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NNSS TRANSLOCATION TEST IN THE KANTO DISTRICT WITH SPECIAL APPLICATION TO ANTARCTIC RESEARCH PROGRAM

Kazuo SHIBUYA[1] and Yukihito KITAZAWA[2]*

[1]*National Institute of Polar Research, 9-10, Kaga 1-chome, Itabashi-ku, Tokyo 173*
[2]*Department of Earth Sciences, Ibaraki University, 1-1, Bunkyo 2-chome, Mito 310*

Abstract: Simultaneous receiving of NNSS satellites was made at the three stations, Tsukuba, Dodaira and Tokyo during 19-26 May 1984 and performance of translocation was tested with changed combination of reference points. The Cartesian coordinates of these receiving points are determined within 2 m error in each axis by the precise ephemeris point-positioning and the test is made by estimating the radial offset between the translocation solution point and the above-specified standard point. Since the translocation is based on the broadcast ephemeris system and the standard points are in the precise ephemeris system, coordinate transform is required before direct comparison of the station coordinates. Tests showed that LeRoy's transform formula is appropriate in the region concerned, though 1.5-2.0 m transform discrepancy has to be considered. Since the translocation is a relative positioning technique of one station with respect to the other reference point, the error in the reference point may affect on the accuracy of translocation solution. Tests showed that when the "bug station" with large coordinate errors was used as the reference point, the involved errors transferred to the coordinates of the translocation solution point. It is thus imoprtant to use the reference point which is determined as accurate as possible. When the translocation is made relative to thus determined standard point, the convergence of the translocation solution can be obtained with a smaller number of satellite passes. The necessary passes to maintain 5 m absolute accuracy including uncertainty in the coordinate transform are found to be reduced under 10; the number is about one-fourth of the necessary passes for the same convergence in case of broadcast ephemeris point-positioning. The operational difficulty in the Antarctic field research programs may be lightened because staying time at one receiving site will be reduced within 24 h by an application of translocation technique.

1. Introduction

Since there is no time-invariant configuration map of the Antarctic ice sheet, accurate and speedy positioning is the main difficulty in geophysical studies on Antarctica. The NNSS (Navy Navigation Satellite System) positioning can give us reasonable estimate of the station coordinates of the receiving point without any reference point, and so it has widely been utilized for oversnow navigation and for the study of ice sheet flow (*e.g.* YOUNG, 1979; SHIBUYA, 1986) and the cartography of outcrop areas (SOUTHERD, 1983).

* Present address: Ishikawajima-Harima Heavy Industries Co., Ltd., Space Development Division, 229, Tonogaya, Mizuho-machi, Nishitama-gun, Tokyo 190-12.

The convergence of NNSS point-positioning can be estimated by taking the accepted satellite pass number N as a parameter. For example, SHIBUYA (1985) made the performance experiment of a two-wave NNSS receiver JMR-1 (JMR INSTRUMENTS INC., 1977) around Syowa Station, East Antarctica, and showed that the convergent process of the iterative point-positioning using broadcast ephemeris can be approximated by

$$\Delta d \sim \frac{15}{\sqrt{N}}, \qquad (1)$$

where Δd is the radial offset measured in meter between the iterative station location and the final convergent solution point. When the absolute accuracy of broadcast ephemeris point-positioning (hereafter denoted as BE positioning) is estimated by referring to the precise ephemeris point-positioning (hereafter denoted as PE positioning) result, the error estimate $\Delta\delta$ in meters is found to obey LEROY's (1982) formula:

$$\Delta\delta = \frac{37}{\sqrt{N}}. \qquad (2)$$

Equation (2) indicates that, if we require 5 m accuracy in BE positioning, a total of 55–60 suitable passes has to be recorded and post-processed. In order to receive 60 suitable satellite passes, it often requires to stay 2.5–3 days at the receiving site in the Antarctic region or 4–5 days in mid-latitude with the 5 satellites' configuration.

The Japanese Antarctic Research Expedition (JARE) has been operating concentrated earth science surveys in the Sør Rondane Mountains, East Queen Maud Land. Antarctic summer field season is so limited (December 1–January 31) that staying 3 days at one receiving site is a strong restriction for field operation. If the necessary satellite passes can be reduced without degrading the 5 m absolute accuracy, it must be a benefit for all Antarctic research programs.

From this viewpoint, we made the performance experiment of the broadcast ephemeris translocation (hereafter denoted as only translocation) by selecting appropriate receiving stations with the known coordinate values (called standard point) in the Kanto district. We calculate the radial offset between the translocation solution point and the standard point and see how many passes are required to maintain 5 m accuracy, and see if the errors in the station coordinates of the reference point may transfer to the translocation solution point.

2. Receiving Experiment

Figure 1 illustrates station locations in the receiving experiment. The two-wave NNSS receiver JMR-1 was installed at the geodetic control points of the Dodaira Astronomical Observatory (DDR) and of the National Institute of Polar Research (NIPR), Tokyo. Another type of the two-wave receiver JMR-4A was installed at the geodetic control point of the Astronomical Laboratory in the Geographical Survey Institute, Ministry of Construction (TSK). JMR-4A is an improved type of JMR-1 for more accurate recovery of time frame (ICENBICE and LOILER, 1979). The coordinate values of these standard points are summarized in rows 7–8 of Table 1. For all of the sta-

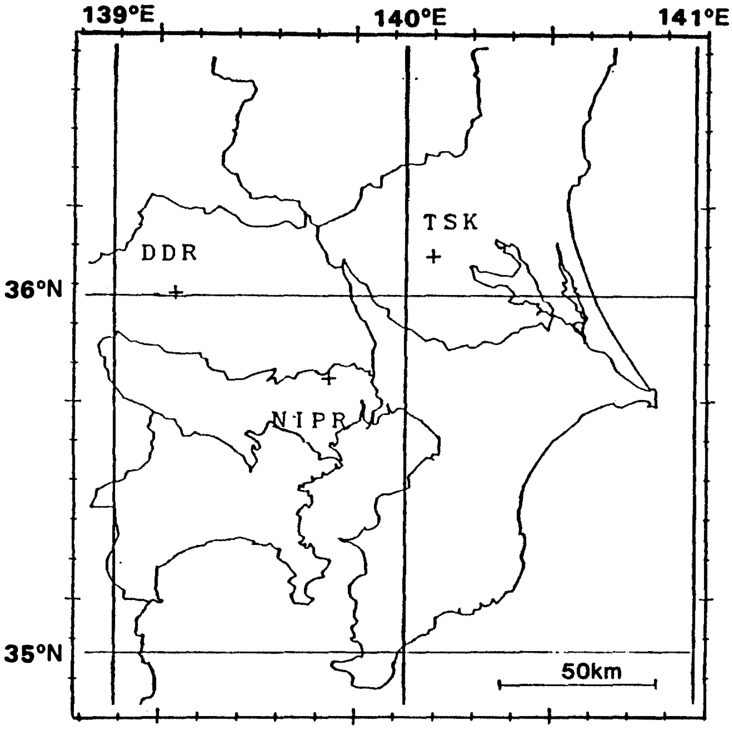

Fig. 1. Station locations in the translocation experiment. Abbreviations and coordinate values are given in Table 1.

Table 1. Summary of translocation experiment. Receiving antenna was located just above the station mark.

Row number	Station name		Tsukuba	Dodaira	Tokyo
1	Station abbreviation		TSK	DDR	NIPR
2	Period of observation (in 1984)		May 19–26	May 21 and 23	May 19–26
3	Receiver type (Serial number)		JMR-4A 60S12360	JMR-1 77–175	JMR-1 20M01483
4	Receiver delay		380 μs	370 μs	420 μs
5	Antenna offset		0.59 m	0.38 m	0.59 m
6	Accepted pass number		71	20	93
7	Geodetic coordinates of the station mark on the Bessel Ellipsoid	ϕ λ h	36°06′08.902″N 140°05′27.060″E 25.06 m	36°00′15.764″N 139°11′38.873″E 856.85 m	35°45′01.777″N 139°43′12.391″E 42.97 m
8	Cartesian coordinates of the station mark in the Tokyo Datum	X Y Z	−3957054.84 m 3309685.27 3737028.02	−3910131.16 m 3375833.64 3728718.12	−3953047.53 m 3350043.63 3705416.36
9	Data source of the geodetic coordinates		Komaki and Kaidzu (1983)	Komaki and Kaidzu (1983)	Itabashi *et al.* (1986)

tions, receiving antenna was set just above the station mark and the offset of the electric phase center is given in row 5. The geometry of these 3 stations provides a north/south line of 40 km and an east/west line of approximately 80 km. It is noted that the height difference betweeen DDR and NIPR (TSK) attains to 810–830 m.

A preamplifier/noise filter was applied and received signals were fed through a 30 m coaxial cable to the receiver. Integrated Doppler counts of 4.6 s time frame were recorded on a digital cassette tape and post-processed according to the analysis programs which will be described in later section. During 7 days receiving experiment at TSK, a total of 71 passes was accepted for post-processing. Likewise, 20 passes for DDR and 93 passes for NIPR remained, respectively (row 6 in Table 1). The reason for the smaller number of accepted passes at DDR is the failure of power supply and the unexpected cut of the coaxial cable by a mouse in the receiving experiment.

Though 29–31 passes are going to rise above the horizon each day, the accepted passes for post-processing are limited around 10 because of their overlapping pass geometry. Since the satellite Doppler positioning is essentially based on the principle of hyperbolic navigation, a satellite pass too high will result in an unstable solution. On the other hand, a sufficient number of Doppler counts cannot be obtained from a satellite pass that is too low and refraction errors in the ionosphere and troposphere may become too large to be adequately corrected for such a pass. NNSS positioning therefore adopts the maximum elevation angle as a pass acceptance criterion and here rejects the passes outside the range of 15–75°.

The receiving experiments were made on rainy and cloudy days. There was a pass of the weather front during the experiment. Since the refraction in the troposphere affects the positioning results especially for height solution (KOMAKI, 1981; SHIBUYA, 1985), surface synoptic data (air temperature, atmospheric pressure and relative humidity) were acquired every 3 h and used for correction after the formulae by BROWN and TROTTER (1973).

3. Broadcast and Precise Ephemeris Point-Positioning

The mathematical formulation of BE positioning is explained, for example, in MOFFETT (1971). The integrated Doppler counts in a certain time frame (nominally 4.6 s) can be correlated with the change of slant range between the satellite position and the receiving station. After receiving sufficient number of integrated Doppler counts, the observation equation for iterative approximation to the most probable coordinates of a receiving electric phase center can be formulated in a linearized first-order equation of the form

$$\boldsymbol{Ax}+\boldsymbol{L}-\boldsymbol{V}=\boldsymbol{O}, \tag{3}$$

where

$$\boldsymbol{A}=(\partial \boldsymbol{F}/\partial \boldsymbol{x})\boldsymbol{x}_0, \tag{4}$$

and $\boldsymbol{F}$ represents one-dimensional vector of m elements

$$f_{k,\,k-1}=g_k-g_{k-1} \quad (k=1,\, m), \tag{5}$$

with

$$g_k=\frac{1}{\lambda_0}\frac{(\boldsymbol{S}_k\cdot\Delta\boldsymbol{r})}{|\boldsymbol{S}_k|}+t_k(F_L+\Delta f_L)+\frac{|\boldsymbol{S}_k|}{\lambda_0}+\frac{R_k}{\lambda_0}-N_k\,. \tag{6}$$

In eq. (4), $\boldsymbol{x}_0$ is the apriori estimates for the coordinates of the receiving point and the local frequency offset. Since detailed expression of eq. (6) is given in SHIBUYA (1985), it is not repeated here.

Table 2. Summary of BE positioning at the three receiving stations.

Row number	Station name		TSK	DDR	NIPR
1	Number of accepted passes		71	20	93
2	Number of north going passes		36	7	44
	Number of south going passes		35	13	49
	Number of passes west of station		36	10	47
	Number of passes east of station		35	10	46
3	Final station coordinate	X	−3957192. 39 m	−3910269. 70 m	−3953183. 32 m
	in the broadcast	Y	3310210. 55	3376360. 29	3350567. 29
	ephemeris system	Z	3737708. 67	3729391. 59	3706096. 81
4	Final geodetic coordinate	ϕ	36°06′20. 330″N	36°00′26. 918″N	35°45′13. 350″N
	on the WGS72 Ellipsoid	λ	140°05′14. 479″E	139°11′26. 572″E	139°42′59. 984″E
		h	68. 79 m	901. 04 m	84. 11 m
5	Standard deviation of latitude estimate		0. 30 m	0. 43 m	0. 22 m
	Standard deviation of longitude estimate		0. 41	0. 61	0. 29
	Standard deviation of height estimate		0. 39	0. 53	0. 27
	Combined standard deviation		0. 64	0. 92	0. 45
6	Estimate of variance factor		1. 006	0. 923	0. 999
	Total degree of freedom		1676	480	2182

Table 2 summarizes the BE positioning of the receiving points listed in Table 1. The processing is made using SP-2G program (JMR Instruments Inc., 1982) and according to the same parameter setting as adopted by Shibuya (1985). The solution coordinates are reduced to each station mark.

BE positioning is based on an assumption that the position of an NNSS satellite (NAVSAT) at any time is correctly predicted in a satellite datum. This also implies that the satellite transmission of its position can always be corrected to UTC (Coordinate Universal Time) to a microsecond accuracy. However, partly because of unpredicted air drag of the NAVSATs, and partly because of the uncertainty in the adjustment between the local receiver time frame and the satellite transmitted time frame, the broadcasted satellite position has an unavoidable error of up to 20–40 m, which directly affects the positioning accuracy. The discrepancy between eq. (2) and eq. (1) can be considered as coming not only from the different definition of the satellite coordinate system (see Section 4) but also from the uncorrected bias errors which cannot cancel out statistically even by receiving a number of satellite passes.

Contrary to the predicted orbital information from broadcast ephemeris, precise ephemeris data are based on post-fitted satellite position, which is determined with 48 h Doppler data collected from over 20 stations of the TRANET distributed around the world. Table 3 (rows 1–2) gives a summary of PE positioning of the receiving point. The station coordinates of TSK and DDR are determined by Komaki and Kaidzu (1983) using 148 and 80 passes respectively, while those at NIPR are determined in this

Table 3. Summary of PE positioning at the three receiving stations. Their transformed coordinates to the broadcast ephemeris system are also given by the inverse of eq. (9).

Row number	Station name		TSK	DDR	NIPR
1	Station coordinate in the precise ephemeris system	X	−3957192.73 m	−3910268.34 m	−3953183.02 m
		Y	3310212.87	3376358.23	3350567.64
		Z	3737707.08	3729396.57	3706096.12
2	Geodetic coordinate on the NWL9D Ellipsoid	ϕ	36°06′20.278″N	36°00′27.117″N	35°45′13.355″N
		λ	140°05′14.417″E	139°11′26.599″E	139°42′59.966″E
		h	59.53 m	892.30 m	73.96 m
3	Transformed station coordinate in the broadcast ephemeris system	X	−3957191.87 m	−3910267.49 m	−3953182.15 m
		Y	3310212.14	3376357.49	3350566.90
		Z	3737708.85	3729398.34	3706097.92
4	Geodetic coordinate on the WGS72 Ellipsoid	ϕ	36°06′20.323″N	36°00′27.162″N	35°45′13.401″N
		λ	140°05′14.417″E	139°11′26.599″E	139°42′59.966″E
		h	69.40 m	902.17 m	83.83 m
5	Standard deviation of latitude estimate		0.33 m	0.23 m	0.15 m
	Standard deviation of longitude estimate		0.42	0.23	0.22
	Standard deviation of height estimate		0.63	0.40	0.18
	Combined standard deviation		0.83	0.56	0.32
6	Accepted pass number		148	80	89
7	Data source		KOMAKI and KAIDZU (1983)	KOMAKI and KAIDZU (1983)	This study

study using 89 passes. The orbital error of precise ephemeris, which comes mainly from uncertainties in the earth's gravitational field and effects of variation in atmospheric density, is considered within 2 m in each axis, and the obtained coordinate values in Table 3 are accurate to 2 m in each axis in the associated Cartesian coordinate system.

4. Coordinate Transform between the Precise Ephemeris System and the Broadcast Ephemeris System

As discussed earlier, BE positioning result is less accurate than PE positioning result and has uncorrected bias errors. The BE positioning in Table 2 thus cannot be taken as the standard point and the accuracy of BE positioning results has to be estimated in terms of the precise ephemeris coordinates.

When we call a standard point hereafter, it means the station mark specified by the PE-positioned coordinate values in Table 3 (row 2 or 3). When we call a reference point hereafter, it means the fixed point for translocation application. The reference point may not necessarily be the standard point in the simulation test but may be the given point by the BE-positioned coordinates.

The definition of the broadcast ephemeris system parameters such as gravitational constant, station coordinate sets, etc., is different from that of the precise ephemeris system, and it is necessary to establish the coordinate transform formula from the BE positioning result to the PE positioning result. In performance testing of the BE translocation, we obtain the convergent solution in the broadcast ephemeris coordinates and transform it into the precise ephemeris coordinates using the most appropriate transform formula, then calculate the radial offset between the standard point and the transformed solution point.

The transform from the broadcast ephemeris coordinates to the precise ephemeris coordinates can be formulated by the BURSA-WOLF model (HOAR, 1982) as

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_P = \begin{pmatrix} \delta x_0 \\ \delta y_0 \\ \delta z_0 \end{pmatrix} + (1+\varepsilon) \begin{pmatrix} 1 & -\theta_z & \theta_y \\ \theta_z & 1 & -\theta_x \\ -\theta_y & \theta_x & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_B , \tag{7}$$

where the notations of the 7 parameters are the same in SHIBUYA (1985) and is self-explanatory from Fig. 2. Since there is no established set of 7 parameters for Japan, we are going to see the applicability of ANDERLE's (1976) formula

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_P = (1+8.27\times 10^{-7}) \begin{pmatrix} 1 & 1.26\times 10^{-6} & 0 \\ -1.26\times 10^{-6} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_B , \tag{8}$$

or LEROY's (1982) formula

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_P = \begin{pmatrix} 0 \\ 0 \\ -2.6 \end{pmatrix} + (1+2.2\times 10^{-7}) \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_B . \tag{9}$$

Table 4 summarizes the application of the above two formulas to the BE positioning in row 3 or Table 2, taking the standard point in row 1 of Table 3 as the reference

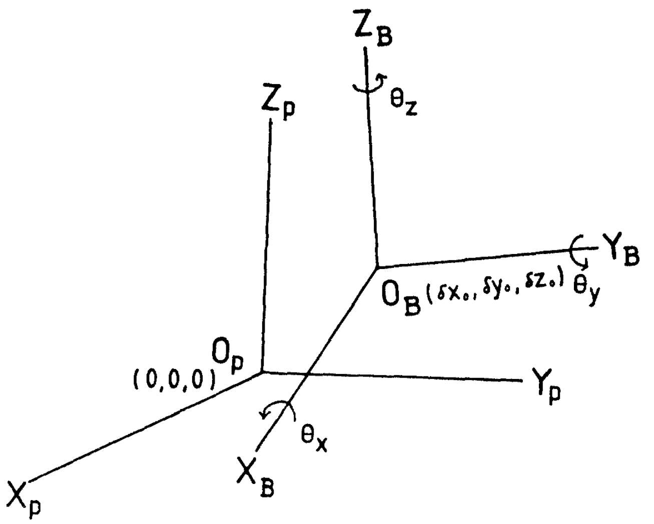

Fig. 2. Coordinate datum relation between the precise ephemeris system (suffix P) and the broadcast ephemeris system (suffix B). The earth's geocenter is taken equal to the origin of the precise ephemeris system.

Table 4. Summary of offsets by Anderle's and LeRoy's formulae to see the applicability of transform from broadcast to precise ephemeris coordinate systems. The offsets in case of no transform are given for comparison.

Station name[1]	Transform formula	$\Delta\phi$ (m)	$\Delta\lambda$ (m)	Δh (m)	Δd (m)[2]
	Anderle's formula	−2.29	5.04	−4.82	7.34
TSK	LeRoy's formula	−0.18	−1.46	0.59	1.59
	No transform	−2.29	−1.46	0.46	2.75
	Anderle's formula	5.54	7.23	−4.29	10.11
DDR	LeRoy's formula	7.65	0.78	1.13	7.77
	No transform	5.54	0.78	1.00	5.68
	Anderle's formula	−0.61	6.01	−5.08	7.90
NIPR	LeRoy's formula	1.51	−0.52	0.32	1.63
	No transform	−0.61	−0.52	0.20	0.83

1) Reference point is taken equal to the standard point in Table 3 (row 1).
2) $(\Delta d)^2=(\Delta\phi)^2+(\Delta\lambda)^2+(\Delta h)^2$.

point. The offsets between the transformed BE positioning to precise ephemeris coordinates and the standard point for latitudinal direction $\Delta\phi(=r\mathrm{d}\phi)$, for longitudinal direction $\Delta\lambda(=r\cos\phi\mathrm{d}\lambda)$ and for height Δh in meters are tabulated for each station TSK, DDR or NIPR, respectively. According to eq. (2), the convergence of BE positioning is considered better than ± 1.9 m for DDR, ± 1.0 m for TSK or ± 0.4 m for NIPR in each direction. The resultant offset however is larger than the predicted value for most of the direction when Anderle's formula is applied. On the other hand, LeRoy's formula gives the offset mostly within ± 1.5 m for each direction except rather large $\Delta\phi=7.77$ m at DDR.

Anderle's formula is characterized by the westward rotation with respect to the Z axis (0.26″ longitudinal shift) and is based on the satellite Doppler tracking data in North America. LeRoy's formula is characterized by the Z shift of -2.6 m of the geocenter and is based on similar experiment by more than 50 stations all over the world. The results in Table 4 indicate that westward rotation is not adequate and LeRoy's formula should be applied for the coordinate transform in the region concerned.

5. Translocation

Though PE positioning is most accurate, the number of usable satellites is usually limited to one or two. Their satellite code numbers are not known beforehand at the receiving site. The receiving period to have 5 m absolute accuracy then cannot necessarily be shortened even if we apply PE positioning. This difficulty may be avoided by an application of "translocation" technique. According to WELLS (1976), the term "translocation" was named by WESTERFIELD and WORSLEY (1966). The technique uses simultaneous Doppler data from separate stations to determine the relative position of one station with respect to another. Among the BE positioning error sources, the effect of ephemeris errors is correlated between stations simultaneously tracking a satellite pass and the errors may be reduced by the following model procedure.

5.1. Mathematical model

The formulation of translocation for multistation relative positions are given by KOUBA and BOAL (1976) of program GEODOP, or outlined by BRUNELL *et al.* (1982) of program GP-1S. The model includes 3 degrees of freedom for orbit relaxation, and other parameters which are specific of the i-th ($i=1\cdots n$) receiving point. Instead of eq. (3), the linearized observation equation is expressed as

$$Ax+Cy+L-V=0, \tag{10}$$

where A becomes the enlarged matrix of eq. (4) to multistation (n stations) coordinates and C is composed by the following terms; (1) $a\Delta E$, Δa and $\Delta\eta$ are the orbital relaxations for the along-track, across-track and out-of-plane components, respectively. (2) Δf_i is the local frequency offset at the i-th station. (3) $\Delta\mu_i$ is the correction to the 10% apriori estimate of tropospheric refraction and the change of path length by refraction is added for the update total pass length. (4) Δt_i is the correction to the apriori estimate of receiver time synchronization. Design matrices of A and C are schematically illustrated in Fig. 3. In constituting L of eq. (10), integrated Doppler counts $N_{i,k}$ which correspond to the i-th station and the k-th time frame can be divided into two portions with random measurement error $\nu_{i,k}$ and systematic measurement error $\zeta_{i,k}$.

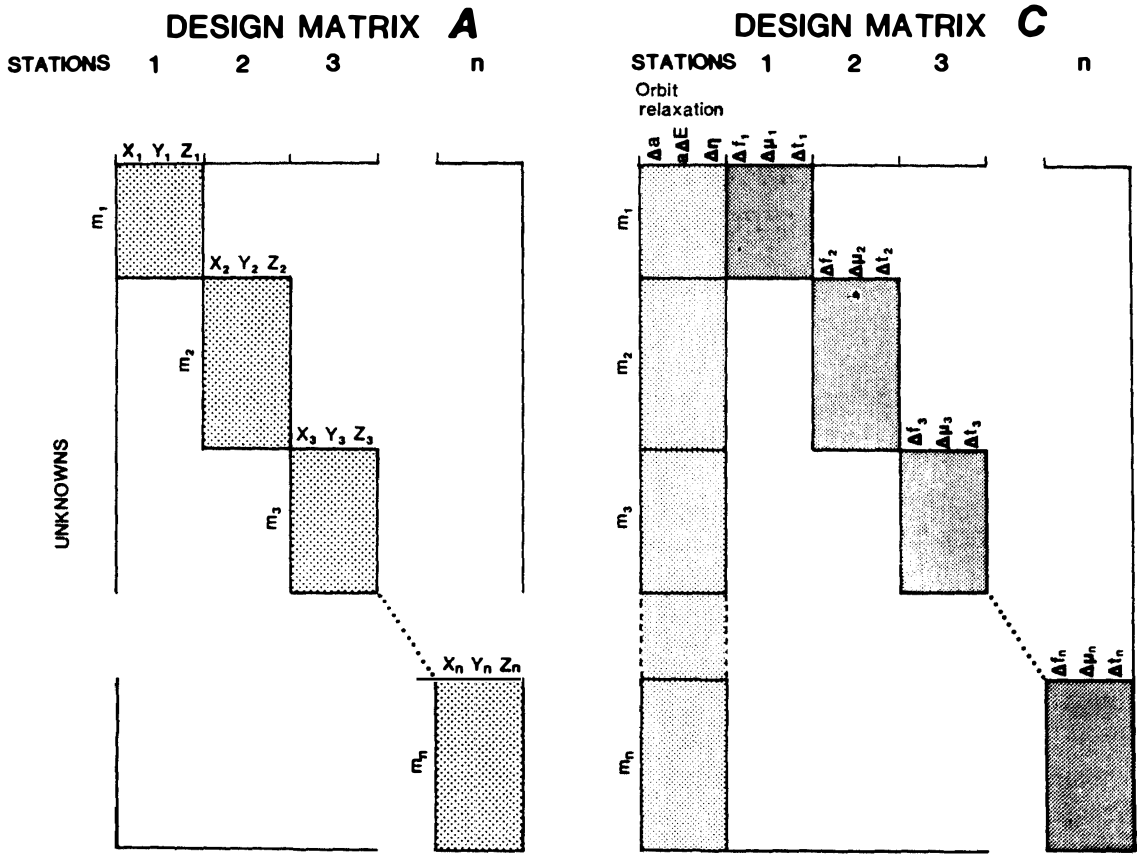

Fig. 3. Design matrices of A and C. A is $(q, 3n)$ matrix and C is $(q, 3n+3)$ matrix, where $q=\sum_{i=1}^{m} n_i$. Only the submatrices of shaded portion have non-zero elements.

The later error has been modeled as a function of 5 parameters as

$$\zeta_{i,k}=f(\Delta E, \Delta a, \Delta \eta, \Delta t_i, \Delta \mu_i). \tag{11}$$

The used software program for our translocation experiment is an industry product of the GP-1S program by JMR INSTRUMENTS INC. (1982). The program solves the least squares estimate for $\boldsymbol{x}$ as

$$\begin{aligned}\boldsymbol{x}= &-[(\boldsymbol{A}^t\boldsymbol{P}\boldsymbol{A}+\boldsymbol{P}_x)-\boldsymbol{A}^t\boldsymbol{P}\boldsymbol{C}(\boldsymbol{C}^t\boldsymbol{P}\boldsymbol{C}+\boldsymbol{P}_y)^{-1}\boldsymbol{C}^t\boldsymbol{P}\boldsymbol{A}]^{-1}\\ &*[\boldsymbol{A}^t\boldsymbol{P}\boldsymbol{L}-\boldsymbol{A}^t\boldsymbol{P}\boldsymbol{C}(\boldsymbol{C}^t\boldsymbol{P}\boldsymbol{C}+\boldsymbol{P}_y)^{-1}\boldsymbol{C}^t\boldsymbol{P}\boldsymbol{L}],\end{aligned} \tag{12}$$

and the least squares estimate for $\boldsymbol{y}$ as

$$\boldsymbol{y}=-(\boldsymbol{C}^t\boldsymbol{P}\boldsymbol{C}+\boldsymbol{P}_y)^{-1}*(\boldsymbol{C}^t\boldsymbol{P}\boldsymbol{L}+\boldsymbol{C}^t\boldsymbol{P}\boldsymbol{A}\boldsymbol{x}). \tag{13}$$

In eqs. (12) and (13), $\boldsymbol{P}$ is the weight matrix of the observations of the diagonal form

$$[p_{ii}]=\frac{\sigma_0^2}{\sigma_i^2}, \tag{14}$$

where σ_0^2 is the apriori variance of unit weight and σ_i^2 is the variance of the observation. Here the matrix notations $\boldsymbol{A}^t$, $\boldsymbol{A}^{-1}$ for example denote transpose matrix and the inverse matrix of $\boldsymbol{A}$, respectively, and $[p_{ii}]$ indicates i-th diagonal element of $\boldsymbol{P}$. The matrices $\boldsymbol{P}_x$ and $\boldsymbol{P}_y$ are the weight matrices assigned to the apriori unknown vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ respectively. As for the variance-covariance matrices for the estimated parameters $\boldsymbol{x}$ and $\boldsymbol{y}$, they are given respectively by

$$\Sigma_x=[(\boldsymbol{A}^t\boldsymbol{P}\boldsymbol{A}+\boldsymbol{P}_x)-\boldsymbol{A}^t\boldsymbol{P}\boldsymbol{C}(\boldsymbol{C}^t\boldsymbol{P}\boldsymbol{C}+\boldsymbol{P}_y)^{-1}\boldsymbol{C}^t\boldsymbol{P}\boldsymbol{A}]^{-1}, \tag{15}$$

$$\Sigma_y=[(\boldsymbol{C}^t\boldsymbol{P}\boldsymbol{C}+\boldsymbol{P}_y)-\boldsymbol{C}^t\boldsymbol{P}\boldsymbol{A}(\boldsymbol{A}^t\boldsymbol{P}\boldsymbol{A}+\boldsymbol{P}_x)^{-1}\boldsymbol{A}^t\boldsymbol{P}\boldsymbol{C}]^{-1}. \tag{16}$$

In this translocation test, station coordinates of the reference point are weighted by the combined standard error estimates (row 5 of Table 2 or 3), while those for the translocation point are allowed for 25 m adjustment shift. After solving eq. (10), the obtained latitudinal, longitudinal and heignt differences are added to the corresponding coordinate of the reference point to obtain the translocation solution coordinates.

5.2. *Importance of reference point in translocation*

Translocation is a relative positioning technique of one station (called translocation point) with respect to the other reference point (called fixed station). When there is a bias error in the coordinates of reference point, the error may transfer to the translocation point and may degrade the positioning accuracy. Figure 4 schematically illustrates such possibility. In Fig. 4, standard points of DDR and TSK are denoted by solid circles. Their coordinate values are transformed from the precise ephemeris system to the broadcast ephemeris system according to the inverse of LeRoy's formula (results are shown in rows 3–4 of Table 3) and used as the standard point for translocation. The translocation solution by the procedure described in 5.1 is then transformed to the precise ephemeris system by eq. (9), and the offset from the standard point in the precise ephemeris system (row 1 or 2 of Table 3) is calculated. Let the BE-positioned

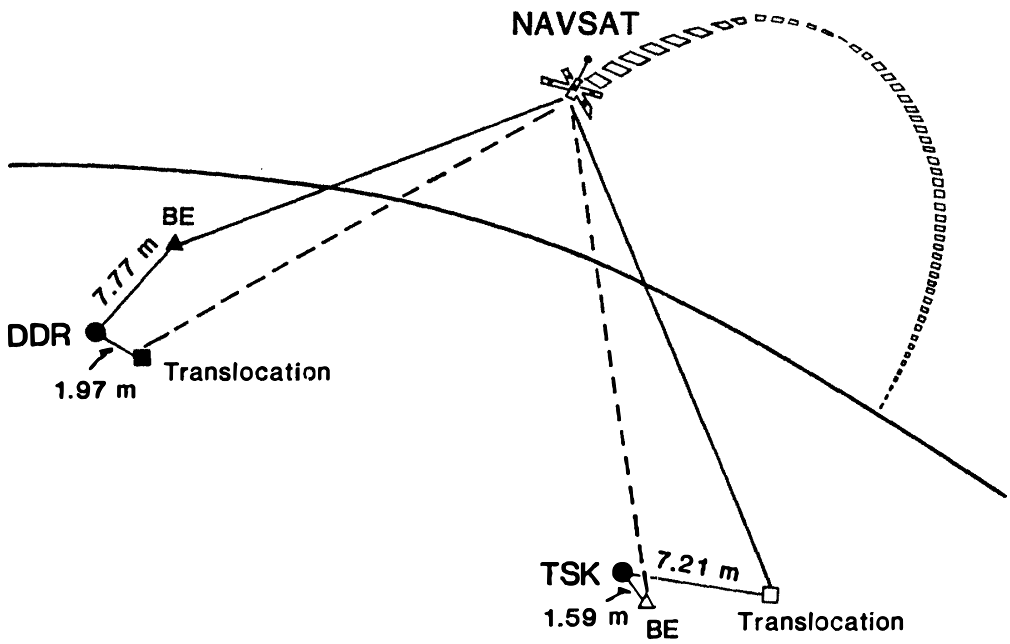

Fig. 4. Translocation of DDR with respect to TSK, and vice versa. For simulation test, the reference point is taken equal to BE positioning station, etc., whose notations are explained in the article.

DDR (solid triangle) be the reference point. The BE-positioned DDR is 7.77 m distant from the standard point. Then, the translocated TSK, which is denoted by open box and is indicated by solid slant range pairs in Fig. 4, is 7.21 m distant from the standard point of TSK. The resultant offset is much larger than 1.59 m offset of the BE-positioned TSK (open triangle). Conversely, when we take BE-positioned TSK as the reference point, the translocated DDR, which is denoted by solid box and is indicated by broken slant range pairs, has much improved offset of 1.97 m as compared with the BE-positioned offset of 7.77 m.

Table 5 summarizes the offset in the coordinates of the translocation point by the simulation test with changed reference point. The offset Δd_B of case 1 corresponds to the example of solid triangle and open box pairs in Fig. 4, and the BE-positioned

Table 5. Transfer of bias error in the reference point to the translocation point. Column 4 indicates pass numbers for calculating Δd_B and column 6 indicates those for calculating Δd_P.

Column number	1	2	3	4	5
Case number	Translocation point	Reference point	Accepted passes	Δd_B (m)	Δd_P (m)
1	TSK	DDR	13	7.21	1.09
2	TSK	NIPR	53	2.14	2.77
3	DDR	TSK	13	1.97	1.24
4	DDR	NIPR	11	0.83	1.87
5	NIPR	DDR	17	10.22	2.76
6	NIPR	TSK	53	2.04	2.64

DDR has 7.21 m offset against the standard point, while Δd_P of 1.09 m shows the offset by the translocation with reference to the standard point itself. Cases 2–6 for Δd_B give similar results for one translocation point (column 1) against one reference point (column 2). It is clear that the translocation of TSK and NIPR with reference to DDR has exceptionally large offset. On the other hand, Δd_P is mostly within 2 m offset except somewhat larger value for the combination of TSK and NIPR (around 2.7 m in cases 2 and 6). The changed combination of translocation and reference points may thus reveal the "bug station". In this net adjustment, BE-positioned DDR is obviously the bug station and the bias error has transferred to the translocation point to degrade the accuracy.

The inclusion of orbit relaxation for three degrees in the modeling of eq. (10) is usually called semishort arc technique (*e.g.*, HOTHEM and EDLER, 1982). The orbital constraints used in this semishort arc adjustment for along track, across track and radial components are 25, 10 and 5 m, respectively. The allowance of larger value for along-track component comes from the fact that unpredicted variation in air drag of the satellite is the main source of errors in the broadcasted ephemeris. Even when we put loose or strict limitations for the above three parameters, the resultant latitude, longitude and height differences were almost unchanged, so that the translocation offset was not improved significantly. In a practical scheme of translocation application, it is thus very important to have the reference point, the station coordinates of which are as accurately determined as possible.

5.3. *Minimum pass number to have 5 m accuracy in translocation*

The NAVSATs circle at a height of 1100 km and the error sources in eq. (10) from ionospheric and tropospheric refractions become less correlated according as the station separation becomes longer. According to WELLS (1976), the character of translocation techniques changes as the station separation changes from short (less than 1 km) to medium (up to a few hundred km) to long. At medium separations in this experiment, enforcing rigorous data simultaneity is expected to improve accuracy significantly with less accepted satellite passes.

For a special application to the Antarctic research program, let us examine the efficiency of translocation in reducing the necessary satellite passes, taking the positioning accuracy criterion as a parameter. Table 6 summarizes the results of such experi-

Table 6. The number of necessary accepted passes in the translocation test when the accuracy criterion is taken as a parameter.

Row number	Station combination	Δd (m)			
		5.0	4.5	4.0	3.5
1	NIPR-TSK	2 passes	4 passes	5 passes	9 passes
2	NIPR-DDR	3	4	4	4
3	DDR-NIPR	3	4	4	4
4	TSK-NIPR	4	5	8	9
5	NIPR	11	12	14	19
6	TSK	10	12	12	12

ment. NIPR-TSK in row 1 indicates that, when the translocation of NIPR was made with reference to the standard point TSK, the necessary accepted passes must increase from 2 to 9 according as the accuracy criterion Δd (offset between the translocated NIPR and the standard NIPR) becomes strict from 5 to 3.5 m allowance. Rows 2–4 show similar results with changed combination of reference points as in Table 5. For comparison, necessary accepted passes for BE positioning of NIPR and TSK are shown in rows 5–6. Though the necessary accepted passes (19 for NIPR and 12 for TSK) are comparatively smaller against the predicted 40–50 passes by eq. (2), this may be a mere coincidence.

When we put 1.5 m net uncertainty in the positioning of standard point and in the coordinate transformation by eq. (9), it is necessary to have 3.5 m convergence for the translocation offset in order to maintain 5 m absolute accuracy. The number of necessary passes under 10 (Table 6) is about one-fourth of the necessary passes (40–50) for BE positioning. Though the number of receiving passes have to be somewhat larger in order to have enough acceptable passes, staying time at the receiving site will be reduced to one day, which makes the field operation much easier.

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