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ATTEMPT TO DETECT COMBINATION TONES BETWEEN THE EARTH'S FREE OSCILLATION AND THE EARTH TIDE, II —BISPECTRAL ANALYSIS—

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Abstract: Two elastic oscillations at frequencies f_1 and f_2 in an elastically nonlinear material can produce combination tones at the sum (f_1+f_2) and the difference (f_1-f_2) frequencies. If the nonlinearity in the earth's interior is sufficiently large, such phenomena can be observed in the earth's oscillations. Analyzing long-period seismograms recorded by a superconducting gravimeter installed at Syowa Station in Antarctica, we try to detect the combination tones between the earth's tides and the spheroidal free oscillations. The method employed here is bispectral analysis. Since the bispectrum represents the coherency among three components with frequencies of f_1, f_2 and f_1+f_2, it is a powerful tool to analyze the combination tones. The results suggest that the spheroidal oscillations ${}_0S_0$ and ${}_0S_5$ modes probably interact with the ter-diurnal and diurnal tides, respectively.

key words: nonlinear elasticity, the free oscillation, the earth tide, combination tone, bispectrum

1. Introduction

Since modern seismology based on linear elastic theory (Hooke's law) has achieved great success, very little attention has been paid to the nonlinear elasticity of rocks in the earth. However, there is abundant evidence indicating that large nonlinearity of the elasticity exists at least in crustal rocks. One well known manifestation of the nonlinearity is the dependence of the rock elasticity on the applied stress or strain (*e.g.* BIRCH, 1960). Another example is the result of precise field observations of the changes in elastic wave velocities. It is shown that the velocities of the rocks are not constant (linear) but are functions of the strain (*e.g.* DE FAZIO *et al.*, 1973). The strain on the order of 10^{-8} generated by earth tides gives rise to wave velocity changes that are on the order of 10^{-4}. The coefficient of nonlinearity defined as $K=\Delta c/c/\theta$ is on the order of 10^4, where c and Δc are the wave velocity and its change, respectively, under the volume change θ. Such a large coefficient is explained by the presence of compliant defects in the rocks, such as cracks and joints. The existence of such compliant defects may not be limited to the crust. One probable region, for example, is the D" region above the core-mantle boundary. Recent studies on the region suggest that it has characteristics which are different from other parts of the mantle but rather similar to those of the crust

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(*e.g.* WYSESSION, 1996). In particular, seismic anisotropy which may be attributed to the existence of melt inclusions is observed in the D" region (*e.g.* KENDALL and SILVER, 1996). If the melt inclusions work in a similar way to the compliant defects in the crust, elastic nonlinearity is also expected in the bottom of the mantle. Of course, since changes of the wave velocities are small, the resultant effect on seismic signals may be very weak. However, recent improvements in seismic observational instruments such as the SCG (superconducting gravimeter) make it possible to detect very small signals such as the incessant earth's free oscillations (NAWA *et al.*, 1998). The results give an expectation that signals generated by the nonlinearity, if any, might be detected. If we can detect the effect, it means that we obtain a new type of physical property to describe the earth's interior, the nonlinearity of the medium.

When two oscillations with different frequencies are excited in a nonlinear elastic body, the nonlinearity causes them to interact and generate new oscillations called combination tones (*e.g.*, LANDAU and LIFSHITZ, 1974). Therefore, if the nonlinearity in the earth's interior is sufficiently large, combination tones of the earth's free oscillations can be generated (ZADRO and CAPUTO, 1968; TANAKA *et al.*, 1997; after the latter paper was published, TANAKA *et al.* knew that ZADRO and CAPUTO had done a pioneering work (ZADRO and CAPUTO, 1968) to investigate the combination tones among the earth's free oscillations.).

TANAKA *et al.* (1997) investigated the combination tones between the earth tides and the earth's free oscillations (from now on refered to as TF-modes; Tide and Free oscillation combination tones). Since the amplitudes of the earth tides are very large, it is expected that such combination tones will be more efficiently excited than those generated by only the free oscillations themselves. TANAKA *et al.* (1997), as a first attempt, employed autocorrelation analysis of the spectra of seismic records. The analysis shows that peaks of the autocorrelation functions could be explained by the possible existence of the TF-modes among the earth's free oscillation modes.

The purpose of the present paper is to confirm the previous results by another method. In this study, we employ bispectral analysis (HINICH and CLAY, 1968).

2. Bispectra

In this section, we briefly describe bispectrum analysis, following mainly HINICH and CLAY (1968). Consider a discrete stationary process $\{x_n\}$ with a sampling interval of Δt, where $x_n = x(n\Delta t)$ $(n=1, 2, \cdots)$. Let $X(f)$ and $B(f_1, f_2)$ denote the Fourier spectrum and the bispectrum of the process, respectively.

The bispectrum $B(f_1, f_2)$ is defined as the two dimensional Fourier transform of a second order autocovariance function.

$$B(f_1, f_2) = \sum_{\sigma}\sum_{\tau} \rho_{\sigma,\tau} \exp[-2\pi i(f_1\sigma + f_2\tau)],$$

where

$$\rho_{\sigma,\tau} = E(x_t x_{t+\sigma} x_{t+\tau}).$$

This is equivalent to the ensemble mean of three Fourier transforms

$$B(f_1, f_2) = E\big(X(f_1)X(f_2)X^*(f_1+f_2)\big),$$

where * denotes the complex conjugate. These equations are similar to the equations relating the power spectrum, Fourier spectrum, and the autocovariance function.

If $\{x_n\}$ is a Gaussian process, its bispectrum is always zero, since the autocovariance is zero for all σ and τ. On the other hand, if the process is nonlinear, the bispectrum is not identically zero.

Consider that there are prominent components in $\{x_n\}$ with frequencies f_1 and f_2 and phases $\phi(f_1)$ and $\phi(f_2)$. If the process is generated through a nonlinear system, the combination component at f_1+f_2 appears and its phase is

$$\phi(f_1+f_2) = \phi(f_1) + \phi(f_2) - \theta,$$

where θ is nearly constant over time; the phase is coherent with the sum of the phases of the original components. Such coherence gives a non-zero value of $B(f_1, f_2)$, because

$$X(f_1)X(f_2)X^*(f_1+f_2) = |X(f_1)X(f_2)X(f_1+f_2)| \exp(-i\theta).$$

Since the phase θ is nearly constant, the result of multiplication of the three Fourier transforms is also nearly constant and then its ensemble mean is not zero.

A normalized equivalent of the bispectrum called the bispectrum skewness is defined as

$$\rho(f_1, f_2) = \frac{|B(f_1, f_2)|}{\big(S(f_1)S(f_2)S(f_1+f_2)\big)^{1/2}}, \qquad (1)$$

where $S(f)$ is the power spectrum of $\{x_n\}$. If the component at $f=f_1+f_2$ is originally a combination tone between the components at f_1 and f_2, the skewness $\rho(f_1, f_2)$ equals 1.

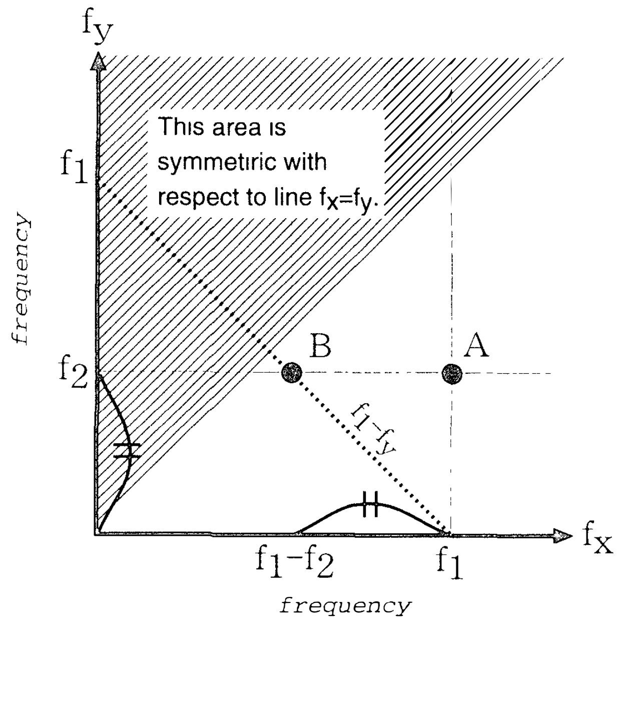

Fig. 1. How bispectral skewness is interpreted. An obvious peak A (f_1, f_2) shows both the existence of three waves at f_1, f_2, f_1+f_2, and the waves are coherent. Simultaneously, a peak B(f_1-f_2, f_2), which is at the same combination level with peak A, may appear.

Since the bispectrum skewness is a function of the two frequencies, f_1 and f_2, it can be depicted as a map on the plane (f_1, f_2). Because of the symmetry properties of $B(f_1, f_2)$, such as $B(f_1, f_2)=B(-f_1, -f_2)$, we follow the custom for the skewness ρ (f_1, f_2) to be plotted in only one octant of the plane (f_1, f_2). If there is a combination component at frequency f_1+f_2, a peak of skewness at the coordinate (f_1, f_2) appears (point A in Fig. 1). On the other hand, if there is a combination component at frequency f_1-f_2, a peak appears at the coordinates (f_1-f_2, f_2) (point B in Fig. 1). This peak comes from the fact that the components at f_1-f_2 and f_2 must be coherent with the component of $(f_1-f_2)+f_2=f_1$.

3. Data and Analysis

The bispectrum method we introduced in Section 2 focuses on a specific frequency band which contains a spheroidal mode. If possible, the ${}_0S_0$ mode without splitting is ideal. Actually, such new event has once occurred and we can utilize the record. Seismograms of the Offshore Eastern Hokkaido, Japan, earthquake (October 4, 1994, M $=8.1$) were recorded by a SCG at Syowa Station in Antarctica. A summary of SCG continuous measurements system at Syowa Station is given in SATO *et al.* (1995). This event efficiently excited the earth's free oscillations, and they were clearly recorded by the gravimeter. Particularly, the amplitude of the ${}_0S_0$ mode was very large; ordinary spectral analysis of the record shows that its clear spectral peak can be recognized for several tens of days after the event (corresponding nearly to 560–610 days in Fig. 3 in NAWA *et al.* (1998)).

Before processing, the record was filtered by an anti-alias low pass filter, and then the original seismogram with a sampling interval of 2 s was resampled with an interval of 60 s. We analyze the data for about 50 days after the earthquake (October 5–November 20).

The bispectra from the finite records is estimated by using the ordinary discrete Fourier transform. We used 37 records with a length of 11 days whose start times lag one day from the previous one. The Fourier transform of each record was taken. The average value of $X(f_1)X(f_2)X^*(f_1+f_2)$ for given frequencies of f_1 and f_2 was computed. It is expected that the average values are good representations of the real bispectra (HINICH and CLAY, 1968). Then, we calculated bispectral skewness by eq. (1).

In this study, we investigate TF-modes in frequency ranges of $f_1=770\sim850\,\mu$Hz and $f_2=0\sim40\,\mu$Hz. The latter frequency range covers the earth's diurnal ($f_2\approx11$ μHz), semi-diurnal ($f_2\approx23\,\mu$Hz), and ter-diurnal tides ($f_2\approx34\,\mu$Hz). On the other hand, in the frequency range from about 650 to 1000 μHz there are only two modes of the earth's free oscillation, ${}_0S_0$ and ${}_0S_5$, whose frequencies are $f_1=814.3\,\mu$Hz and 840.3 μHz, respectively (PREM; DZIEWONSKI and ANDERSON, 1981). As stated above, since the signal of the ${}_0S_0$ mode was well recorded on the seismogram, the f_1 frequency band appears to be the best for the detection of TF-modes.

4. Results

Figure 2 shows the bispectral skewness diagram in the analyzed area. The horizontal dashed and vertical dotted lines indicate the frequencies of the relevant oscillation modes, the earth's tides and free oscillation ${}_0S_0$ and ${}_0S_5$ modes. If there are combination modes whose frequencies are the sums of their frequencies, skewness peaks appear at the point of intersection between the corresponding lines. The inclined dotted lines show the expected frequency of the combination mode whose frequency is the remainder of the relevant oscillations. If a peak appears at an intersection between one of the inclined dotted lines and any one of the horizontal dashed lines, it means that a TF-mode between the tidal component and the free oscillation mode is included in the signal.

Many peaks of skewness are observed on the bispectral skewness diagram, in particular at the ordinate frequencies f_2 of the earth tides (f_2 equals about 11, 23, and 34 μHz). However, excepting some peaks that will be discussed below, few of the peaks may be attributed to TF-modes, since they do not exist on the intersecting points.

One peak representing a TF-mode stands at f_1 = about 840 μHz and $f_2 = 11$ μHz. As shown in Fig. 2 (P1), this peak exists nearly at the intersection of the lines of the ${}_0S_5$ mode and the diurnal tide.

Furthermore, a pair of skewness peaks corresponding to the combination between the ter-diurnal tide and ${}_0S_0$ mode is observed. One peak (P2 in Fig. 2) is at frequency about $f_1 = 780$ μHz, which equals the remainder of the frequencies (814-34) μHz, and the other (P3 in Fig. 2) is at the coordinates (814 μHz, 34 μHz), though its amplitude is relatively small.

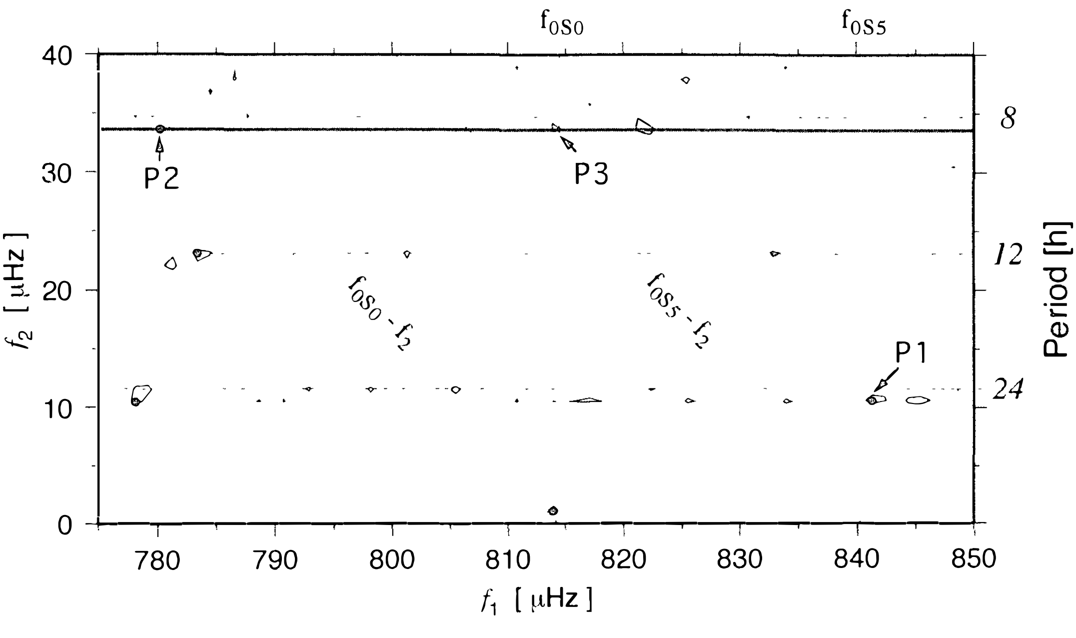

Fig. 2. Bispectral skewness diagram from seismograms of the Offshore Eastern Hokkaido earthquake. Small encircled regions and solid circles represent coordinates whose skewnesses are larger than 0.7 and 0.9, respectively. The shaded line at $f_2 = 33.7$ μHz corresponds to Fig. 3. For explanation of dotted and dashed lines, see text.

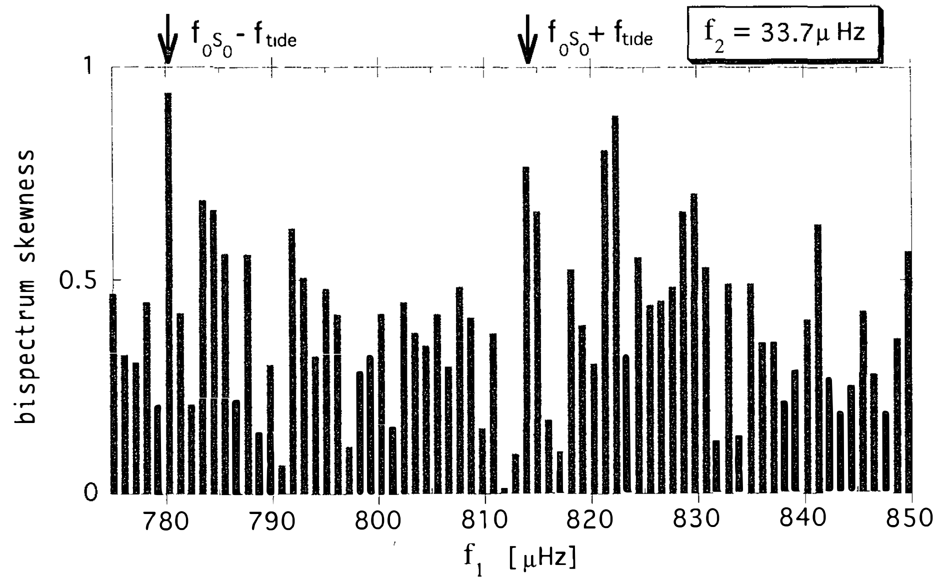

Fig. 3. Profile of bispectral skewness diagram at the frequency of the ter-diurnal $f_2=33.7\,\mu$Hz (shaded line in Fig. 2). Arrows indicate the expected positions of the two combination tones between ${}_0S_0$ modes and the ter-diurnal tide.

Figure 3 shows the profiles of the shaded line in Fig. 2. This skewness is depicted as a function of the abscissa frequency f_1. From visual inspection, the peaks at $f_1=780$ and $814\,\mu$Hz are very sharp. These features probably suggest the existence of the TF-mode.

5. Summary

In this paper, we have applied bispectrum analysis to the detection of combination tones between the earth tides and the earth's free oscillations, *i.e.*, TF-modes. Bispectral analysis is useful in studying combination tones generated through nonlinear processes. On the bispectral skewness diagram, there are clear peaks that correspond to the TF-mode between ${}_0S_0$ mode and the ter-diurnal tide and the ${}_0S_5$ mode and the diurnal tide.

From the results obtained here, combined with a previous study (TANAKA *et al.*, 1997), we argue that combination modes between the earth tides and the earth's free oscillations are excited in the earth. Nevertheless, we also recognize that the persuasiveness of the results still remains weak; the existence of the combination modes must be considered as a hypothesis that needs further testing, for example, why the ${}_0S_0$ mode is easier to couple with ter-diurnal tide than semidiurnal tide.

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