Proc. NIPR Symp. Polar Meteorol. Glaciol., **8**, 208, 1994

DEVELOPMENT OF A COMPUTER SIMULATION SCHEME FOR TWO AND THREE DIMENSIONAL ICE SHEET DYNAMICS MODELS (ABSTRACT)

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We have developed a scheme for two and three dimensional ice sheet dynamics with the model considered by Mahaffy, assuming the basal sliding velocity to be zero. Mahaffy's model is given by $\partial h/\partial t = b - \nabla \cdot \mathbf{q}$ and $\mathbf{q} = -ck\nabla h$, or, $c\nabla \cdot (-k\nabla h) = b - \partial h/\partial t$, where $c = \{(2A)/(n+2)\}(\rho g)^n$ and $k\ (x,y,t) = (\nabla h \cdot \nabla h)^{\frac{n-1}{2}}\ (h - z_0)^{n+2}$. We can lead the dimensionless form, in which $c = 1$.

In the two dimensional model, let $\Omega_1 = [-x_1, x_1]$ which is the land area, and $\Omega_2 = [-x_2, -x_1) \cup (x_1, x_2]$, which is the sea area, where $0 < x_1 < x_2$. We assume that q in Ω_2 is m times larger than in Ω_1 and that the initial shape of h is symmetric about $x = 0$. Let $0 \leq x \leq n\Delta x$, $q_{i,k} = q((i-1/2)\ \Delta x,\ k\Delta t)$ $(i = 0, 1, \ldots, n+1)$ and $h_{i,k} = h\ (i\Delta x,\ k\Delta t)$ $(i = 0, 1, \ldots, n)$, $z_{0i} = z_0\ (i\Delta x)$ $(i = 0, 1, \ldots, n)$. Then $q_{i,k}$ and $h_{i,k}$ are placed alternately. The finite difference representations of Mahaffy's model are $q_{i,k} = -\{(h_{i,k} - h_{i-1,k}) / \Delta x\}^n\{(h_{i,k} + h_{i-1,k}) / 2 + (z_{0i} + z_{0i-1}) / 2\}^{n+2}$ and $h_{i,k+1} = h_{i,k} + \Delta t\{b - (q_{i+1,k} - q_{i,k}) / \Delta x\}$. If $i\Delta x \in \Omega_2$, $mq_{i,k}$ is used instead of $q_{i,k}$. Boundary conditions are $q_{0,k} = (-1)^n\ q_{1,k}$ and $h_{n,k} = 0$.

In the three dimensional model, let Ω be the region of interest and $\partial\Omega$ be the boundary of Ω. For both sides of Mahaffy's model, we multiply the weighting function W_l and integrate in the interior region Ω and apply Green's theorem, $\int_\Omega k\nabla h \cdot \nabla W_l \mathrm{d}\Omega - \int_{\partial\Omega} k\ (\partial h/\partial n)\ W_l \mathrm{d}\Gamma = \int_\Omega (b - \partial h/\partial t)\ W_l \mathrm{d}\Omega$, where $\partial h / \partial n = \nabla h \cdot n$ in which n is the outer normal vector of $\partial\Omega$. We divided the region Ω into N small regions Ω^e. Let M be the number of nodes and assign a number from 1 to M to each node. Let $\hat{h}$ be the approximation of h. $h\ (x,y,t) \simeq \hat{h}\ (x,y,t) = \sum_{m=1}^{M} h_m\ (t) N_m(x,y)$ where $N_m\ (x,y)$ are basis functions which are 1 at the node m and 0 in small regions which do not include the node m. We take N_l as the weighting function. Let $\hat{k} = k\ (\hat{h})$, $\hat{b} = b\ (\hat{h})$, $K_{l,m}\ (\mathbf{h}) = \int_\Omega \hat{k}\{(\partial N_m / \partial x)(\partial N_l / \partial x) + (\partial N_m / \partial y)(\partial N_l / \partial y)\}\mathrm{d}\Omega - \int_{\partial\Omega} \hat{k}\ (\partial N_m / \partial n)\ N_l \mathrm{d}\Gamma$, $C_{l,m} = \int_\Omega N_m\ N_l\ \mathrm{d}\Omega$, $f_l(\mathbf{h}) = \int_\Omega \hat{b} N_l \mathrm{d}\Omega$, $K = (K_{l,m})_{l,m=1,\ldots,M}$, $C = (C_{l,m})_{l,m=1,\ldots,M}$ and $\mathbf{f} = (f_1, \ldots, f_M)^T$. Then we can write the integral equation as $K\ (\mathbf{h})\ \mathbf{h} + C\ (\partial \mathbf{h}/\partial t) = \mathbf{f}\ (\mathbf{h})$. We apply the backward difference method over time, and define $\mathbf{h}^{(n)} = \mathbf{h}\ (n\Delta t)$, $K\ (\mathbf{h}^{(n)})\ \mathbf{h}^{(n)} + C\ \{(\mathbf{h}^{(n)} - \mathbf{h}^{(n-1)})/\Delta t\} = \mathbf{f}(\mathbf{h}^{(n)})$. When $\mathbf{h}^{(n-1)}$ is solved, $\mathbf{h}^{(n)}$ can be solved by an iterative method, substituting an initial value into $\mathbf{h}^{(n)}$. $K^e_{l,m}\ (\mathbf{h})$, $C^e_{l,m}$ and $K_{l,m}\ (\mathbf{h})$ are defined when using Ω^e instead of Ω in $K_{l,m}\ (\mathbf{h})\ C_{l,m}$ and $f_l\ (\mathbf{h})$ respectively. Then $K_{l,m}\ (\mathbf{h}) = \sum_{e=1}^{N} K^e_{l,m}\ (\mathbf{h})$, $C_{l,m} = \sum_{e=1}^{N} C^e_{l,m}$, and $f_l\ (\mathbf{h}) = \sum_{e=1}^{N} f^e_l\ (\mathbf{h})$. Here, $K^e_{l,m} = C^e_{l,m} = f^e_l = 0$ if $l,\ m \notin \Omega^e$. Letting $K^e = (K^e_{l,m})_{l,m=1,\ldots,M}$, $C^e = (C^e_{l,m})_{l,m=1,\ldots,M}$ and $\mathbf{f}^e = (f^e_1, \ldots, f^e_M)^T$, we calculate K^e, C^e and $\mathbf{f}^e$ for each e at first, then find K, C and $\mathbf{f}$.

The two dimensional scheme was applied to the Shirase drainage basin, and the three dimensional to the entire Antarctic ice sheet. The results seem to be satisfactory, although further improvements are to be carried out.

(*Received April 25, 1994*)