

## Impact of satellite gravity missions on glaciology and Antarctic Earth sciences

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**Abstract:** Satellite gravity missions in the 21st century are expected to be beneficial to multi-disciplinary scientific objectives. Especially, the Gravity Recovery And Climate Experiment (GRACE) and its follow-on missions will provide not only data for precise gravity mapping but also time series of global gravity field coefficients at intervals of about 15 days to two months. These data are precise enough to reveal the temporal variations of the gravity fields due to mass redistribution in and on the Earth. From the viewpoint of Earth sciences in the Antarctic region, the data are expected to contribute to studies of ice sheet mass balance and postglacial rebound as well as other geodetic and geophysical problems. These issues have been mainly investigated based on the degree variance analyses of the gravity field so far. In this paper, we briefly review the gravity mission data from the viewpoint of along track geoid height variations which are more direct results of the mass variations, and then discuss some of the issues related to *in-situ* observations.

### 1. Introduction

Starting with the successful launch of the CHAMP (Challenging Mini-satellite Payload) satellite on July 15th, 2001, successive gravity missions of GRACE (Gravity Recovery And Climate Experiment) in 2002, GOCE (Gravity field and steady-state Ocean Circulation Explorer) in 2005, and GRACE-FO (Follow On) in 2007 will open a new era of satellite gravimetry. Historically, a large number of satellite tracking data obtained by primarily SLR (Satellite Laser Ranging) and/or other Doppler tracking techniques, have been employed to determine a geopotential model, *i.e.*, a set of coefficients of the solid spherical harmonic expansion of the static earth's gravity field (the Stokes' coefficients). However, the coefficients obtained by ground base tracking techniques are restricted to only long wavelength (low degrees) parts of the gravity fields because of the limitation of spatial coverage of the ground base tracking system and the non-gravitational forces which disturb the free fall motion of the satellite.

The dedicated satellite gravity missions are designed to overcome these problems by employing a concept of High-Low Satellite-to-Satellite Tracking (H-L SST) and a three-axis accelerometer or an equivalent drag-free control mechanism. CHAMP is the first satellite based on this concept (Geo Forschungs Zentrum (GFZ), 2002; The CHAMP Mission, <http://op.gfz-potsdam.de/champ/index-CHAMP.html>) and a preliminary model of the Earth gravity field has already been released (Reigber *et al.*,

2002). Although the improvements made by CHAMP over the current standard gravity model, e.g. EGM-96 (Lemoine *et al.*, 1998), lie at harmonic degrees less than 25, it fully proves the conceptual validity of future gravity missions.

Further improvements of the gravity field recovery can be made by Low-Low Satellite-to-Satellite Tracking (L-L SST) or Satellite Gravity Gradiometry (SGG) together with the H-L SST configuration. The first L-L SST mission is the US-German GRACE (Center for Space Research (CSR), 2002; GRACE, <http://www.csr.utexas.edu/grace/>) and the SGG mission will be the GOCE of ESA (European Space Agency) (C. Grill, 1999; From Eötvös to mGal, <http://www.cis.tugraz.at/mggi/goce/goce0-root.html>). These missions will dramatically improve the accuracy of the Stokes' coefficients up to degree and order around 200 to 250 which correspond to a spatial half-wavelength of 100 km to 80 km. Moreover, GRACE will provide a time series of Stokes' coefficients and it will enable us to study temporal variations of the Earth's gravity fields due to mass redistribution in and on the Earth. These new data sets will contribute to the understanding of various problems in the Earth sciences, *i.e.*, meteorology, hydrology, oceanography, glaciology as well as the solid Earth sciences (NRC, 1997; Wahr *et al.*, 1998; Fukuda, 2000). Especially, in the Antarctic region and/or in Greenland, total ice mass balance and postglacial rebound are problems of top priority, because both are major uncertainties in predicting global climate changes and sea level rise. The data sets will impose a strong constraint on these issues in terms of total mass conservation (Wahr *et al.*, 2000).

The Antarctic region is also expected to become a promising CAL/VAL (Calibration/Validation) field (Shum *et al.*, 2001). From this point of view, it is also important to make clear the relation between the satellite data and ground base (*in-situ*) observations. Many simulation studies conducted so far (*e.g.*, Wahr *et al.*, 1998; Fukuda and Foldvary, 2001) were based on the Level 2 data products which include the coefficients of the geopotential field, the satellite position and velocity, and related geophysical products. However, the Level 2 data are essentially frequency domain quantities and they are not so convenient for direct comparison with *in-situ* observations. Hence, in this study, we first present several simulation results for Level-1 data products, which include the line-of-sight range change and its derivatives, assuming a typical GRACE like orbit and its specifications. Then we will discuss future utilization of the satellite gravity data, especially related to *in-situ* observations and/or precise gravity measurements.

## 2. L-L SST

The principle of the L-L SST is rather simple. Two LEOs (Low Earth Orbiters) are in essentially the same orbit and a distance of somewhere between 50 and 500 km apart, and one of the LEOs chases the other. The relative motion or a range rate between the LEOs is measured as precisely as possible by means of a microwave radar link (in case of GRACE) or an optical laser link (in case of GRACE-FO). If the effect of non-gravitational forces acting on the two LEOs can be measured and/or properly compensated for, the relative motion of the center of mass gives information on the Earth's gravitational field.

Let  $V$  be a gravitational potential,  $\hat{v}$  the range rate measured at satellite altitude, and  $E$  the total energy of the LEOs, then the law of conservation of energy is given by

$$\frac{1}{2} \hat{v}^2 - V = E. \quad (1)$$

Decomposing  $V$  and  $\hat{v}$  into a reference component ( $v_m$ ,  $U$ ) and a signal part ( $v$ ,  $T$ ) as follows:

$$\begin{aligned} \hat{v} &= v_m + v, \\ V &= U + T, \end{aligned}$$

and neglecting second-order terms, we finally obtained a simple equation which gives the relation between the range rate  $v$  and geoid height  $N$  as follows:

$$v \cong \frac{T}{v_m} = \frac{\gamma(R+h)}{R} N, \quad (2)$$

where  $R$  is the mean radius of the Earth,  $\gamma$  is the normal gravity or an average value of gravity on the surface of the Earth, and  $h$  is the altitude of the satellite orbit (Jekeli and Rapp, 1980). Note that the geoid height  $N$  at the satellite altitude is ambiguous, but we define it, from analogy of the famous Bruns' formula (Heiskanen and Moritz, 1967), so as to be equal to the disturbing potential  $T$  at the satellite altitude divided by the normal gravity at the Earth's surface. It is also noted that the L-L SST, which is essentially based on measurement of the along track range rate  $v$ , is only an approximate measure of the geopotential difference; the eq. (2) gives approximate relations, accordingly. This issue is discussed in detail by Jekeli (1999).

GRACE, which was successfully launched on March 18th, 2002, is the first experiment of the L-L SST configuration. It employs a K-band inter-satellite radar link, and measures phase differences with the sampling rate of 10 Hz. These raw data (Level 0 data) will be processed at Jet Propulsion Laboratory (JPL) to produce so-called Level 1 data, *i.e.*, biased range and its derivatives, every 5 s. The accuracy of the range rate  $v$  will be better than  $1 \mu\text{m/s}$ , which corresponds to the geoid error of 0.78 mm if we assume a nominal orbit altitude  $h=450\text{ km}$  in eq. (2). In the following chapters, we will give some simulation results of the gravity field variations in terms of geoid height variations. Here, it is noted that 1 mm geoid height ( $N$ ) variation corresponds to  $1 \mu\text{m/s}$  range rate ( $v$ ) which can be observed by GRACE; and that the sensitivity of the GRACE-FO, which will employ Satellite-to-Satellite laser Interferometry (SSI) for the range rate measurements, is better than the sensitivity of GRACE by 2 to 3 orders of magnitude.

### 3. Simulation of gravity field variations

#### 3.1. Static gravity field

We first tried to simulate the static gravity field. For this purpose, we calculated geoid heights at altitude 450 km as well as at ground level using the EGM 96 geopotential model. Figures 1a and b show the geoid heights at the surface of the Earth and at 450 km height, respectively. Although geoid signals of degree  $l$  are damped by the factor  $(R/(R+h))^l$  at satellite altitude  $h$ , Fig. 1b has a similar spatial pattern to that of Fig. 1a. This is because the geoid has more power at longer wavelength.

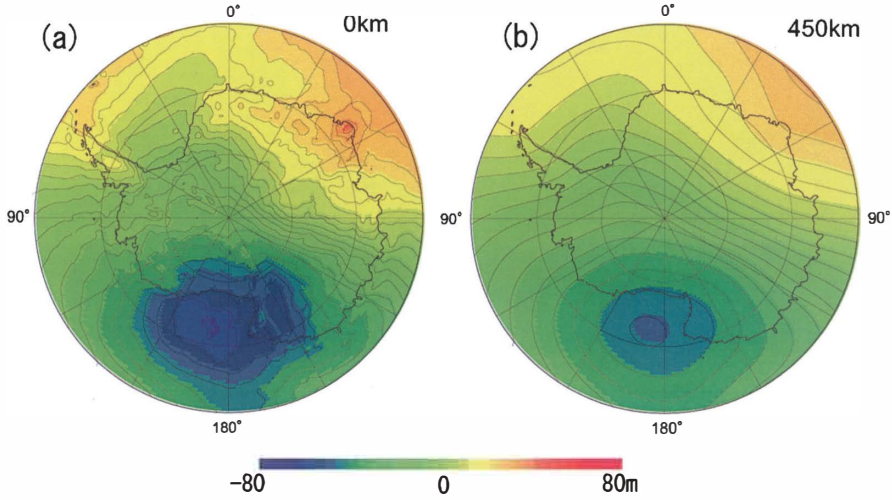


Fig. 1. Geoid heights calculated using the EGM-96 model. (a) At the surface of the Earth, (b) at a height of 450 km. Contour interval is 5 m.

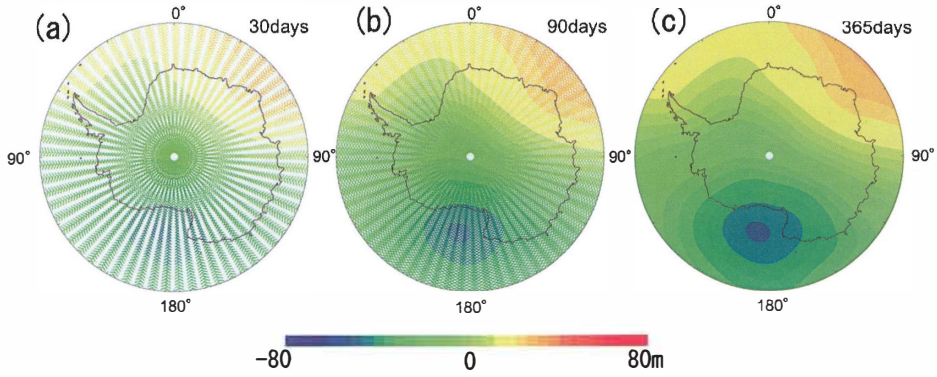


Fig. 2. Gravity field recovery after (a) 30 days, (b) 90 days, and (c) 365 days. A GRACE like orbit of altitude = 450 km, inclination = 89 degrees and eccentricity = 0.005, is assumed.

We next simulated the gravity field recovery after 30 days, 90 days and 365 days of measurements, assuming simple Kepler motion of the satellite with GRACE-like orbit parameters, *i.e.*, altitude of 450 km, inclination of 89 degrees and eccentricity of 0.005. The simulation has been practically conducted by calculating the satellite position at every 10 s, and interpolating the geoid height at the satellite position from the grid values of Fig. 1b. Figures 2a, b and c show the mapping results after 30 days, 90 days and 365 days, respectively. Though Fig. 2a has some spatial gaps, this does not directly mean that 30 days of sampling is not enough for the studies of gravity field variations, because the satellite measures gravity effects not only from its foot print but also from

surrounding areas. Figure 2c, on the other hand, shows that the pattern of Fig. 1b is completely recovered from the 365 days of mapping.

The importance of the static gravity field recovery lies mainly in two respects. First, as is well known from the traditional gravity anomaly, the gravity data reflect the shape of the bedrock and/or the Moho discontinuity. This issue will be discussed later. Second, though less familiar, the geoid provides the reference surface for the gravity-driven ice flow. Recent space geodetic technologies, such as GPS, SAR (Synthetic Aperture Radar) interferometry, and/or ICESAT (Ice, Cloud and land Elevation Satellite) GLAS (the Geoscience Laser Altimeter System) (Schutz, 1998) laser altimetry enable us to map very precise ice sheet topography. However, the positions obtained by those techniques are referred to the Earth's center of the mass, while the ice flow is driven by pressure gradients parallel to the geoid. This is completely analogous to the relation between sea surface dynamic topography and ocean geoid.

The gravity mission data will contribute to the static gravity fields recovery in longer wavelengths than 100 km. However, for shorter wavelengths, the attenuation of gravity signals due to high satellite altitude terribly degrades the performance of the satellite measurements. At shorter wavelengths, combination with airborne gravimetry will be important (Bell *et al.*, 1998).

### 3.2. Temporal variation of the gravity field

It is known that hydrologic, atmospheric and oceanic mass variations cause temporal geoid variations of the order of several mm to 10 mm. These mass variations remain essentially unknown, and therefore, they are the main targets of the satellite gravity missions. Among these phenomena, the atmospheric variation is relatively well measured, and re-analysis or operational analysis data sets are released from several organizations. Thus, we employed the surface pressure data for the purpose of demonstrating temporal variation of the geoid heights due to mass redistribution in the fluid envelope of the Earth.

Let us suppose that atmospheric mass variation is approximated by surface density anomaly  $\Delta\rho(\theta, \lambda, t)$ , then a time series of the variations of Stokes' coefficients ( $\Delta C_{l,m}$ ,  $\Delta S_{l,m}$ ) can be given by the following equation (Chao and Au, 1991):

$$\begin{pmatrix} \Delta \overline{C_{l,m}}(t) \\ \Delta \overline{S_{l,m}}(t) \end{pmatrix} = \frac{3}{4\pi} \frac{1+k'_l}{2l+1} \frac{1}{R\rho_{\text{ave}}}_{\text{Earth}} \iint \Delta\rho(\theta, \lambda, t) P_{l,m}(\cos\theta) \begin{pmatrix} \cos m\lambda \\ \sin m\lambda \end{pmatrix} d\sigma, \quad (3)$$

where  $k'_l$  is the load Love number of degree  $l$  (Farell, 1972), and  $\rho_{\text{ave}}$  is the average density of the Earth. Once the Stokes' coefficients are obtained, arbitrary gravity field quantities can be easily calculated by composition of the coefficients.

The surface pressure data employed are advanced operational analysis data sets between January 1st to December 31st in 1999 provided by ECMWF (European Centre for Medium-Range Weather Forecasts). To calculate the surface mass density variations, we first subtract the average value of the year at each grid point from the original surface pressure values. Then the deviations from the averages are divided by a mean gravity value ( $9.8 \text{ m/s}^2$ ) to convert the unit from surface pressure to density anomaly  $\Delta\rho(\theta, \lambda, t)$ .

In calculation of the atmospheric pressure effect on the gravity field, we must pay



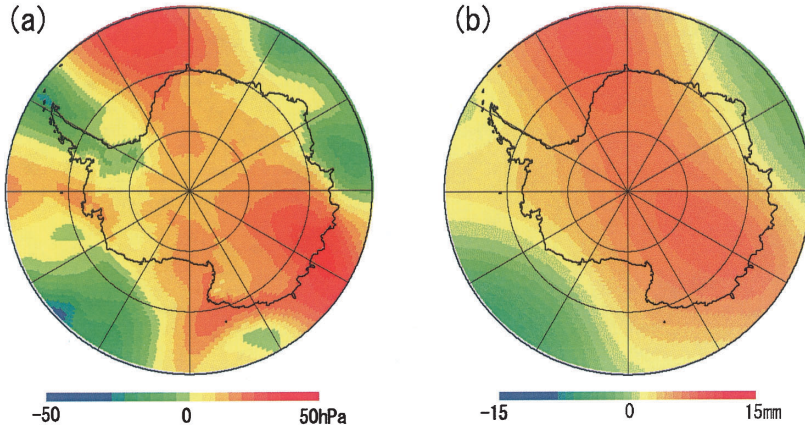


Fig. 3. Geoid height variations due to atmospheric effects. (a) Surface pressure variations and, (b) corresponding geoid height variations.

attention to the assumption of the oceanic response. Usually, one of two basic cases, *i.e.*, inverted barometer (IB) or non-inverted barometer (NIB), is assumed, while the reality is between those two extreme cases (Fordvary and Fukuda, 2001). In this study we assumed the NIB hypothesis because we included the pressure effect every 6 hours, which seems too fast for the IB response.

Figure 3a shows a snap shot of the surface pressure variation on June 14th, 1999 when a high pressure area was starting to cover the whole of Antarctica. Figure 3b is the corresponding geoid height variation map. Because the calculation of the geoid is a kind of low pass filtering, the long wavelength feature of Fig. 3a is emphasized in Fig. 3b. These figures show that a peak-to-peak surface pressure variation of about 70 hPa causes about 15 mm geoid variation in the spatial scale of the Antarctic continent. This amount of geoid variation can be easily observed by GRACE, not to speak of GRACE-FO or a SSI mission which has better sensitivity by 2 to 3 orders than that of GRACE. Therefore the atmospheric effects should be carefully removed before using the gravity mission data for the other applications. This in turn means that a gravity mission, especially an SSI mission, will have the capability of a very precise barometer (resolution of 0.01 hPa or better), although its practical utilization remains for the future.

### 3.3. Loading effects of the ice sheet mass

Next, we estimated the gravity effects of loading mass on a certain area. For this purpose we assume the two cases shown in Figs. 4a and b. In the case shown in Fig. 4a, a mass of 1-m equivalent water thickness is loaded on a  $5^\circ \times 10^\circ$  area,  $70^\circ$ – $75^\circ$ S and  $35^\circ$ – $45^\circ$ E. In the case shown in Fig. 4b, the same thickness of water mass is loaded on a  $1^\circ \times 2^\circ$  area,  $70^\circ$ – $71^\circ$ S and  $39^\circ$ – $41^\circ$ E. For both cases, the same amount of negative mass corresponding to the loaded mass is homogeneously distributed outside the area to ensure total mass conservation.

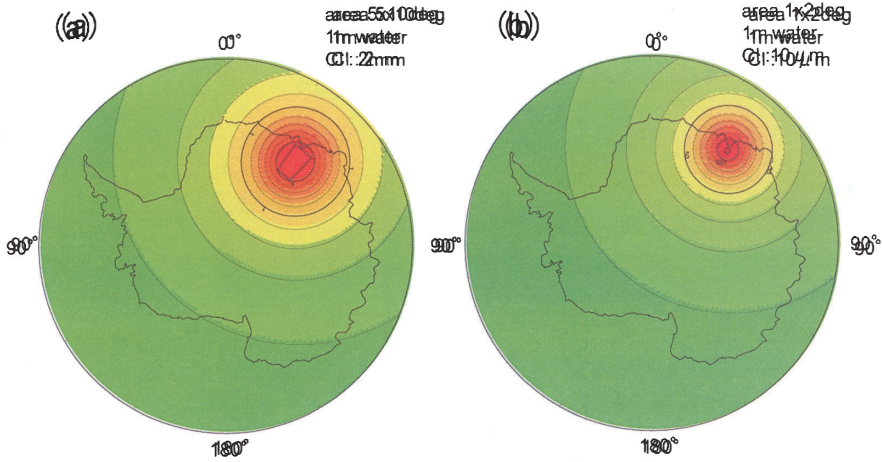


Fig. 4. Geoid height changes due to ice sheet mass loading. The loaded areas are shown by rectangles, and the ice thickness is set to 100 cm equivalent water. The contour intervals are 2 mm for (a), and 10  $\mu$ m for (b).

The contour lines in the figures show the geoid height variations due to mass loading at the satellite altitude, where contour intervals in Figs. 4a and b are 2 mm and 0.01 mm, respectively. The GRACE mission can easily observe the geoid variations in Fig. 4a. Because the sensitivity limit of GRACE is about 1 mm geoid variation, GRACE can detect variations of about 10 cm water thickness at this spatial scale. The detection of the variations in Fig. 4b or one order smaller variations will be the sensitivity limit of an SSI mission, for example GRACE-FO.

The calculation of the geoid height anomalies due to density contrast is almost the same as in the case of the loading effects. The only difference is omission of the load Love number terms. The loading terms are negligibly small on the spatial scale of the figures, so that the results shown in Figs. 4a and b can be easily translated to a stationary geoid signal due to basement topography under the ice sheet and/or deeper in the crustal. If we assume the density contrast between the ice sheet and the basement to be 1.7 g/cm<sup>3</sup>, and that between the crust and the upper mantle to be 0.3 g/cm<sup>3</sup>, then the equivalent thickness of the ice sheet or the crust is 0.5 m and 3 m respectively. These values can be considered to be the sensitivity of the GRACE mission in the half-wavelength of 1000 km when detecting underground density structures.

#### 4. Linkage with surface gravity measurements

As discussed in Section 3, GRACE and GRACE-FO can detect a 10 cm ice sheet mass change in 1000 km and 100 km spatial wavelengths, respectively. Assuming an infinite Bouguer plate, the 10 cm ice sheet mass change will cause gravity changes of about 4  $\mu$ gal on the Earth's surface. This amount of gravity change can be detected by a superconducting gravimeter (SG). However, the SG at Syowa Station is essentially not portable. Thus it is practically impossible to use the SG data for the purpose of

calibration of the satellite gravity mission data. Note that this does not decrease the importance of the SG observations. The SG is a unique tool which can detect the time variation of the gravity field continuously and very precisely. Therefore the SG data are important for modeling the time variation of the gravity field.

Regarding glaciological applications as well as calibration of the satellite data, very high precision field measurements are much more important. Traditional spring type gravimeters, *e.g.* LaCoste and Lomberg and/or Scintrex meters, are insufficient in their accuracy and stability. Thus absolute gravity measurements are essential for these studies. A recent absolute gravimeter, *e.g.*, the FG-5, has attained the accuracy of  $1-2\mu\text{gal}$  under laboratory conditions (Amalvict *et al.*, 2001), while the accuracy of a portable type absolute meter, the A-10, reaches around  $10\mu\text{gal}$ . This value is equivalent to a 25 cm ice sheet mass change. Thus, if the number of measurement points can be increased and appropriate spatial coverage attained, the data will be compared/merged with the satellite gravity data.

When we observe a gravity change on the ice sheet, we must take care that the measurement points themselves are moving due to the ice sheet flow. Therefore, precise positioning of the points and the measurements of gravity gradients around the points are required. Considering these issues, we propose a recommended gravity measurement site, which consists of one absolute gravity point and several surrounding relative gravity points, as configured in Fig. 5. In each site, absolute gravity measurements and precise GPS measurements should be conducted at the absolute gravity point; meanwhile, precise relative gravity measurements can be carried out to determine the horizontal gravity gradient at the site using a spring type gravimeter with a kinematic GPS positioning system. The size of the site should be determined considering the

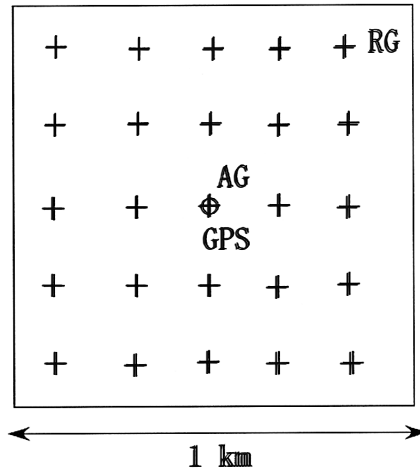


Fig. 5. A schematic illustration of a proposed gravity measurement site. The site consists of an absolute gravity measurement point ( $\odot$ : AG), a GPS point ( $\odot$ : GPS) and several relative gravity points ( $+$ : RG) which are employed to determine the horizontal gravity gradient around the absolute point.



speed of the ice sheet flow and the degree of the gravity gradient, but it would be less than  $1 \text{ km}^2$ , and 25 relative gravity points at maximum should be enough to determine the horizontal gravity gradient.

The absolute gravity measurements and precise GPS measurements will take one day or so to ensure the required accuracy of better than  $10 \mu\text{gal}$  and  $1 \text{ cm}$ , respectively. During these continuous measurements, which can be conducted basically without operators, the relative gravity measurements can be carried out. Consequently, a combination of absolute and relative gravity measurements as well as precise GPS positioning at a site can be completed within a couple of days, preferably by two surveyors.

If such measurements are conducted over several hundred km span with intervals of 10–50 km, the data should contribute not only to the CAL/VAL of the satellite data, but also to more detailed studies of ice sheet movements.

## 5. Concluding remarks

We introduced the basic concept of the satellite gravity missions, especially GRACE based on L-L SST. As already discussed in Wahr *et al.* (2000), the contributions of those missions to the continental scale mass balance is the most promising, because ground based observations can hardly contribute the study of such large scale phenomena. In shorter wavelength variations on regional or local scales, appropriate combinations of absolute gravity measurements, relative gravity measurements and GPS positioning have the essential importance for the utilization of the satellite missions data. Moreover, additional information, such as the compaction rate of the snow, can be obtained only from *in-situ* observations. There is no doubt that satellite gravity missions will bring about a revolution in various disciplinary objectives. However, the gravity mission data obtained are essentially potential field data and their interpretation is a kind of inverse problem. In other words, interpretation and/or analysis of the data require various *in-situ* observations as constraints. The most important point is to find an efficient combination of those different types of data sets, and it consequently requires interdisciplinary collaboration to understand well the phenomena and the characteristics of both satellite and *in-situ* observations.

## Acknowledgments

We would like to thank two anonymous reviewers for their critical reading of the manuscript and useful suggestions for its improvement. This study was partially supported by a Grant-in-Aid for Scientific Research from the Japanese Society for the Promotion of Science (11440132).

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*(Received February 7, 2002; Revised manuscript accepted April 15, 2002)*