicity, normality and monochromaticity (coherence or incoherence), have been investigated on the basis of the VLF/ELF hiss data observed at Moshiri in Japan and at Brorfelde in Denmark. It is then found that magnetospheric VLF/ELF hiss is generally considered as a Gaussian, stationary random signal. However, a difference from the white noise generated by a random noise generator is found such that there are some kinds of structures such as coherent (monochromatic) large amplitude spectral components (wavelets) existing within the hiss band.

1. Introduction

Magnetospheric VLF/ELF emissions can be classified into two main types; (1) unstructured hiss (HELLIWELL, 1965; HAYAKAWA *et al.*, 1986) and (2) structured discrete emissions including chorus and riser (HELLIWELL, 1965; TSURUTANI and SMITH, 1974; BURTIS and HELLIWELL, 1976; HAYAKAWA *et al.*, 1984, 1989) on the basis of emission structures. But, it is quite uncertain whether the two types are essentially different or not, and the link between hiss and chorus is very poorly understood. The ground and satellite VLF measurements have implied that the chorus is often accompanied by a background of the hiss, and, in other words, the chorus seems to be triggered from the hiss band (HELLIWELL, 1965; BURTIS and HELLIWELL, 1976; CORNILLEAU-WEHRLIN *et al.*, 1978; KOONS, 1981). However, there appears to be a question whether or not a coherent signal like chorus can be triggered from a hiss commonly considered as an incoherent noise. In order to obtain a clue to these problems, HELLIWELL *et al.* (1986) have tried a simultation experiment for hiss and the associated chorus triggering by means of the Siple transmission experiment. The detailed information on the statistical properties of magnetospheric VLF/ELF hiss is highly required in order to study the above-mentioned unsolved problems, but no study on this subject has been done so far.

The purpose of the present paper is to elucidate the statistical properties of mag-

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netospheric ELF/VLF hiss such as the stationarity, ergodicity, normality and monochromaticity (coherence or incoherence), based on the computer analyses of time series data of hiss observed. We have used two different kinds of VLF/ELF hiss data observed at Moshiri (geomagnetic latitude=35°; L=1.6) (HAYAKAWA *et al.*, 1975) on a particular day and at Brorfelde in Denmark (55°; L=3.0) (HAYAKAWA *et al.*, 1986) on a different particular day.

2. Data Base

VLF/ELF hiss events were adopted for the study from the two ground stations: (1) Moshiri in Japan (L=1.6) and (2) Brorfelde in Denmark (L=3). The most intense ELF hiss at Moshiri was recorded at 0850–0852 UT on 12th September, 1968 during this hiss event. VLF hiss emissions were observed at Brorfelde for several hours if happened and we took the two minutes data of VLF hiss starting at 2250 and 2320 LT (=UT+1 h) on 7th January, 1979. Sonagrams of ELF hiss at Moshiri and at Brorfelde are presented in Fig. 1. The hiss energy is concentrated to the frequency band from 1.0 to ~2.5 kHz at Moshiri as seen in Figs. 1(a) and 1(b). On the other hand, we find a rather wide-banded VLF hiss centered at frequency around 4 kHz at Brorfelde as in Fig. 1(c). For the sake of comparison with these data of VLF/ELF hiss emissions, we analyzed also a Gaussian white noise passed through a band-pass-filter in a frequency band from 2 to 4 kHz. These wide-band data are sampled at a frequency of 20 kHz by an A/D converter (12 bits), and those digitized data are used for the subsequent detailed computer analyses.

3. Statistical Properties of Magnetospheric ELF/VLF Hiss

3.1. Stationarity and ergodicity

As described before, the routine observations of VLF wide-band signals at Moshiri and Brorfelde were made during two minutes (about 120 s) every hour, and so we analyze the period of about two minutes when VLF/ELF emissions or hiss were most intense. The data of VLF emissions observed at Brorfelde, included an unwanted and rather strong signal at 0.8 kHz, which occurred every 50 ms as seen in Fig. 1(c), due to the leakage of the goniometer modulation. Hence, the VLF data from Brorfelde in Fig. 1(c) are not suitable for the study of stationarity and ergodicity, and so we do not make any tests for them. The data with duration of about 120 s are analyzed and we divide the sample record (time history) into many equal time intervals (or segments) with duration of 1 s. In each interval or segment, we compute a mean value and a mean-square value, and the time sequence of each quantity is subjected to the run test (BENDAT and PIERSOL, 1971) which is a nonparametric approach that does not require a knowledge of the sampling distributions of data parameters. We show briefly the procedure of the run test by taking the mean value as an example. Let it be hypothesized that the sequence of sample mean values ($\bar{x}_1$, $\bar{x}_2$, ..., $\bar{x}_N$) consists of each independent sample values of a stationary random variable with a mean value of μ_x. If this hypothesis is true, the variations in the sequence of sample values will be random and display no trends.

Hence, the number of runs in the sequence relative to the mean value μ_x will be as expected for a sequence of independent random observations of the random variable. The number of runs is found to be 54 for $N=117$ segments in the case of the mean value of ELF hiss at Moshiri. The same run test for the mean square values of ELF hiss at Moshiri yields that the number of runs is 58 for $N=117$ segments. Both of these run tests show that the ELF hiss at Moshiri can be concluded to be stationary at the 5% level of significance. For the sake of comparison, a Gaussian white noise from the random noise generator is subjected to the same run test and the hypothesis of stationary noise is again accepted at the 5% level of significance.

Next, we deal with the ergodicity of the VLF/ELF hiss data. In practice, random data (as described by a stationary physical random process as found above) are generally considered to be ergodic, which means that the time average of the random signal is identical with its ensemble average.

3.2. *Normality (Gaussian nature)*

The dynamic spectrum of ELF hiss at Moshiri is already presented in Figs. 1(a) and 1(b), and Fig. 2 illustrates the corresponding frequency spectrum of the wide-band wave data during a period of 0.5 s at a specific time. The most useful way to test samples for normality is to measure the amplitude distribution of the data, and the amplitude distribution function of the ELF hiss at Moshiri is presented in Fig. 3 in which the vertical scale is arbitrary and linear. The figure clearly implies that the relevant hiss follows a Gaussian distribution and we can conclude that the ELF hiss observed at Moshiri is a Gaussian random signal. Although not shown, the amplitude distribution functions of the VLF hiss at Brorfelde at two different UT's are found to be a mixture of the Gaussian component and the effect from the leakage of the goniometer modulation. So when we remove the effect of the goniometer modulation, we are sure that the amplitude distribution function will be approximately Gaussian.

3.3. *Monochromaticity (coherence or incoherence)*

Figure 4 illustrates the frequency spectrum of VLF hiss observed at Brorfelde during a short duration of 256 samples (or 12.8 ms) by means of two different kinds of methods, FFT (Fast Fourier Transform) and MEM (Maximum Entropy Method). This short interval was fortunately free from the effect of the leakage of goniometer modulation. We first have to mention about the error in estimating the power spectrum by means of the FFT analysis (BENDAT and PIERSOL, 1971). For the process, $x(t)$ defined on a time interval T, the spectral density has the form,

$$G_x(f, T)=\frac{2}{T}|X(f, T)|^2$$

where $X(f,T)$ is the direct Fourier transformation of the function $x(t)$ on a finite time interval T,

$$X(f, T)=\int_0^T x(t)e^{-i2\pi ft}dt\,.$$

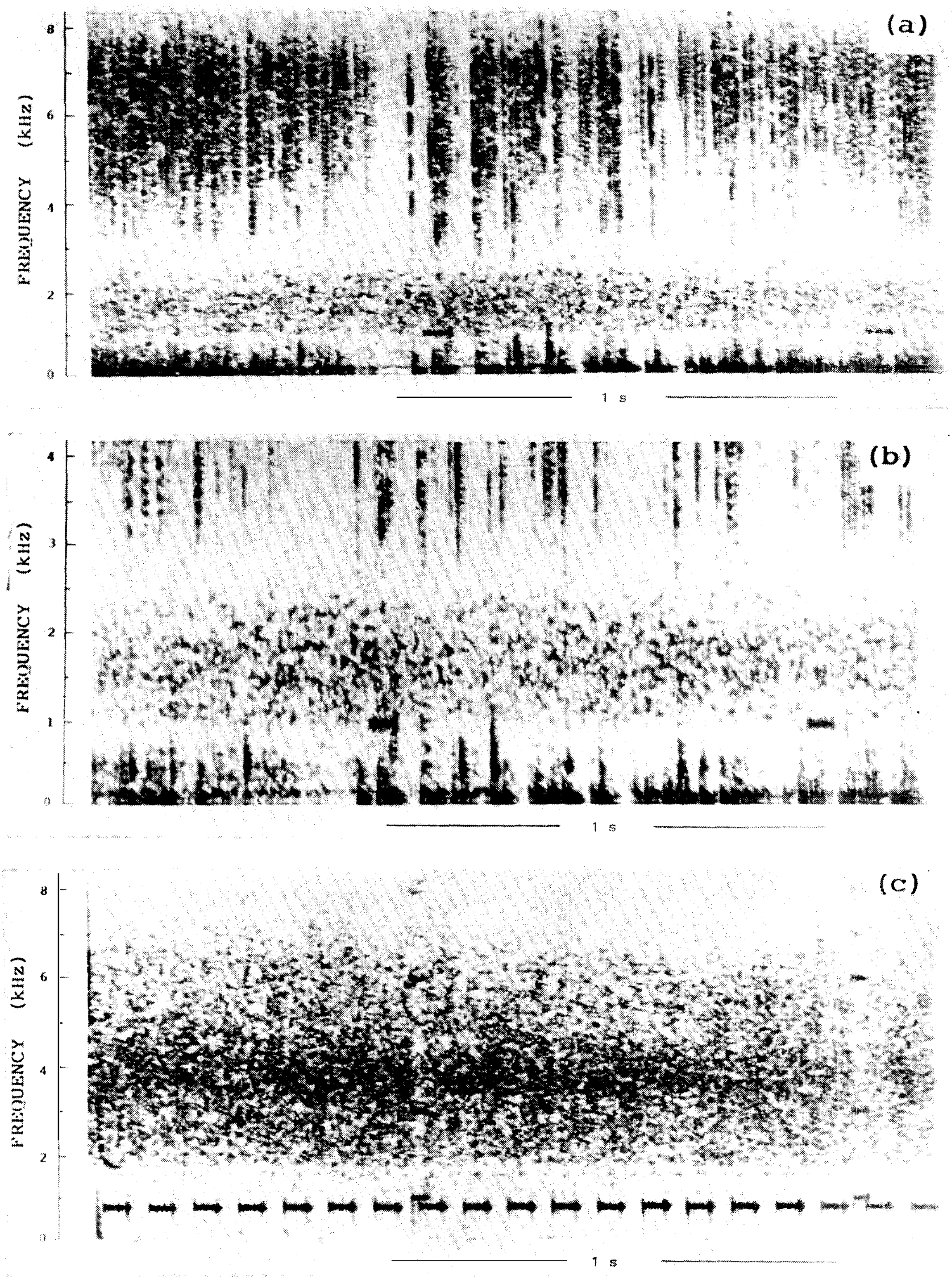

Fig. 1. (a) The sonagram of ELF hiss observed at Moshiri at 0850 UT on 12th September, 1986. The frequency range is 0–8 kHz. (b) The same sonagram as above with an expanded frequency range of 0–4 kHz. We can see strong ELF hiss. (c) The sonagram of VLF hiss observed at Brorfelde at 2250 LT (=UT+1 h) on 7th January, 1979. The frequency range is 0–8 kHz.

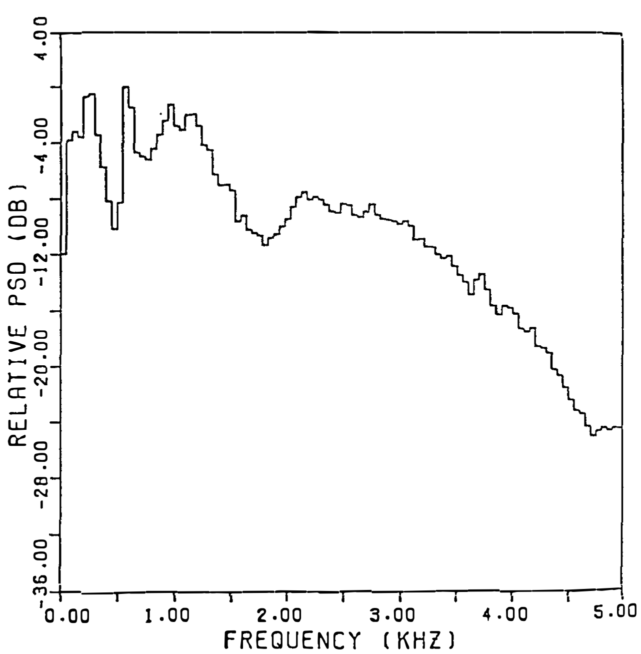

Fig. 2. The frequency spectrum of ELF hiss observed at Moshiri during a short time interval of 0.5 s.

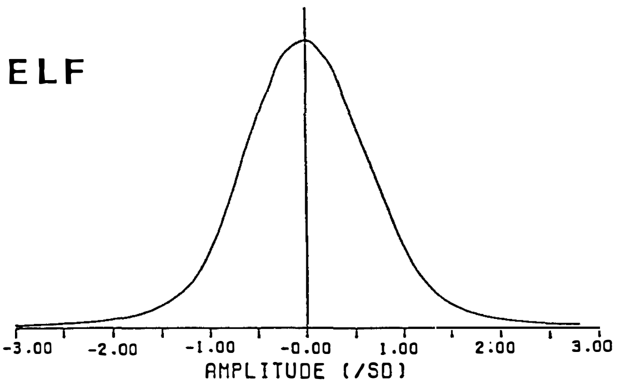

Fig. 3. The amplitude distribution function of ELF hiss at Moshiri. SD means the standard deviation.

Each frequency component of the estimate $G_x(f)=\lim_{T\to\infty} E[G_x(f, T)]$ (where E means the ensemble average) has the sample distribution

$$G_x(f, T)/G_x(f)=\chi_n^2/2$$

where χ_n^2 obeys the χ^2 distribution with n degrees of freedom. The normalized error, determining the random error of the estimate obtained, has the form,

$$\varepsilon_r=\sigma[G_x(f, T)]/G_x(f)=\sqrt{2/n}$$

where σ is the standard deviation. In the case under study of direct FFT, n is equal to 2 and then $\varepsilon_r=1$ (100%), *i.e.* the standard deviation is equal to the quantity being estimated. Since such a random estimate is not acceptable, it is smoothed with respect to frequency (BENDAT and PIERSOL, 1971). By adopting this procedure, we may loose the frequency resolution, but we can increase the degree of freedom. For each method in Fig. 4, the peak power is normalized to 0 dB. A thin full line refers to the result by the FFT with $n=2$, a thin broken line to that by the same FFT

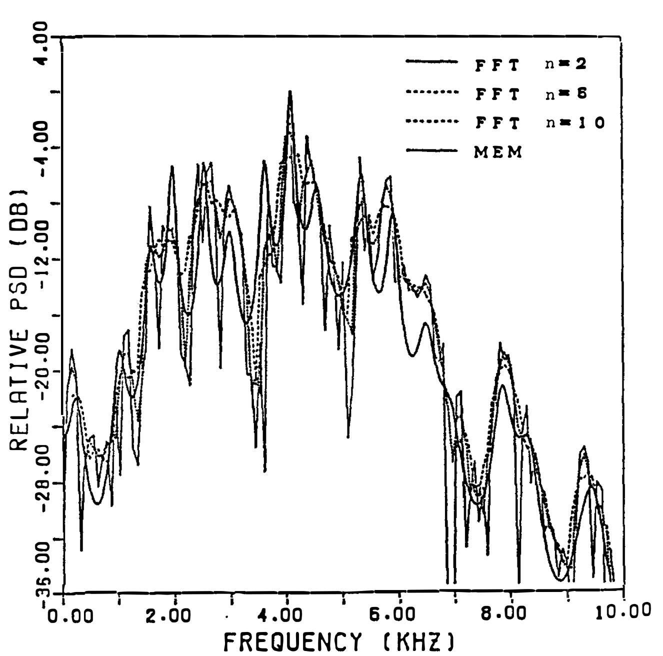

Fig. 4. The power spectra for VLF hiss observed at Brorfelde by means of two methods; FFT with different degrees of freedom and MEM. The duration of the data is 12.8 ms from the VLF hiss data at 2250 LT.

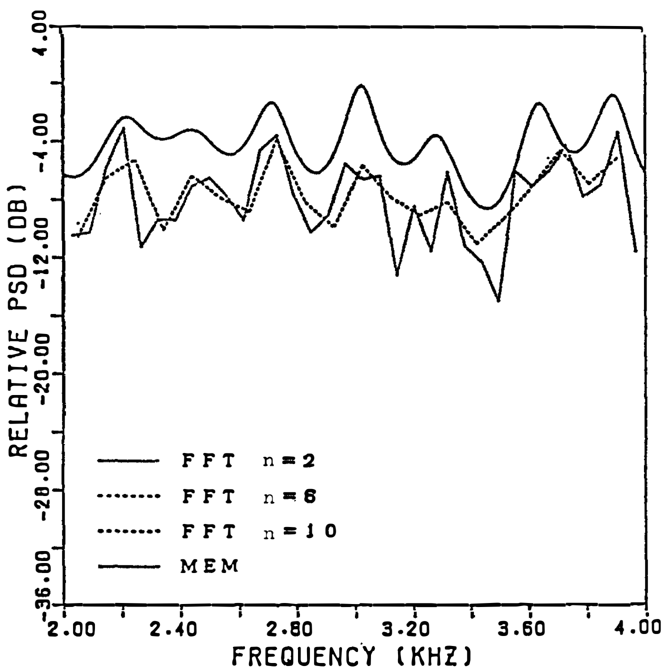

Fig. 5. The power spectra by different methods for the white noise from random noise generator. The duration of the data is 51.2 ms.

with $n=6$, and a thick broken line to that by the same FFT with still larger $n=10$. The thick full line refers to the result by the MEM. It is likely from the figure that the curve by the MEM is just like the curve connecting smoothly the power spectra estimated by the FFT methods. We can notice some kinds of structures in the power spectrum with the MEM in Fig. 4. For the sake of comparison, the similar analyses have been carried out for a Gaussian white noise during the time period of 51.2 ms, and the result is presented in Fig. 5. It is found from the figure that there are several peaks in the power spectra, but the difference between the local maxima and minima is only within 4 dB. Figure 6 is the variation of the local intensity peaks in the power spectrum estimated by means of the MEM for the VLF hiss at Brorfelde at 2250 LT. The frequency range is from 2.0 to 7.0 kHz, and the frequency spacing between two adjacent dots is 50 Hz. Strong intensities above the numeric 6 (above 10 is in-

Table 1. Occurrence histogram of the duration of local intensity maxima within the hiss band.

Starting time of analysis (s)	Analyzing blocks (1 block = ~200 ms)	Duration (ms)					
		19.2	25.6	32.0	38.4	44.8	51.2
0	10	114	46	14	3	2	3
20	9	113	31	13	3	1	
40	10	130	44	10	4	3	1
60	10	126	39	7	1	2	1
80	9	132	30	9	3	2	
100	10	104	38	13	2	3	

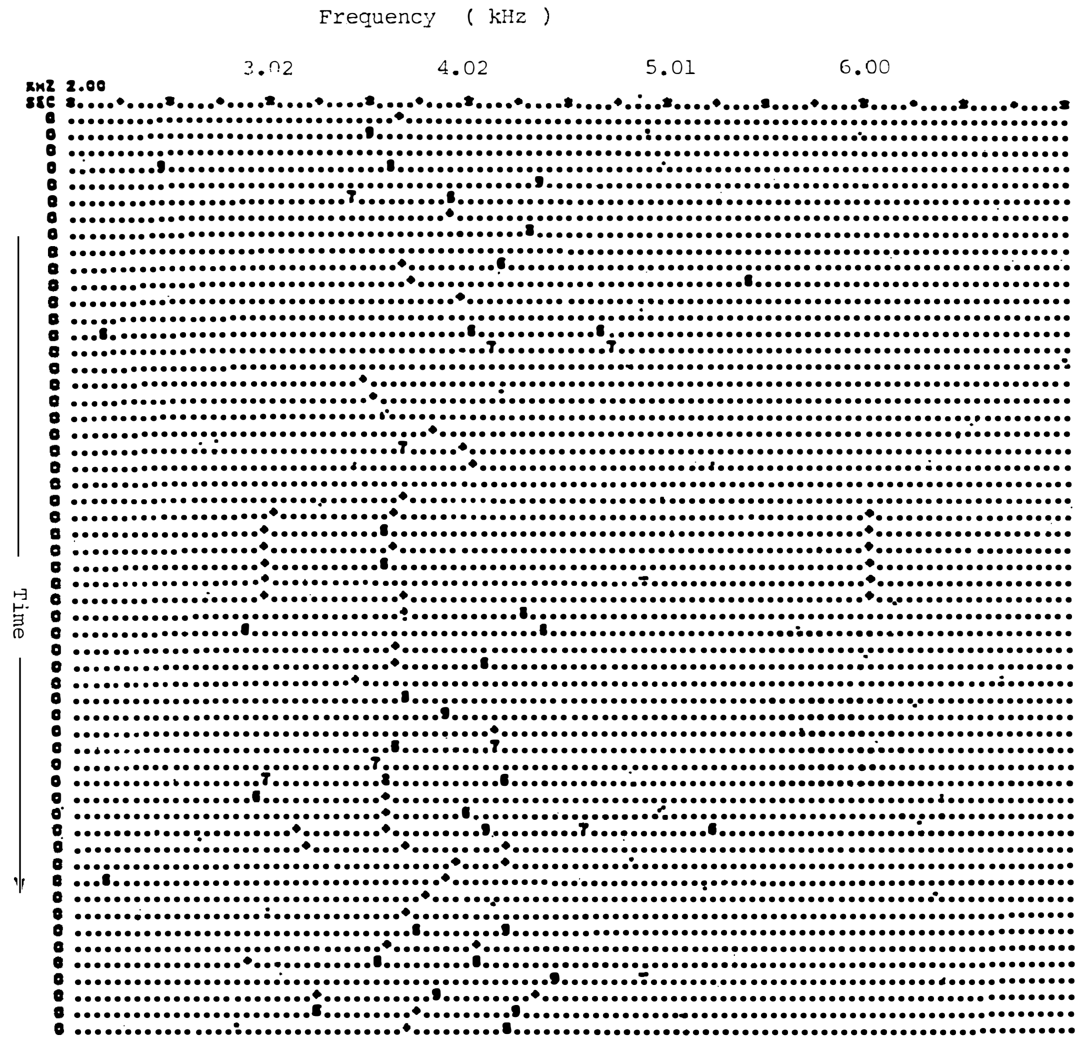

Fig. 6. The variation of local intensity peaks in the power spectrum of the hiss, determined by the MEM.

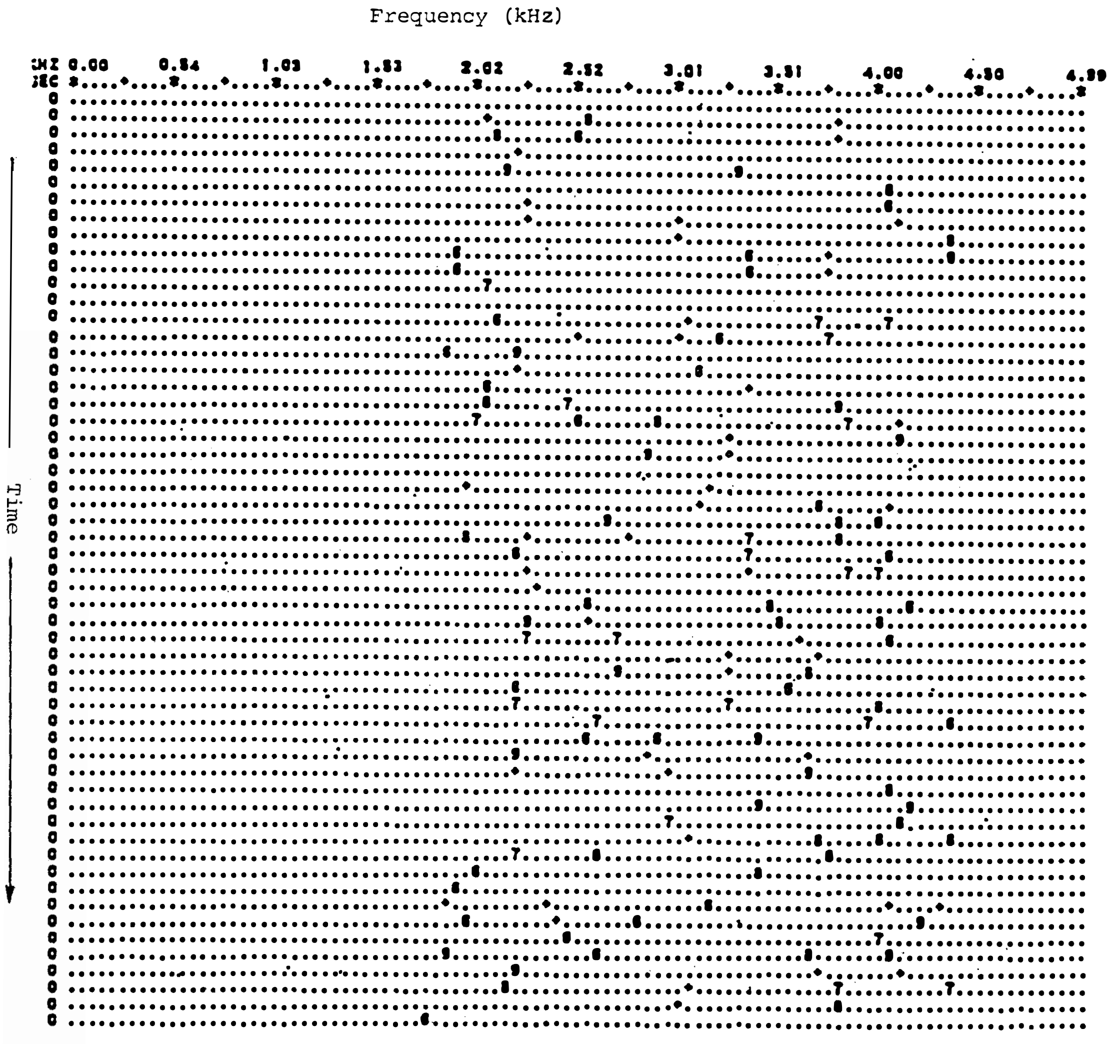

Fig. 7. The same as Fig. 6, but for the white Gaussian noise.

dicated by +) are only selected as the local maxima. It appears that some structures exist even in the hiss band. The corresponding figure for the white noise is given in Fig. 7. There are so many such peaks in the white noise, but we can say that there is no enduring structure within the white noise, as compared with the situation in Fig. 6. Now we investigate the duration of each local intensity maximum in the hiss spectrum, and the summary for VLF hiss at Brorfelde at 2250 LT is given in Table 1. Similar kinds of results are obtained for the VLF hiss at Brorfelde at a different time and also for the ELF hiss at Moshiri. It is seen from this table that there are many local maxima with duration of the order of ~20 ms, and on occasions some local maxima have duration of 40–50 ms. This finally suggests that such coherent wavelets (or monochromatic waves) with duration of the order of a few tens of milliseconds frequently exist even in the hiss band. It may be possible that a chorus could be triggered by such a kind of coherent, structured amplitude fluctuations within the hiss band. This hypothesis has been suggested for the interpretation and mechanism of hiss-triggered chorus based on the analyses of the wave data observed on board GEOS 1 satellite (HATTORI *et al*., 1989). In Figs. 6 and 7, the frequency averaging over 50 Hz is used so as to obtain an accurate estimation of the power spectra, and so the frequency resolution is reduced. On the contrary, when we want to enhance the frequency resolution, the estimation of the power spectra becomes less accurate. Even so, there seem to exist coherent (monochromatic) large amplitude spectral components with duration of the order of a few tens of milliseconds within the hiss band which is so far considered to be very incoherent. This is in sharp contrast with the property of the white noise from the random noise generator.

4. Conclusion

The statistical properties of magnetospheric ELF/VLF hiss have been studied on the basis of the data observed on the ground. It is found that ELF hiss observed at Moshiri and VLF hiss observed at Brorfelde are generally concluded to be Gaussian, stationary random signals. ELF hiss at Moshiri is found to be more stationary than that at Brorfelde. The most important difference of magnetospheric ELF/VLF hiss from the white noise generated by a random noise generator, is that the frequency spectra of the hiss band exhibit large amplitude spectral components, which can be considered as coherent (monochromatic) wavelets with duration of the order of a few tens of milliseconds in the case of our analyses with the frequency resolution of 50 Hz.

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