Modelling Glacial Isostatic Adjustment in Firedrake

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Melting ice sheets transfer water from land into ocean basins. The resulting sea-level rise is, however, highly spatially non uniform and time dependent due to complex feedbacks between viscoelastic deformation of the solid Earth in response to these evolving surface loads and coupled perturbations in the gravitational field and rotation axis. Together, these processes are referred to as Glacial Isostatic Adjustment (GIA) and accurate models of GIA are crucial for robust interpretation of both modern and paleo measurements of sea-level change and ice-mass balance (Whitehouse, 2018).

A limitation with many existing GIA modelling codes is their inability to incorporate lateral variations in Earth structure. Nevertheless, there is mounting evidence for the presence of significant lateral changes in mantle viscosity, for example beneath West Antarctica, that give rise to complex interactions between rates of surface rebound, sea-level change and ice retreat (Nield et al., 2018). Understanding these processes requires development of a new generation of GIA codes capable of handling such variations in rheology at increasingly fine spatial and temporal evolution.

In this presentation, we will introduce a new project to model GIA using the Firedrake finite element framework (Ham et al., 2023). Firedrake leverages automatic code generation to create a separation of concerns between employing the finite-element method and implementing it. This approach maximises the potential for collaboration between computer scientists, mathematicians, scientists and engineers and enables sophisticated high performance simulations. A key advantage of Firedrake is the automatic availability of sensitivity information through the adjoint method, allowing us to investigate inverse problems (Farrell et al., 2013). We aim to develop an open-source tool highly suited to the challenge of modelling complex Earth structure in GIA. We envision that future applications might include, but are not limited to, investigating non-linear and transient rheologies, feedbacks between sea-level and glacier dynamics, and reducing uncertainty on sea-level projections into the future.



Figure 1. Workflow of finite element GIA modelling.

References

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