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# A MULTIPLE SCATTERING MODEL FOR THE ATMOSPHERE-SNOW SYSTEM

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***Abstract:*** A multiple scattering model for radiative transfer in the atmosphere-snow system is constructed by applying the Mie theory to the snow layer composed of ice grain particles. The spectral albedo from 0.2 to 3.0 $\mu$m wavelength is examined as a function of effective grain size, solar zenith angle and snow optical depth with the single snow layer model. The upward radiances at the top and base of the atmosphere are also examined with the single atmospheric layer + single snow layer model.

The spectral variation of albedo is mainly dependent on the imaginary part of the refractive index of ice. The albedo decreases at all wavelengths with increase of grain size; its degree is generally large in the infrared region and small in the visible region. The albedo increases with increase of solar zenith angle and optical depth of snow. These results are consistent with previous work. The upward radiance distribution in the solar principal plane shows that snow reflectance gradually deviates from the Lambertian with the increase of solar zenith angle.

## 1. Introduction

Theoretical studies on the optical properties of snow cover began with the Schuster two-stream model by Dunkle and Bevans (1956). They obtained the spectral albedo of snow cover for three particle sizes. Barkstrom (1972) analytically formulated and solved the multiple scattering problem in a snow layer based on isotropic scattering in a plane-parallel, semi-infinite and gray medium. Bohren and Barkstrom (1974) improved the work of Barkstrom (1972) by formulating the radiative transfer in snow grains with geometrical optics. Wiscombe and Warren (1980) calculated the spectral albedo of snow cover from 0.3 to 5.0 $\mu$m wavelength with diffusely or directly incident radiation by using the Eddington "approximation" for multiple scattering, together with the Mie theory for single scattering. Warren and Wiscombe (1980) also derived the spectral albedo of a snow layer which contains absorptive impurities by using the same model as their previous work.

In previous models, snow cover has been treated as a homogeneous layer, and atmospheric effects except cloud cover have not been considered. The calculated parameters with these models are restricted to albedo or radiant flux density. In this study, a multiple scattering model for the radiative transfer in the atmosphere-snow system is constructed by applying the Mie theory to a snow layer composed of ice grain particles. Both the atmosphere and snow cover can be divided into any layers in this model. Moreover, not only the surface albedo of snow cover, but also the planetary albedo at the top of the atmosphere, radiant flux density and radiance at each boundary of the atmosphere and snow cover can be calculated at any wavelength.

## 2. Model

The atmosphere-snow system generally consists of inhomogeneous atmospheric and snow layers. Radiative transfer in snow layers is treated in the same manner as a multiple scattering model in the atmosphere. Doubling-adding method without polarization is employed for multiple scattering, which is a basis of the model. Layers of the atmosphere and snow cover are treated as being plane-parallel. The snow grains are assumed to be spherical ice particles, and phase functions for single scattering are calculated by the Mie theory. The atmospheric layers can be composed of air molecules, aerosols and cloud particles. Figure 1 shows the structure of this model. Input parameters are wavelength, size distribution of snow grains, refractive index of ice, composition of the atmospheric and snow layers and so on. Output parameters have already been described in the introduction.

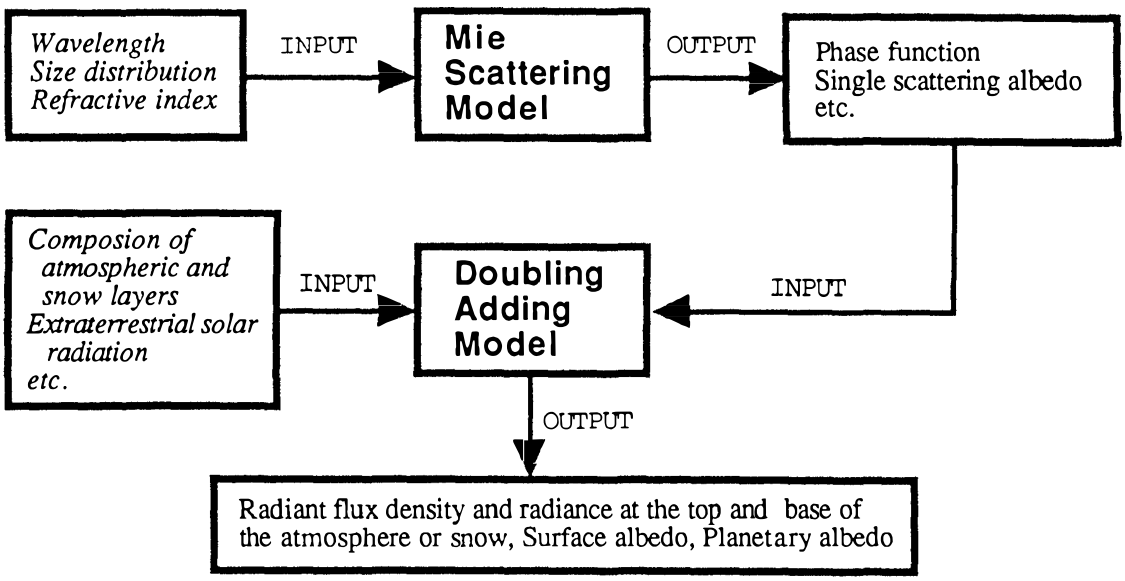


*Fig. 1. Structure of a multiple scattering model for the atmosphere-snow system.*

## 3. Results

Calculations are performed by the single snow layer model with the single atmospheric layer + single snow layer model. When the doubling procedure is continued until the snow optical depth $\tau$ becomes 800, albedo almost converges to an upper limit in the visible and near infrared regions.

### *3.1. Snow grain size distribution and phase function*

Snow grains are assumed to be spherical ice particles which follow the standard gamma size distribution of HANSEN (1971). We employed four effective radii $r_{\rm eff}$=10, 100, 500 and 1000 $\mu$m following this size distribution as shown in the inset of Fig. 2. Phase functions for single scattering are calculated in the

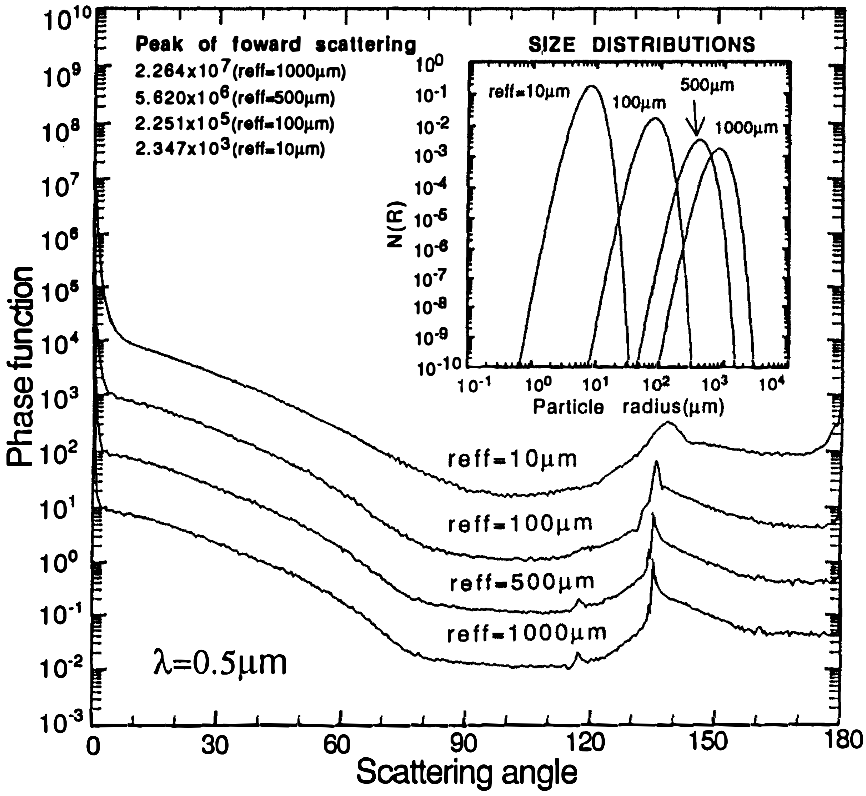


*Fig. 2. Standard gamma size distributions for four effective radii $r_{eff}$=10, 100, 500, 1000 $\mu$m and examples of phase function for wavelength $\lambda$=0.5 $\mu$m. The curves of phase function except $r_{eff}$=1000 $\mu$m are successively displaced upward by a factor of 10.*

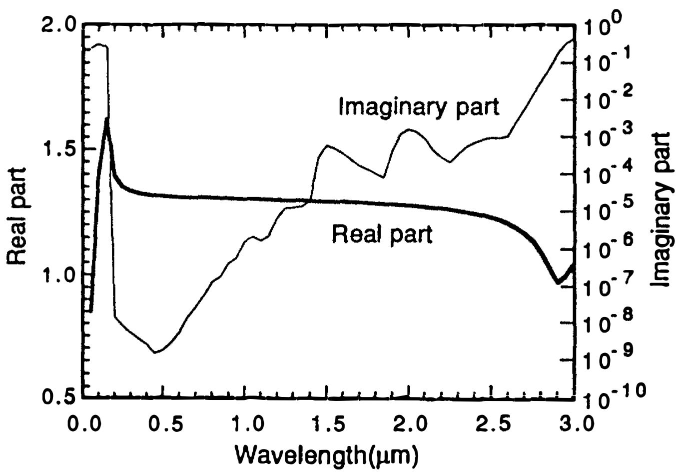


*Fig. 3. Refractive index of ice as a function of wavelength. The real and imaginary parts are shown by thick and thin lines, respectively.*

wavelength region $\lambda=0.2\sim3.0$ $\mu$m with 0.1 $\mu$m increments by the Mie theory. Then, we use the refractive index compiled by WARREN (1984) as shown in Fig. 3. Figure 2 shows the phase functions for four size distributions at $\lambda=0.5$ $\mu$m. The peak of forward scattering in phase function for large size distribution is very sharp. This influences the calculation accuracy of radiance, as described later.

### 3.2. *Spectral albedo*

The spectral albedos for four size distributions calculated by the single snow layer model are shown in Fig. 4. The spectral variation of albedo is mainly dependent on the imaginary part of the refractive index of ice (Fig. 3). Albedo decreases in the near infrared region where radiation absorption by ice is large, and increases in the visible region where radiation absorption by ice is small. When effective grain radius increases, albedo decreases at all wavelengths; the decrease is generally large in the infrared region and small in the visible region. This is due to the dependence of the decrease of single scattering albedo on the imaginary part of refractive index with increase of particle size.

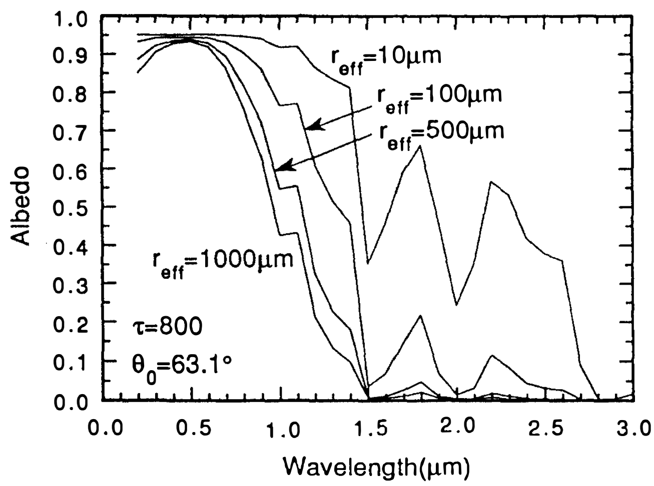


*Fig. 4. Spectral albedos of snow for four size distributions from 0.2 to 3.0 μm wavelength. The solar zenith angle $\theta_0$ is 63.1°, and snow optical depth $\tau$ is 800.*

### 3.3. *Albedo dependence on the solar zenith angle*

The albedo dependences on solar zenith angles for four size distributions calculated by the single snow layer model are shown in Fig. 5, where snow optical depth $\tau$ is 800; albedos with smaller optical depths ($\tau=12.5\sim400$) are also shown only for the effective radius $r_{eff}=100$ $\mu$m. From Fig. 5 we find: 1) The albedo decreases with increase of effective grain radius; 2) The albedo increases with increase of solar zenith angle; 3) The albedo increases with increase of snow optical depth and converges. The first result suggests that snow aging will reduce the albedo, since grain size generally increases with snow age as mentioned by WISCOMBE and WARREN (1980). The second result is pointed out by some observational studies (*e.g.*, YAMANOUCHI, 1983). The third result means that

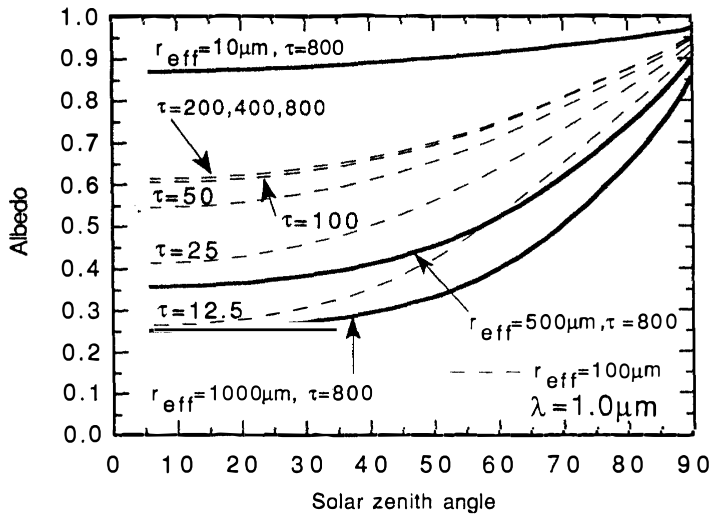


*Fig. 5. Snow albedo dependences on solar zenith angle for four size distributions at wavelength λ=1.0 μm. For snow layers with effective radius $r_{eff}$=10, 100 and 1000 μm, albedos of snow optical depth τ=800 are shown by solid lines; and for snow layers with $r_{eff}$=100 μm, albedos of τ=12.5~800 are shown by dashed lines.*

albedo increases with snow depth and becomes independent of the optical depth in the case of sufficient snow depth. The albedo convergence rate for the increase of snow optical depth is also examined for each size distribution at other wavelengths. It is found that the albedo convergence rate is generally accelerated by increase of grain size, and albedo converges in the near infrared region more quickly than in the visible region

### *3.4. Upward radiance*

The upward radiance distributions in the solar principal plane at the top and base of the atmosphere are shown in Fig. 6. These are calculated by the single Rayleigh atmospheric layer + single snow layer ($r_{eff}$=500 μm) model. The radiance difference between the top and base of the atmosphere increases with the increase of solar zenith angle ($\theta_0$) and/or viewing zenith angle ($\theta$). This is due to the influence of the atmospheric path radiance and extinction under such geometric conditions. The upward radiance distribution shows that snow reflectance gradually deviates from the Lambertian with the increase of solar zenith angle.

In should be noted that although the radiance distributions should be essentially smooth curves, some irregularities of radiance value appear in Fig. 6. This is because the peak of forward scattering in the phase function cannot be expressed accurately by a Fourier series expansion in the doubling-adding model. The radiant flux density is corrected by the renormalization approximation of HANSEN (1971). However, this problem with radiance could not be removed with this method.

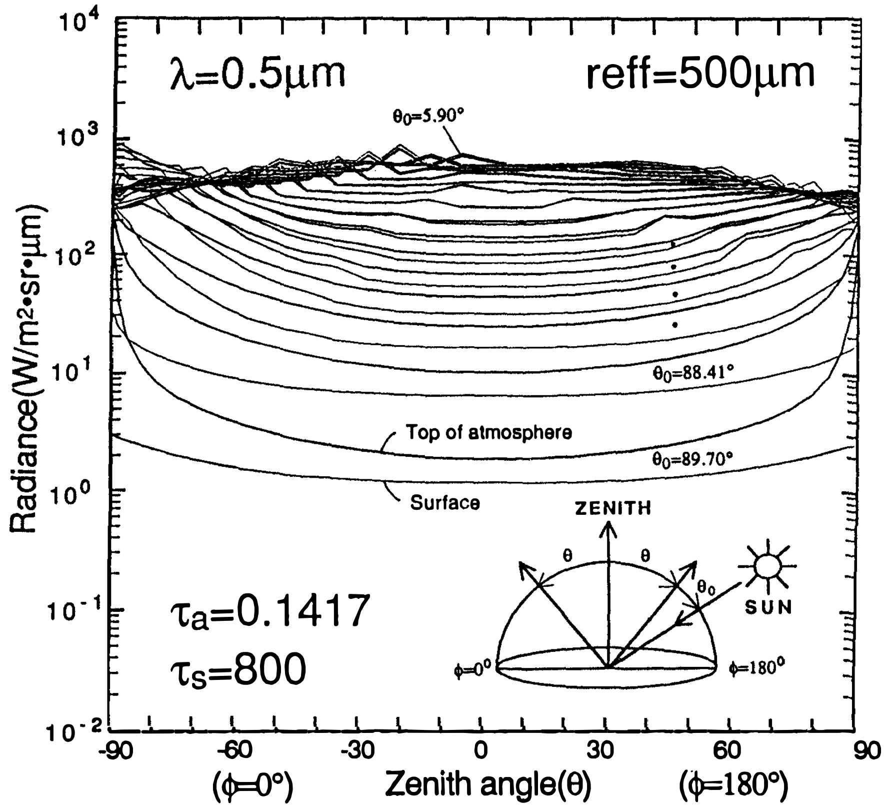


*Fig. 6. Radiance distributions in the solar principal plane at the top and base of the atmosphere. These distributions are calculated for $\lambda=0.5$ μm, $r_{eff}=500$ μm, the atmospheric optical depth $\tau_a=0.1417$ and snow optical depth $\tau_s=800$. Azimuth angles $\phi=0°$ and $\phi=180°$ are away from and toward the sun, respectively.*

## 4. Summary

A multiple scattering model for radiative transfer in the atmosphere-snow system is constructed by applying the Mie theory to a snow layer composed of ice grain particles. The calculated results for albedo and radiance show the following: The spectral variation of albedo is mainly dependent on the imaginary part of the refractive index of ice. The albedo decreases at all wavelengths with increase in grain size; degree is generally large in the infrared region and small in the visible region. The albedo increases with increases of solar zenith angel and snow optical depth. These results are consistent with previous works. The upward radiance distribution shows that snow reflectance gradually deviates from the Lambertian with increase of solar zenith angle. There remains the problem that the peak of forward scattering in phase function for large grain like snow cannot be expressed accurately in the multiple scattering scheme.

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