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DIFFUSION COEFFICIENTS DERIVED FROM THE LAGRANGIAN STATISTICS

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Abstract: Diffusion coefficients are derived from trajectory analyses. Derived horizontal diffusivity in the lower stratosphere varies with season and confirmed the results obtained in YAMAZAKI (Proc. NIPR Symp. Polar Meteorol. Glaciol., **1**, 39, 1987). Large horizontal diffusivity appears in the region of enhanced wave activity and/or weak zonal wind. The horizontal and vertical diffusivities in the troposphere are large in middle latitudes. The pattern and magnitude of derived diffusivity generally agree with the results of GFDL General Circulation/Transport Model by PLUMB and MAHLMAN (J. Atmos. Sci., **44**, 298, 1987). Dispersion from the antarctic troposphere is very rapid and the residence time over Antarctica is about 3 days.

1. Introduction

YAMAZAKI (1987a, hereafter referred to as Y87) described transport characteristics in the lower stratosphere during the southern winter-to-summer transition period by a trajectory analysis. Dispersion of the air parcels that were initially placed at the same latitude and height was studied. The dispersion in the meridional plane can be divided into two parts, *i.e.*, the movement of the center of mass and the variance around the center of mass. The latitudinal and vertical components of diffusion coefficients, K_y and K_z, can be derived from the time evolution of the variance during 10 days as follows:

$$K_y = 1/2\frac{\partial}{\partial t}\overline{y^2}\ , \qquad K_z = 1/2\frac{\partial}{\partial t}\overline{z^2} \tag{1}$$

where y is a latitudinal distance from the center of mass and z is a vertical distance from the center of mass; $\overline{(\ )^2}$ denotes a variance of ( ). In this study the distributions of K_y and K_z estimated by the least squares method are shown.

Data and method are the same as in Y87 with some additional calculations. The diffusion coefficient thus obtained can be used for one- or two-dimensional tracer transport models of minor constituents.

2. Southern Winter-to-Summer Transition Period

In addition to cases in Y87 (Table 1 of Y87), September 30–October 10 and November 10–20 cases are included in this study to complete the period (September 10–December 30), though the additional cases have more than one missing date. Missing data are filled by a linear interpolation with time.

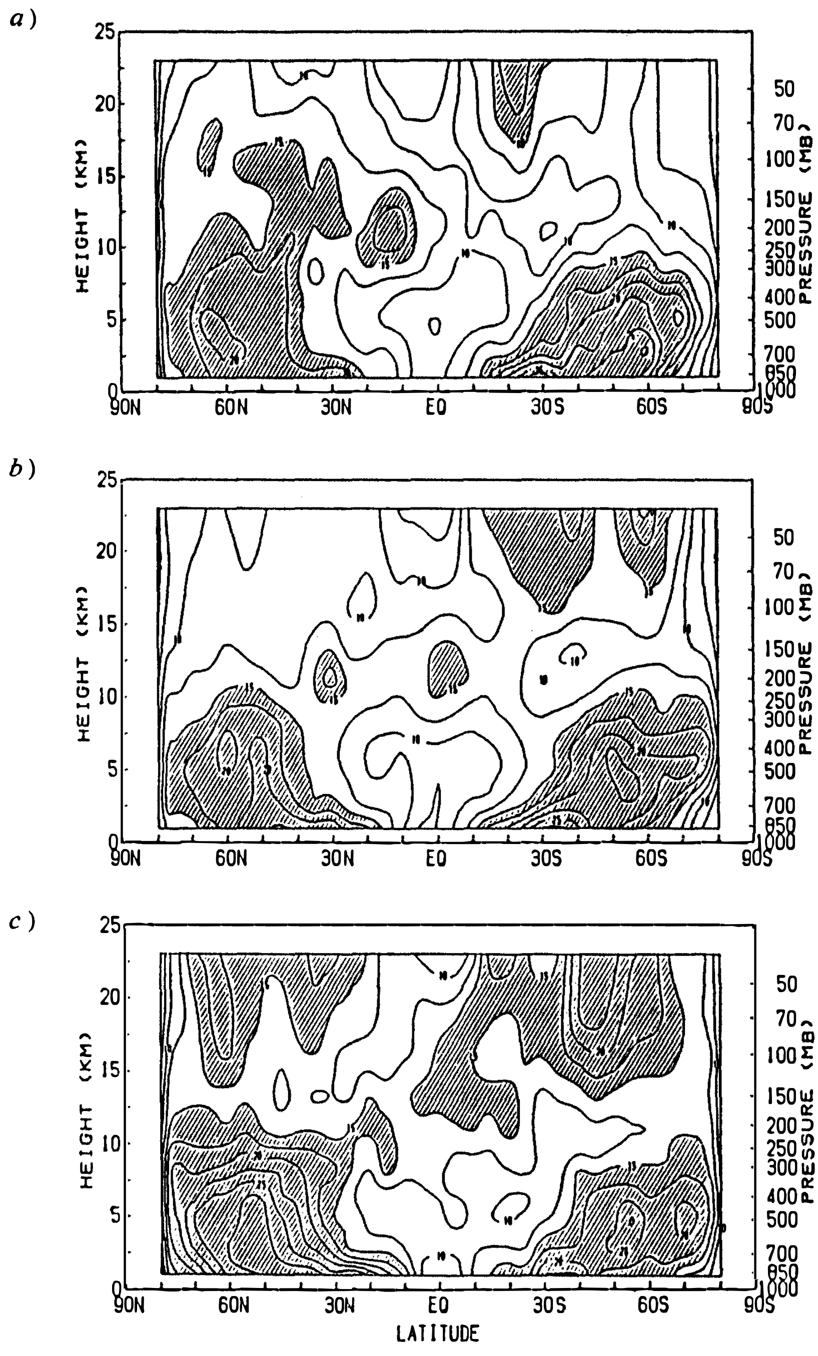

Fig. 1. Derived horizontal component of diffusion coefficient K_y. Contour interval is 2.5×10^5 m^2 s^{-1}. Values greater than 15×10^5 m^2 s^{-1} are shaded. a) September 10–October 10, 1892 (3-case average). b) October 10–November 30 (5-case average). c) November 30–December 30 (3-case average). One case=10 days.

Figure 1 shows the derived diffusion coefficient K_y during the southern wineter-to-summer transition period. The value of diffusion coefficient is plotted at the initial position of air parcels in the meridional plane in Fig. 1 and the subsequent figures, though the center of mass moves during 10 days. Figure 1a corresponds to the period when the south polar vortex was intense (stage 1), Fig. 1b, when the vortex was being encroached (stage 2); and Fig. 1c, after the breakdown of the vortex (stage 3).

In the lower stratosphere during stage 1, small values of K_y are found in the south polar region. Large values are located at about 20°S. The large values of K_y spread poleward during stages 2 and 3 in the southern stratosphere. Large values of K_y also appear in the northern stratosphere during stage 3, which corresponds to the enhanced planetary wave activity (cf. Fig. 4 of YAMAZAKI, 1987b). However, large values in the southern stratosphere during stage 3 do not correspond to the enhanced wave activity. The large values in the southern stratosphere are probably related to the weak zonal wind there. Let us consider a very simple 2-dimensional case. Suppose that the wind field is steady and expressed as follows:

$$u=\mathrm{d}x/\mathrm{d}t=u_0\,, \tag{2a}$$

$$v=\mathrm{d}y/\mathrm{d}t=v_1 \cos kx\,, \tag{2b}$$

where u, v are the eastward and northward velocities, respectively, u_0, v_1 is uniform zonal wind and the amplitude of eddy meridional velocity, respectively, and k is the wave number. The trajectory of the air parcel is easily obtained as follows:

$$x=u_0t+x_0\,, \tag{3a}$$

$$y=v_1/ku_0(\sin kx-\sin kx_0)+y_0\,, \tag{3b}$$

and the variance is

$$\overline{y^2}=v_1/(ku_0)(1-\cos ku_0t) \tag{4}$$

where (x_0, y_0) is the initial ($t=0$) position. The trajectory is sinusoidal and the latitudinal extent of movement of air parcels originally placed on $y=y_0$ line is large when zonal wind is weak and/or eddy velocity is large. In the real atmosphere, the wind field is no longer steady and there exist small-scale eddies. Moreover, a Rossby wave breaking (irreversible deformation of large-scale waves to small-scale waves) can occur where the zonal wind is small. Therefore coming back to the original latitude is less likely for the parcels which have moved far from the original latitudes. Thus, the K_y estimated in the present study is expected to be large where zonal wind is weak.

Note also that the derived K_y depends on time. In a very short time, $\overline{y^2}$ is proportional to t^2. Namely standard deviation is a linear function of t. This can be seen in Fig. 5 of Y87 for the first 1 or 2 days. On the other hand, when the time interval is too large, say 1 year, the parcels would spread globally and the derived K_y has little meaning. Therefore a proper time interval should be chosen.

In the troposphere, K_y is large at middle latitudes and large K_y extends to the subtropical lower troposphere. Another relatively large K_y lies in the tropical upper troposphere. In the tropical middle and lower troposphere, K_y is quite small. Small K_y in the tropics makes the mass exchange between the two hemispheres slow. Small K_y layers in middle latitudes lie at about 150 mb in both hemispheres.

The distribution of vertical component of diffusion coefficient K_z is shown in Fig. 2 for an 8-case average during the period September 10–November 30, 1982. Since K_z does not show large temporal variations as in K_y, the average of 8 cases is shown.

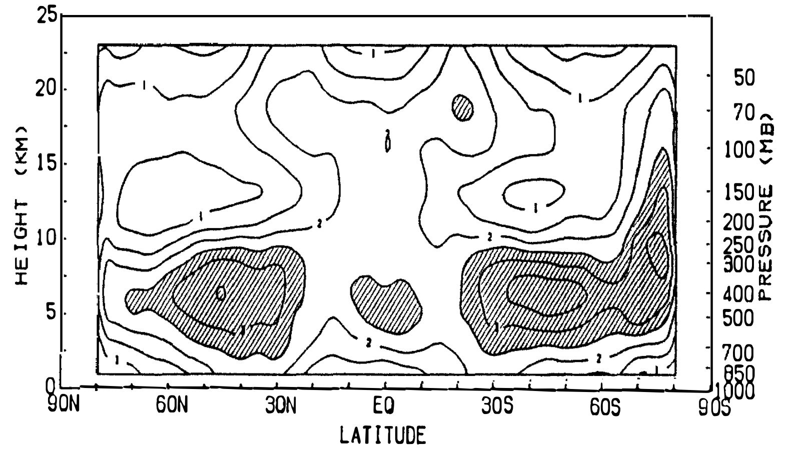

Fig. 2. Derived vertical component of diffusion coefficient K_z. Contour interval is 0.5 $m^2 s^{-1}$. Values greater than 2.5 $m^2 s^{-1}$ are shaded. September 10–November 30, 1982 (8-case average).

Large values are found in the midlatitude middle troposphere. Secondary maximum is located in the tropics. Another maximum is found over Antarctica. This Antarctic maximum is probably related to the baroclinic zone around Antarctica. As mentioned before, K_z is also plotted at the initial position of air parcels. The center of mass that was initially located over Antarctica moves 10 days later to the circumpolar baroclinic zone around 60°S as seen in Fig. 7 of Y87. Therefore large values of K_z over Antarctica in Fig. 2 should reflect the high cyclonic activity around Antarctica.

The layers of low values of K_z are located at about 150 mb (near the tropopause level) in middle latitudes of both hemispheres. K_y is also small there (Fig. 1). Thus the mass exchange between the troposphere and stratosphere is slow compared with that within the troposphere.

3. Northern Winter-to-Summer Transition Period

A stratospheric warming occurred in the Northern Hemisphere during late March through early April in 1982 (Yamazaki, 1987b). Ten-day trajectory analyses are made from the initial times of 12Z March 25, 12Z March 31, and 12Z April 5 of 1982. The derived K_y and K_z are shown in Figs. 3 and 4.

In the northern stratosphere, large values of K_y move poleward from the March 25 case to the April 5 case. As a result, large values are found in the northern lower stratosphere in Fig. 3. In the southern stratosphere, where zonal wind is moderate westerly of about 10–20 m/s, K_y is generally small. In the troposphere, large values in the northern extratropics are noticeable, though the pattern of K_y distribution in the troposphere during the period is rather similar to that during the southern winter-to-summer transition period (Fig. 1).

Values of K_z in the northern troposphere are larger than those in the southern troposphere. Besides that, there seems to be no significant difference between Fig. 2 and Fig. 4.

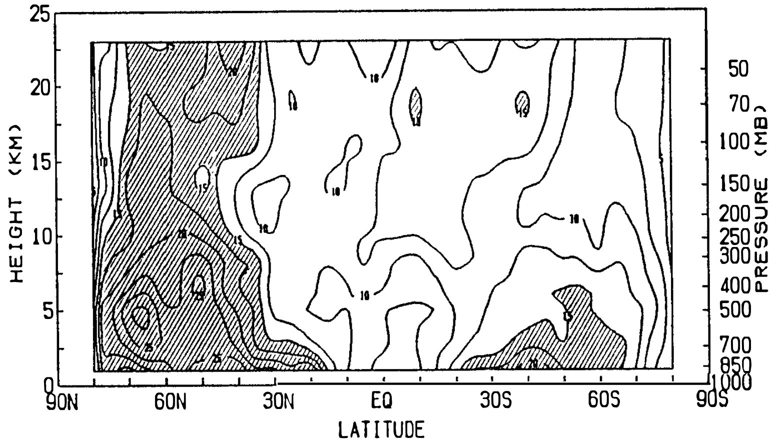

Fig. 3. Same as in Fig. 1 except for the period, March 25–April 15, 1982 (3-case average).

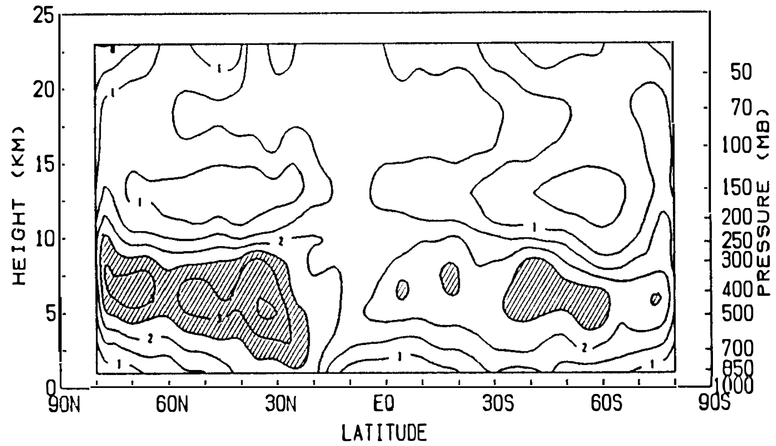

Fig. 4. Same as in Fig. 2 except for the period, March 25–April 15, 1982 (3-case average).

4. Dispersion of the Air Parcels in the Antarctic Troposphere

Figure 5 shows the meridional projection of air parcels whose initial positions are at 70°S–80°S and 200–500 mb. Its initial time is 12Z October 20, 1982. The diffusion takes place mainly along the isentrope, though the direct meridional circulation exists over Antarctica. In the antarctic region, the picture of direct circulation is misleading as far as the actual air mass transport is concerned. The diffusion process is very rapid and a portion of parcels reaches the near-surface layer at 40°S after 3 days. The average residence time of air over Antarctica seems to be about 3 days. In a sense, the tropospheric air over Antarctica is closely tied to the air in the lower troposphere at middle latitudes and the characteristic exchange time scale is a few days.

5. Discussion

KIDA (1983) derived K_y and K_z in the lower stratosphere by the trajectory analyses of the hemispheric GCM results for perpetual winter conditions. His and our methods

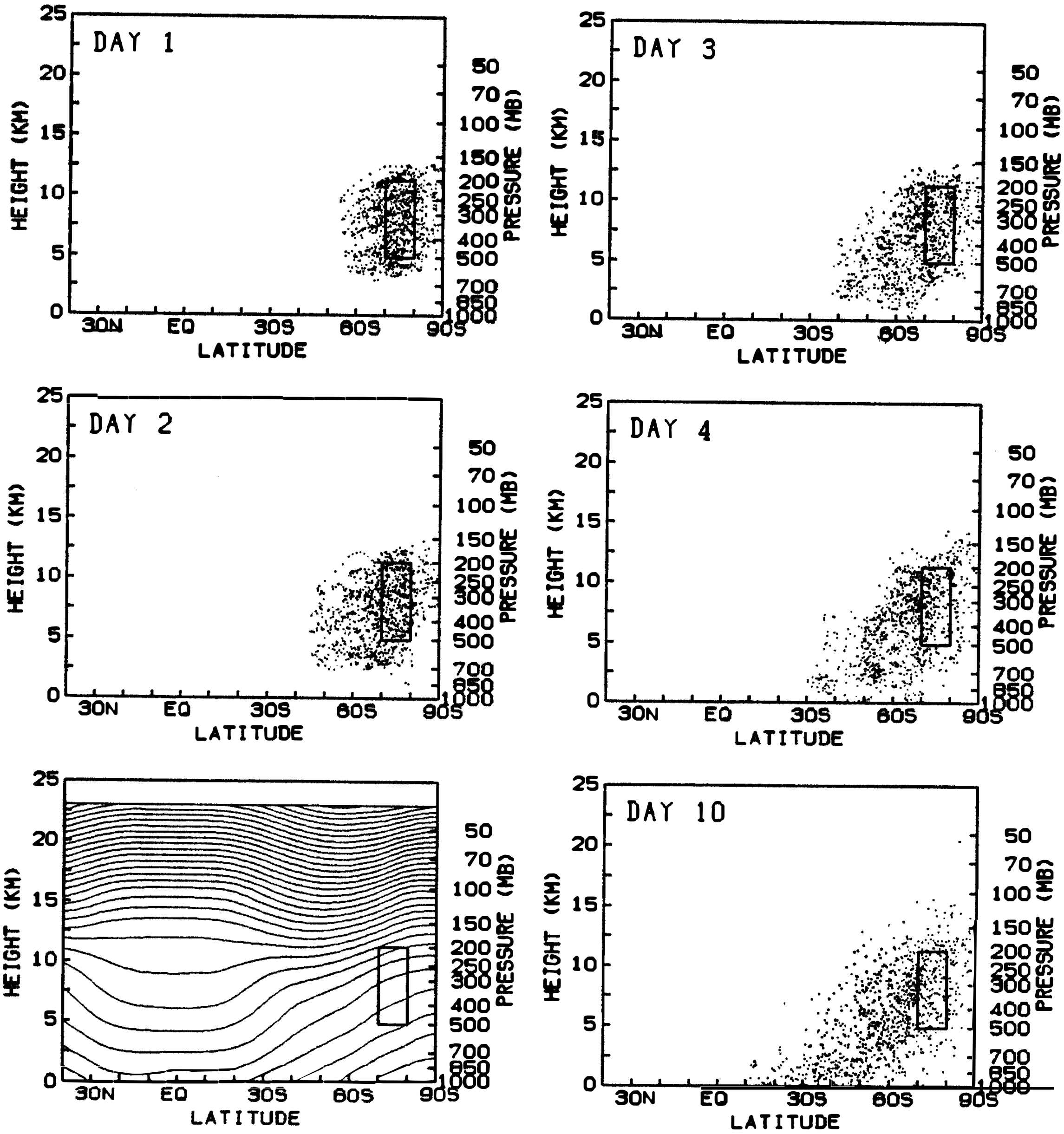

Fig. 5. Latitude-height distributions of air parcels at days 1, 2, 3, 4, and 10 (from top left to right bottom). The initial positions are at 200–500 mb, 70–80°S (shown by rectangles). The initial time is 12Z October 20, 1982. The bottom left panel is the 10-day mean zonal mean potential temperature. The contour intervals is 10 K.

to derive diffusion coefficients are almost the same. He obtained $K_y \sim 0.3 \times 10^6$ m^2 s^{-1}, and $K_z \sim 0.1$ m^2 s^{-1} as typical values in the lower stratosphere. The global averaged values in the present study are $K_y \sim 1.2\text{–}1.4 \times 10^6$ m^2 s^{-1}, $K_z \sim 1.4\text{–}1.7$ m^2 s^{-1}. Our estimate is about 4 times larger than KIDA's value for K_y and one order larger for K_z. Even considering differences of methods between this study and KIDA's, such as GCM data vs. real data, our 10-day trajectory and KIDA's 30-day trajectory. KIDA's values are too small. The reason why KIDA's values are small probably lies in weak planetary wave activity and/or strong zonal westerly wind in his GCM simulation.

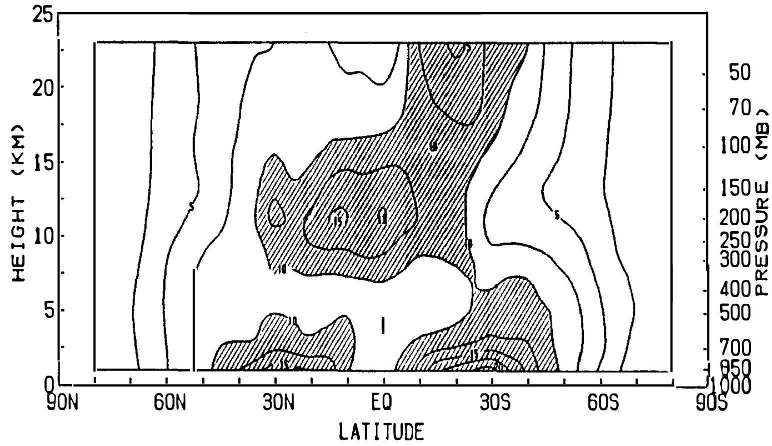

Fig. 6. Derived horizontal diffusivity K_y multiplied by $\cos^2\phi$. The contour interval is 2.5×10^5 $m^2 s^{-1}$. Values greater than 10^6 $m^2 s^{-1}$ are shaded. September 10–November 30, 1982 (8-case average).

Recently PLUMB and MAHLMAN (1987, hereafter referred to as PM), performed two tracer experiments with the GFDL GCM. One tracer has the density gradient mainly in latitudinal direction and the other in vertical direction. They derived diffusion tensors from two sets of the zonal mean density gradient and the eddy flux. In order to compare our results with their values, distribution of K_y multiplied by $\cos^2\phi$, where ϕ is latitude; they used $a \sin\phi$ as a latitudinal coordinate, where a is the Earth's radius. The results for the 8-case average during September 10–November 30, 1982 is shown in Fig. 6. An overall agreement is good. The features such as large K_y in the midlatitude lower troposphere and in the tropical upper troposphere are seen in both Fig. 6 and Fig. 8c of PM, though the latitudinal locations of the maximum in the lower troposphere are different between them. The magnitudes in both studies are also similar. In the highlatitude lower stratosphere, however, the magnitude in PM is more than 2 times smaller than that in the present study. The region of large K_y stretches across the tropical upper troposphere and the subtropical southern stratosphere in both studies. This region follows the region of weak zonal mean winds. PM suggested that it is a manifestation of the "surf zone," which is a region of breaking planetary waves.

The magnitude of K_z is about 3–4 $m^2 s^{-1}$ in our study, whereas PM derived 5–10 $m^2 s^{-1}$. Large values in PM are probably due to the inclusion of subgrid-scale vertical mixing. In our study, parcels are advected by grid-scale motion without any subgrid-scale mixing.

PM used the GCM-generated wind field which is free from observational errors, while the present study cannot be free from observational errors. On the other hand, GCMs have systematic biases compared with the real atmosphere. To estimate diffusion coefficient with the present method, the length of time-integration must be chosen properly, as mentioned in section 2. When PM's method is applied, the time-integration must be long enough for the tracer density to reach the equilibrium state. It will be very hard when real data are used.

Near the tropopause level (at about 150 mb) in middle latitudes, the estimated

K_z is as small as 1 m^2 s^{-1} or less. However, 1-dimensional models of tracer gasses usually adopt much smaller values of K_z (order of 0.1 m^2 s^{-1}) near the tropopause level for explaining the vertical distribution of tracer gases (*e.g.*, MATSUNO and SHIMAZAKI, 1981). However, in 2-dimensional models, the value is about 1 m^2 s^{-1} (*e.g.*, HIDALGO and CRUTZEN, 1977, or Fig. 12 of NODA, 1988). The reason for this discrepancy is not clear.

The estimated K_y in the tropical middle and lower troposphere is as small as 1×10^6 m^2 s^{-1} or less. However, the region of small K_z is narrow (about 20°) and the dispersion time scale across the equator is a few months which seems to be shorter than the estimated time scale of 1 year from the distribution of tracer gases. One possible reason is that the length of time integration (10 days) might be short in the tropics. NODA (1988) also mentioned this point. Another reason is as follows. Generally, the center of mass of parcels in the subtropical lower troposphere moves to the poleward region where the K_y is large (Fig. 7 of Y87). Thus the derived K_y in the subtropical latitudes might be overestimated. However, the estimated diffusion coefficients in the present study are used for a 2-dimensional tracer model and the calculated latitudinal distributions of Kr^{85} and chlorofluorocarbon accord with the observations fairly well (TANAKA *et al.*, 1988; T. NAKAZAWA, pers. commun.). This suggests that the errors are not serious. The mechanism of interhemispheric transport of tracer gasses has not yet been clarified and warrants further studies.

6. Summary

Obtained features are summarized as follows.

(1) Strong horizontal mixing ($K_y \sim 2$–3×10^6 m^2 s^{-1}) in the midlatitude lower stratosphere. Small mixing (less than 1×10^6 m^2 s^{-1}) in the tropical middle and lower troposphere. Strong mixing ($\sim 1.5\times 10^6$ m^2 s^{-1}) in the tropical upper troposphere.

(2) In the lower stratosphere, intensity of horizontal mixing varies with season. Strong mixing ($K_y \sim 1.5$–2×10^6 m^2 s^{-1}) is generally associated with enhanced planetary wave activities and/or weak zonal winds.

(3) Intense vertical mixing ($K_z \sim 3$ m^2 s^{-1}) in the midlatitude middle troposphere. Secondary maximum (~ 2.5 m^2 s^{-1}) in the tropical middle troposphere. Weak vertical mixing (less than 1 m^2 s^{-1}) at about 150 mb.

(4) Dispersion from the antarctic troposphere is very rapid and the residence time over Antarctica is about 3 days.

This study provides useful information on the large-scale transport process.

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