OCCURRENCE CHARACTERISTICS OF Pc 1 PULSATIONS OBSERVED AT JAPANESE OBSERVATORY NETWORK

Shigeru FUJITA and Takeshi OWADA

Kakioka Magnetic Observatory, 595, Kakioka, Yasato-machi, Niihari-gun, Ibaraki 315–01

Abstract: Employing Pc 1 pulsation data obtained at the Japanese Observatory Network in an interval of 1976 to 1984 covering almost one solar cycle, it is confirmed that the Pc 1 pulsations appear more frequently in the period of the minimum solar activity than in the period of the maximum activity, in winter than in summer and in the nighttime than in the daytime. The attenuation rate of the magnetic signal intensity at this network is also obtained in the period of the minimum solar activity. In addition, diurnal variation in the lower boundary frequencies of Pc 1 events are analysed. As for the variations in occurrence of the low-latitude Pc 1 pulsations related to seasonal and solar-activity, the observational results are essentially consistent with the duct propagation theory based on the International Reference Ionosphere. Attenuation rates obtained at the network fall in the same order of magnitude as that suggested by the theory. In the final part of this paper, we consider geometrical damping of the wave field which depends on spatial spread of an incident wave in high latitudes.

1. Introduction

Since International Magnetospheric Study in 1976–1978, the Kakioka Magnetic Observatory has been continuously observing magnetic pulsations with frequencies lower than 1 Hz at four coordinated magnetic observatories: Memambetsu (MMB), Kakioka (KAK), Kanoya (KNY) and Chichijima (CBI) as shown in Fig. 1 (KAKIOKA MAGNETIC OBSERVATORY, 1984). The network which consists of those observatories is called the Japanese Observatory Network. Using pulsation data observed at this network in an interval of 1976–1984 (covering almost one solar cycle), we investigated properties of the low-latitude Pc 1 pulsations: the occurrences of pulsations at MMB, the attenuations rates of Pc 1 signals derived from the multi-station observations and the local-time dependence of the lower boundary frequencies of Pc 1 events at MMB. However, the time-accuracy of the observation system (about 1 s) prevents us from examining propagation speed of the pulsation.

As the Pc 1 pulsations have many groups of dynamic spectral forms in high latitudes (KOKUBUN, 1970; FUKUNISHI *et al.*, 1981), it is important to mention what kind of the Pc 1 pulsations are frequently observed in low latitudes. After KAWAMURA (1970) and KUWASHIMA *et al.* (1981), mainly the Periodic Emissions are observed in low latitudes. However, we cannot confirm this fact with our data because our quick-look chart of the dynamic spectrum (1.5 cm/1 h) does not have sufficient resolution to present fine spectral structures of Pc 1 signals.

Turning to the high-latitude phenomena, KUWASHIMA *et al.* (1981) showed that

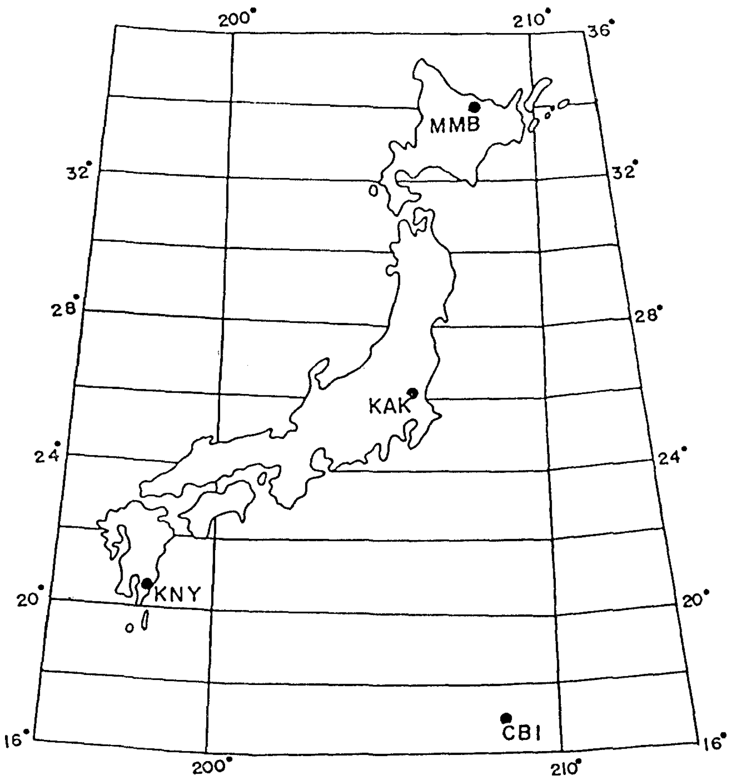

Fig. 1. The Japanese Observatory Network consists of four magnetic observatories operated by the Kakioka Magnetic Observatory. Geomagnetic and geographic coordinates of these observatories are:

		Geomagnetic (1980 IGRF)		Geographic	
		latitude	longitude	latitude	longitude
Memambetsu	(MMB)	34.4°	210.0°	43.9°N	144.2°E
Kakioka	(KAK)	26.4°	207.6°	36.2°	140.2°
Kanoya	(KNY)	20.9°	199.7°	31.4°	130.9°
Chichijima	(CBI)	17.4°	208.7°	27.2°	142.3°

It is very distinctive that MMB, KAK and CBI are located on the same geomagnetic meridian line.

the diurnal variation of the Periodic Emissions in high latitudes (Syowa in Antarctica) is not so conspicuous as that in low latitudes. SAITO (1969) insisted the same diurnal variation in the geomagnetic latitude of about 60°. HAYASHI *et al.* (1981) revealed that the periodic Pc 1 is observed at the plasmapause latitude which is located at the lower latitude in the nighttime than in the daytime. After all, the high-latitude Periodic Emissions occur at all local times without so distinguished diurnal variation. Moreover, Figs. 3 and 4 of KAWAMURA *et al.* (1983) suggest respectively no distinctive seasonal and annual variations in occurrence of the Periodic Emissions. (However, since their data covered only a half of a solar cycle, we need in the future to confirm this fact using data in an interval longer than one solar cycle.)

In this analysis, we will re-examine the results obtained by many scientists on the low-latitude Pc 1 pulsations (TROITSKAYA, 1964; SAITO, 1969; KAWAMURA, 1970; FRASER, 1975a,b; ALTHOUSE and DAVIS, 1978; KUWASHIMA *et al.*, 1980; KAWAMURA *et al.*, 1981); they revealed that the low-latitude Pc 1 pulsations appear mainly in the nighttime, in winter and in the period of the minimum solar activity, which is very different from the occurrence property of the high-latitude Pc 1 pulsations. This difference can be attributed to difference in the condition of ionospheric duct propagation suggested by MANCHESTER (1966) and C. GREIFINGER and P. GREIFINGER (1968). Moreover, FUJITA (1985) recently studied ionospheric duct propagation based on the International Reference Ionosphere model (the IRI model) (RAWER *et al.*, 1978). Comparison between observed results and theoretical ones will be done in this work.

HAYASHI *et al.* (1981) and INHESTER *et al.* (1984) discovered that the Pc 1 pulsations in high latitudes are spatially localized. This feature of the Pc 1 pulsations may introduce geometrical damping of the wave signal. This possibility will be discussed in the last part of this paper.

2. Data Analysis

Inquiries on the observing apparatus must be referred to 'the blue book' published by the Kakioka Magnetic Observatory (KAKIOKA MAGNETIC OBSERVATORY, 1984), adding particular notice that instruments at all stations are carefully maintained and periodically calibrated. Data thus obtained in magnetic tapes are analyzed with a dynamic spectrum method and consequently events are manually picked up from records of dynamic spectra. The amplitude of the event is measured from a reproduced wave form on a chart with man-made scaling.

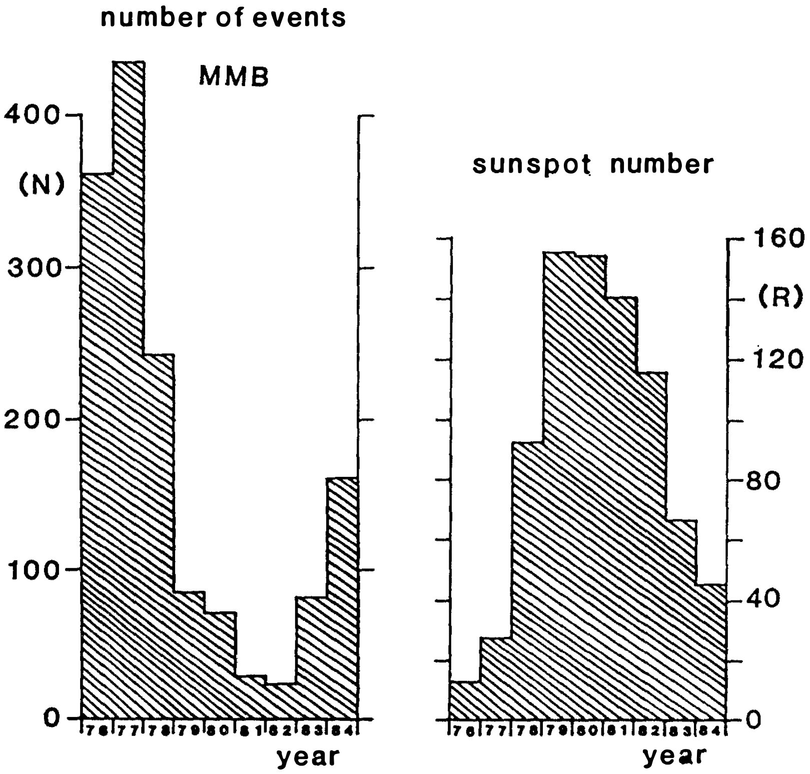

Fig. 2. Annual total numbers of Pc 1 pulsations observed at MMB (left) and annual means of the sunspot number (right).

2.1. Occurrence characteristics

Employing a dynamic spectral analysis of the Pc 1 pulsations observed at MMB, we compare annual changes in the number of Pc 1 events with annual mean values of the sunspot numbers in Fig. 2. We do not show the data at other observatories because the same feature appears there. It is evident that the occurrence is inversely correlated with that means of the sunspot number. Comparing the results by KAWAMURA *et al.* (1983) that the Periodic Emissions occur independently of the solar activity in high latitudes, we can conclude that the Periodic Emissions propagate with smaller attenuation in the period of the minimum solar activity than in that of the maximum activity.

In order to study seasonal variation in the occurrence, all data in 1976 (the minimum solar-active period) and those in 1979–1980 (the maximum solar-active period) are employed. Figure 3 shows that Pc 1 pulsations are observed mainly in winter (November, December and January) and are rare in summer (June and July) in both periods of minimum and maximum solar activity. This feature is consisted with those briefly mentioned by KUWASHIMA *et al.* (1980). Since KUWASHIMA *et al.* (1981) showed constant occurrence of the high-latitude Periodic Emissions in all seasons, the emissions are propagating with smaller attenuation in winter than in summer.

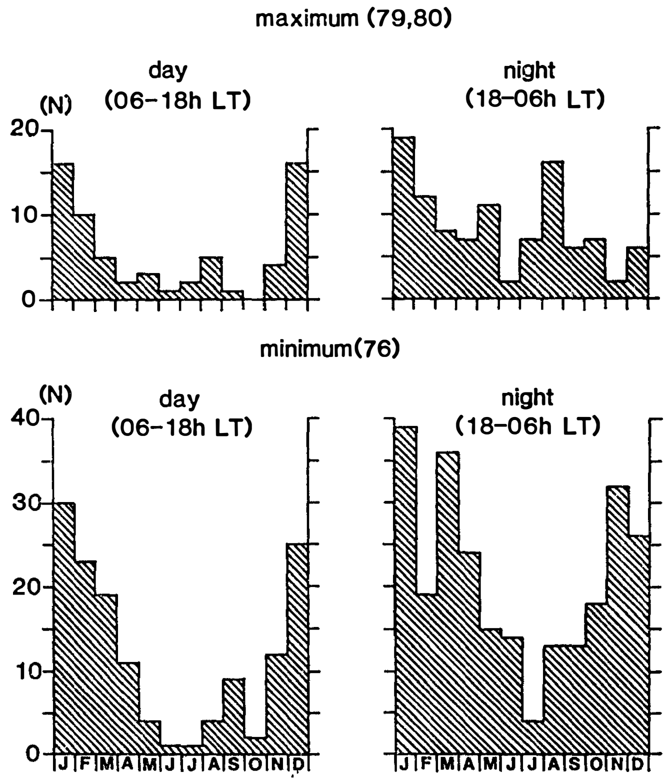

Fig. 3. Seasonal variation in occurrence of the Pc 1 pulsations observed at MMB.

The diurnal variation of the occurrence is also important (Fig. 4). This figure shows that the Pc 1 pulsations appear frequently in the nighttime in low latitudes as was noticed previously (FRASER 1975a,b; KUWASHIMA *et al.*, 1980; KAWAMURA *et al.*,

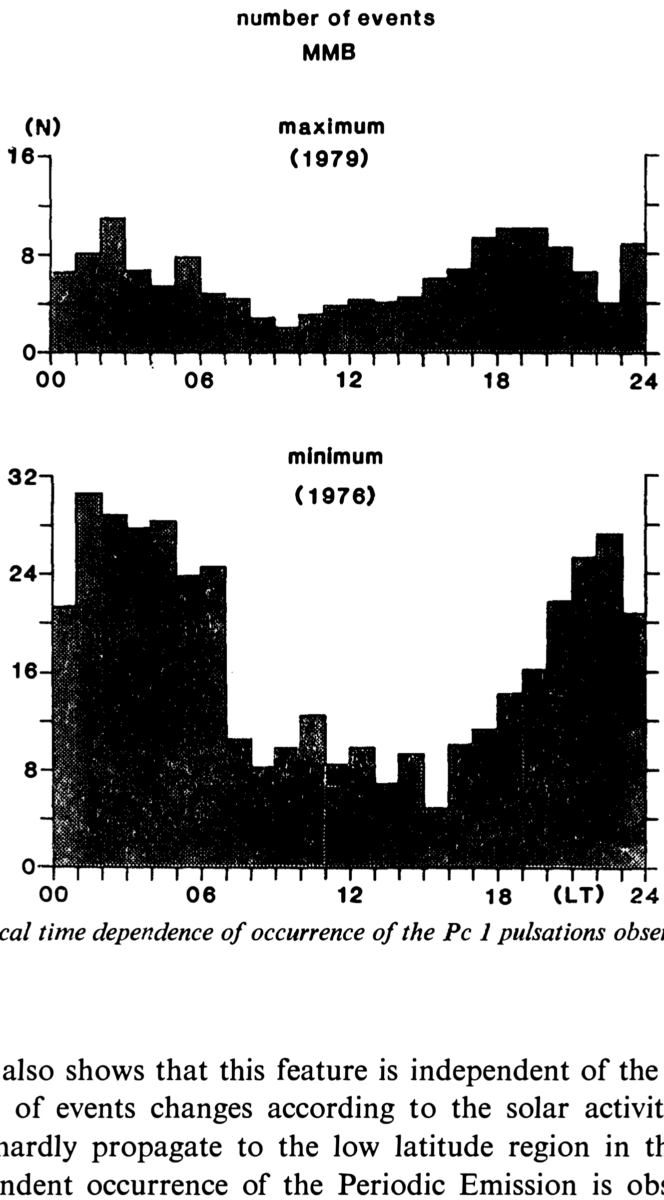

Fig. 4. Local time dependence of occurrence of the Pc 1 pulsations observed at MMB.

1981). Figure 4 also shows that this feature is independent of the solar activity (only the total number of events changes according to the solar activity). Therefore, the Pc 1 pulsations hardly propagate to the low latitude region in the daytime because local time-independent occurrence of the Periodic Emission is observed in high latitudes (KUWASHIMA *et al.*, 1981).

2.2. *Comparison between observation and theory*

Attenuation rates of the ducted wave based on the IRI model is shown in Fig. 5 after FUJITA (1985). Comparison between the results in Fig. 5 and the observations indicates good correspondence as follows:

(1) The theoretical attenuation rate of the ducted wave is decreased in the nighttime, thus, we can explain the fact that Pc 1 pulsations appear mainly in the nighttime. This tendency is independent of season and of solar activity.

(2) In the nighttime, the attenuation is larger in summer than in winter. As the low-latitude Pc 1 pulsations occur in the nighttime, this winter-summer asymmetry of the attenuation is consistent with the observed results.

(3) In the nighttime we have smaller attenuation in the minimum solar activity period, which is harmonized with the observed feature in Fig. 2.

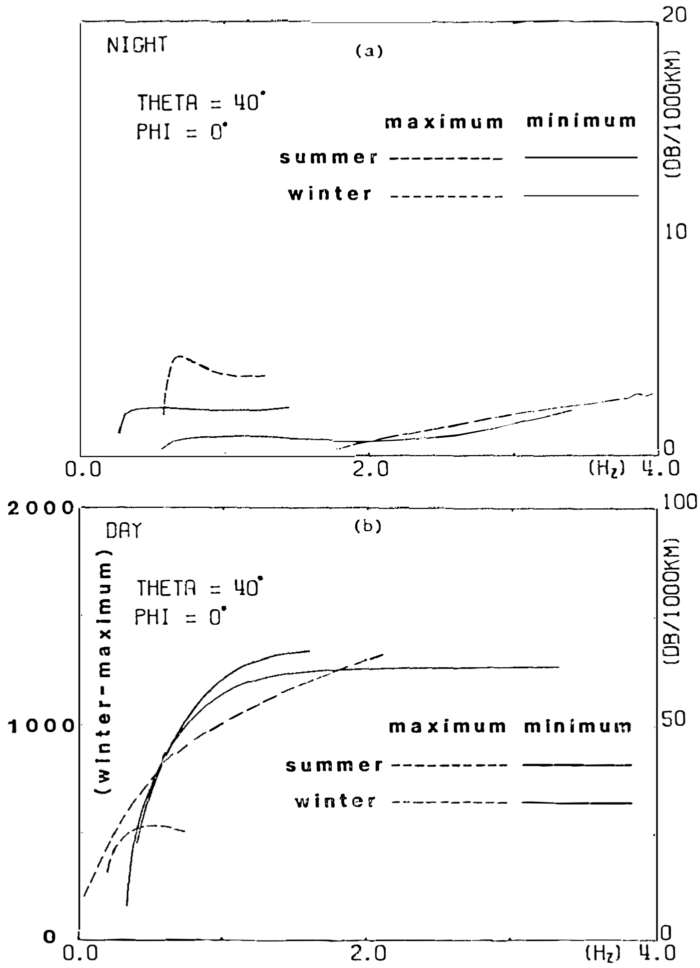

Fig. 5. Calculated attenuation rates (a: nighttime, b: daytime) in the case of meridional propagation of the ducted wave. It is very distinguished that the attenuation is much smaller in the nighttime than in the daytime.

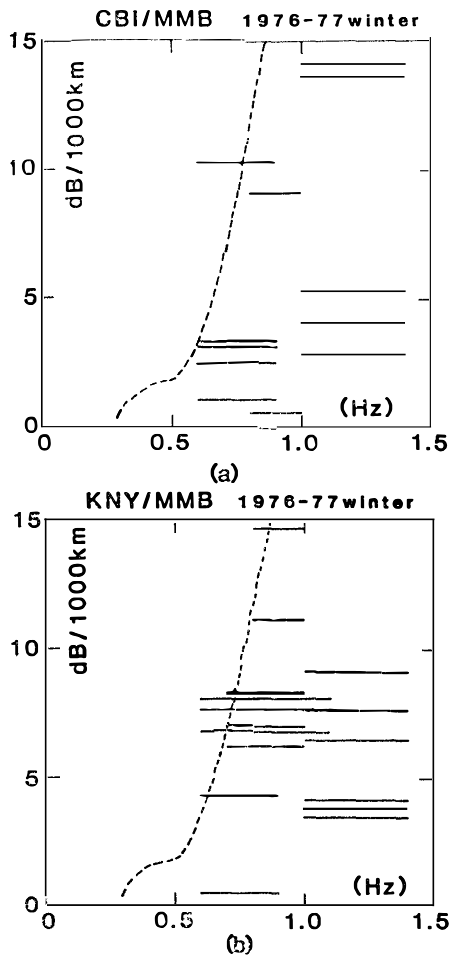

Fig. 6. Attenuation rates observed at observatory pairs of MMB-CBI (a) and of MMB-KNY (b). Broken lines in both figures indicate attenuation rates obtained from theoretical calculation based on the IRI model in the case of meridional propagation.

2.3. Attenuation rates derived from network observation

Attenuation rates of distinct Pc 1 signals which occurred simultaneously at MMB and CBI (Fig. 6a) and those at MMB and KNY (Fig. 6b) are employed for comparison between the duct propagation theory and the observation. All events employed occur in the nighttime of winter in the minimum solar activity period. As for the other conditions, we do not obtain sufficient number of the events with large magnetic amplitude. After comparing the observation and the theory in Fig. 6, we can conclude that the attenuation rate based on the IRI model almost corresponds to the observed one. However, we can also find a tendency that the observed attenuation rate is lower than the theoretical one in some degree. The reason is that the density of neutral particles employed (U.S. Standard Atmosphere, 1976) is probably a little larger than that in the actual condition. Moreover, it can be pointed out that the ionosphere is possibly a little disturbed from the IRI model which presents an averaged feature of the ionosphere, since the Pc 1 pulsations in middle and low latitudes are often observed in the recovery phase of a magnetic storm (WENTWORTH, 1964; KAWAMURA, 1970).

2.4. The lower boundary frequency

The lower boundary frequencies of Pc 1 events at MMB are plotted in Fig. 7 against the local time. It is higher in the nighttime than in the daytime in winter of the minimum solar activity period. However, this behaviour of the lower boundary frequency is not clear in the other conditions. A theoretical lower cutoff frequency is also presented in this figure. It can be seen that there is a fairly good correspondence between the observation and the theory in winter of the minimum solar activity period, although the lower cutoff frequency in winter of the maximum solar activity period is too high to explain the observation (Fig. 7b). This fact may suggest that the IRI model of this condition does not represent the actual ionosphere.

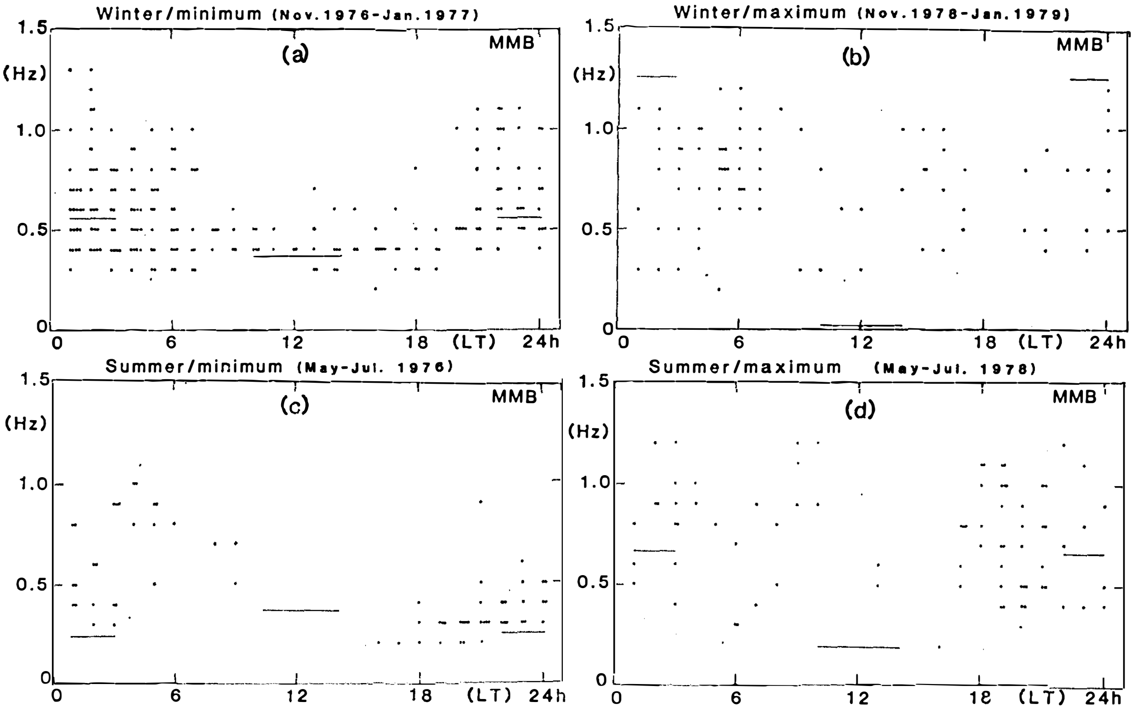

Fig. 7. Lower boundary frequencies of the Pc 1 pulsations observed at MMB are plotted against the local time: (a) for winter of the minimum solar active period, (b) for winter of the maximum period, (c) for summer of the minimum period and (d) for summer of the maximum period. Bars in the figures show the cutoff frequencies of the ducted wave obtained with the IRI model.

3. The Effect of Localization of an Incident Wave

When a short-period hydromagnetic wave propagates in an ionospheric duct and if the Earth is flat, the wave field has geometrical damping in a case of cylindrically concentrated injection of an incident wave but does not have it in a case of longi-

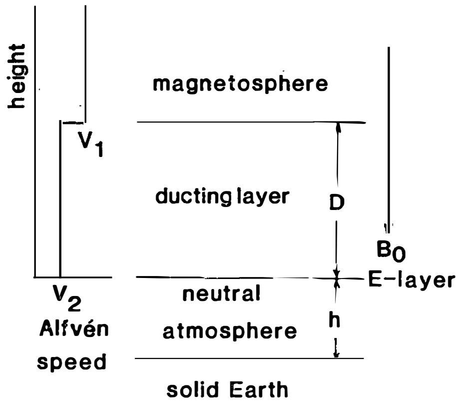

Fig. 8. Schematic presentation of the idealized 5-layer model. The ducting layer with an Alfvén speed smaller than that in the magnetosphere is a wave-guide layer for the ducted wave. The E-layer is a thin layer with height-integrated anisotropic conductivities and the ambient magnetic field is assumed vertical.

tudinally elongating injection. This difference introduces different latitudinal diminution in electromagnetic intensity of the wave. Let us consider this effect in an idealized model with the vertical ambient field (Fig. 8). The magnetic field disturbance ($\delta\boldsymbol{B}$) on the ground is written as:

$$\delta\boldsymbol{B}=\mathrm{rot\ rot}\ (\boldsymbol{z}S), \tag{1}$$

where S is a scalar function representing the poloidal magnetific field disturbance (FUJITA and TAMAO, 1984) and $\boldsymbol{z}=\boldsymbol{B}_0/B_0$ ($\boldsymbol{B}_0$ is the ambient field vector).

(1) Cylindrically concentrated injection

Detailed formulation of S is presented in the Appendix. Estimating $|\delta\boldsymbol{B}_\mathrm{n}|$ in low latitudes ($\delta\boldsymbol{B}_\mathrm{n}$ is a magnetic field disturbance normal to the ambient field) with eqs. (1) and (A-8),

$$|\delta\boldsymbol{B}_\mathrm{n}|\sim\delta B_0|I_1(k_j)||\rho_4 k_j T(k_j)|(r_0/r)^{1/2}\exp(-k_{ji}r), \tag{2}$$

where $\rho_4=(i\omega\mu_0\sigma_G-k_\mathrm{n}^2)^{1/2}$ (σ_G is the electric conductivity in the solid Earth). δB_z is not considered because it is usually excluded for Pc 1 observation on the ground.

As the Pc 1 pulsations are injected in the ionosphere at the latitude of about 65° and the observation point in Japan is located at about 25°, we employ r=4000 km. Typical values of parameters are selected as follows: D=1000 km, $\mu_0 V_2\Sigma_\mathrm{H}=\mu_0 V_2\Sigma_\mathrm{P}=1$ (the height-integrated Pedersen and Hall conductivities), $\mu_0 V_2 D\sigma_G=1000$, $r_0/D=0.1$, $V_2/V_1=0.25$ and $h/D=0.1$ (thickness of the neutral atmosphere). Using

$$\phi_1(k_\mathrm{n})=\delta B_0 V_2(V_1/V_2)(e/\sqrt{2})r_0^3(k_\mathrm{n}r_0)\cdot\exp\left\{-\left(\frac{k_\mathrm{n}r_0}{2}\right)^2\right\}, \tag{3}$$

the factor in r.h.s. of eq. (2), $|I_1(k_j)||\rho_4 k_j T(k_j)|$ (this factor is called the transmission-localization factor), is plotted in Fig. 9 against a normalized frequency ($\omega D/V_2$) for the fundamental ducted wave ($j=1$) with the smallest imaginary part of k_j. Thus, this factor is as large as 10^{-4}. In addition, real and imaginary parts of k_j are also plotted in Fig. 10. As $k_1 D\sim 2+0.1i$ when $\omega D/V_2=2$ ($\omega=1\ \mathrm{s}^{-1}$) (Fig. 10) and $r/D=4$:

$$\begin{aligned}|\delta\boldsymbol{B}_\mathrm{n}|&\sim 10^{-4}(r_0/r)^{1/2}\exp(-k_{1i}r)\delta B_0\\&\sim 10^{-5}\delta B_0\ (r=4D).\end{aligned} \tag{4}$$

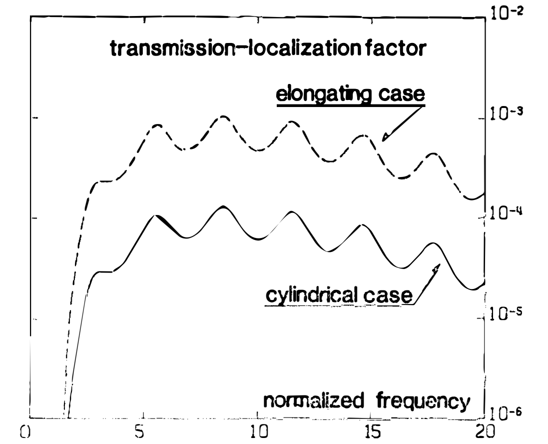

Fig. 9. The transmission-localization factor in the cylindrical case and that in the elongating case against $\omega D/V_2$ for $j=1$ (the fundamental ducted wave).

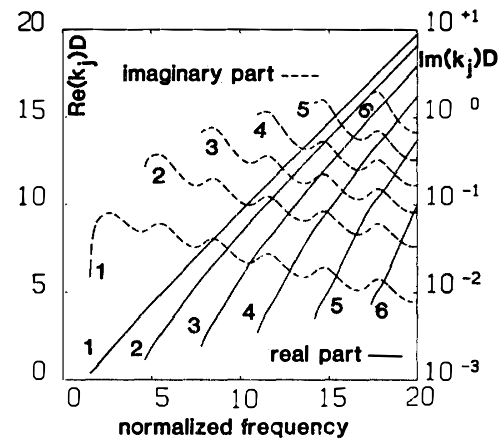

Fig. 10. Real and imaginary parts of k_j.

(2) Longitudinally elongating injection

There is no geometrical damping as explained in the Appendix. Hence,

$$|\delta \boldsymbol{B}_n| \sim \delta B_0 |I_2(k_j)| |\rho_1 k_j T(k_j)| \exp(-k_{ji} r). \tag{5}$$

When

$$\phi_1(k_x) = \delta B_0 V_2 (V_1/V_2) \frac{i k_x x_0^3}{2} \pi^{1/2} \exp\left\{ -\left(\frac{k_x x_0}{2}\right)^2 + 1 \right\}, \tag{6}$$

is employed, the transmission-localization factor with $I_2(k_j)$ for $j=1$ becomes as large as 10^{-3} as shown in Fig. 9 (assuming $x_0/D=0.1$). Therefore,

$$\begin{aligned} |\delta \boldsymbol{B}_n| &\sim 10^{-3} \exp(-k_{1i} x) \delta B_0 \\ &\sim 6 \times 10^{-4} \delta B_0 \ (x=4D), \end{aligned} \tag{7}$$

for the fundamental ducted wave.

Comparing the results of two cases, the former (the cylindrical case) has smaller magnetic field intensity due to geometrical damping when the magnetic field disturbance has the same intensity in high latitudes (the source region) in both cases. Thus, the HM chorus which is observed frequently in high latitudes but is rare in low latitudes may have concentrated injection and Periodic Emission is injected being extended in longitudes. This suggestion is harmonized with constant occurrence of the Periodic Emission for the local time in high latitudes.

On the other hand, many scientists suggested that localization of the source region is not only in the latitude but also in the longitude by means of propagation direction finding based on the polarization method of low-latitude pulsations (ALTHOUSE and DAVIS, 1978; WEBSTAR and FRASER, 1985). However, they did not mention what kind of the Pc 1 pulsation was used in their works. In order to confirm our theoretical suggestion, we need systematic observation of pulsations with a network expanding from a high-latitude region to a low-latitude region. Moreover, since the shape of $\Phi_i(r)$ or $\Phi_i(x)$ controls latitudinal spread of the wave field, determination of Φ_i is also very important.

(After TROITSKAYA (1964), the magnetic field intensity of the Pc 1 pulsation in high latitudes is as large as several nT's and that in low latitudes is about 10^{-2} nT. Thus, the ratio of magnetic field intensity in low latitudes to that in high latitudes is about 10^{-2}–10^{-3}. Besides, the intensity on the ground is about $10^{-1}\delta B_0$ in the idealized model (FUJITA and TAMAO, 1986). Thus, the result of eq. (7) gives the ratio as large as the observation.)

4. Conclusions

The Pc 1 pulsation data obtained in 1976–1984 at Japanese Observatory Network reveal the following characteristics:

(1) Occurrence of the Pc 1 pulsations in low latitudes is consistent with previous works, namely, Pc 1 pulsations in low latitudes appear more frequently in the period of minimum solar activity than in the maximum one, in winter than in summer and in night than in day. This feature is harmonized with theoretical results of the ionospheric duct propagation.

(2) Attenuation rates of the Pc 1 signals observed fall in the same range of order of magnitude as suggested by the duct propagation theory. However, the former is a little smaller than the latter.

(3) The lower boundary frequency of the Pc 1 signal observed at MMB is consisted with the lower cutoff frequency by theoretical calculation. However, there is an apparent discrepancy between the observation and the theory in the case of daytime in winter of the maximum solar active period.

Theoretical consideration on the relation between magnetic intensity in low latitudes and horizontal distribution of the localized incident wave in high latitudes leads us to the result:

(4) Geometrical damping of the wave field is $r^{-1/2}$ for the wave with cylindrically concentrated injection. Whereas, the wave with longitudinally expanding injection does not have geometrical damping. Therefore, the latter model gives larger magnetic field intensity in low latitudes than that obtained with the former model.

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Appendix

(a) Cylindrically concentrated injection

Employing Fourier-Bessel transformation described in FUJITA and TAMAO (1984), $S(r)$ is obtained as:

$$S(r)=\int_0^{\infty}\{s^*(k_n)/g(k_n)\}\,\phi_i(k_n)J_1(k_n r)k_n\mathrm{d}k_n, \qquad \text{(A-1)}$$

where r is a radial distance from the center of injection. g and k_n are, respectively, the dispersion-generating function obtained by boundary conditions of electromagnetic field disturbances at each interface between neighbouring layers and a wave number normal to the ambient magnetic field. $\phi_i(k_n)$ is derived from $\Phi_i(r)$ by means of the Fourier-Bessel transformation (FUJITA and TAMAO, 1984) and

$$s^*(k_n)=-2(V_2/V_1)\mu_0\Sigma_H\exp(-i\omega D/V_1)\{\cosh(\rho_2 D)-(\rho_1/\rho_2)\sinh(\rho_2 D)\}. \qquad \text{(A-2)}$$

This form can be obtained by means of boundary conditions expressed in FUJITA and TAMAO (1984) (although we must add the neutral atmosphere to their 4-layer model). Symbols used in the above equation are: D (width of the ducting layer), V_1 (the Alfvén speed in the magnetosphere), V_2 (the Alfvén speed in the ducting layer), $\rho_1=\{(\omega/V_1)^2-k_n^2\}^{1/2}$ (a vertical wave number of the fast magnetosonic wave in the magnetosphere), $\rho_2=\{(\omega/V_2)^2-k_n^2\}^{1/2}$ (a vertical wave number of the fast magnetosonic wave in the ducting layer). J_1 is the Bessel function with a degree of 1. Since $k_n\{s^*(k_n)/g(k_n)\}\phi_i$ is an odd function of k_n, we can obtain the following form:

$$S(r)=\int_{-\infty}^{+\infty}\{s^*(k_n)/2g(k_n)\}\,\phi_i(k_n)H_1^{(1)}(k_n r)k_n\mathrm{d}k_n. \qquad \text{(A-3)}$$

As $k_n r\gg 1$ in low latitudes,

$$H_1^{(1)}(k_n r)\sim\exp(ik_n r-3\pi i/4)(2/\pi k_n r)^{1/2}. \qquad \text{(A-4)}$$

Bearing in mind that

$$\phi_i(k_n)=\int_0^{\infty}r'\mathrm{d}r'\Phi_i(r')J_1(k_n r'), \qquad \text{(A-5)}$$

eq. (A-3) is transformed into the following form:

$$S(r)\sim\int_{-\infty}^{+\infty}\Phi_i(r')F_s(r')r'\mathrm{d}r', \qquad \text{(A-6)}$$

where

$$F_s(r')=\int_{-\infty}^{+\infty}\{s^*(k_n)/2g(k_n)\}\,H_1^{(1)}(k_n r)H_1^{(1)}(k_n r')\mathrm{d}k_n. \tag{A-7}$$

When the only contribution from a pole of $g=0$ ($k_n=k_j$ for the j-th ducted wave) is considered,

$$S(r)\sim\delta B_0 I_1(k_j)T(k_j)(r_0/r)^{1/2}\exp(ik_j r), \tag{A-8}$$

where $T(k_j)$ is a factor representing ionospheric transmission of the signal:

$$T(k_j)=(V_2/4\pi i)k_n\left(\frac{2}{\pi k_j}\right)^{\frac{1}{2}}\exp\left(\frac{3}{4}\pi i\right)\{s^*(k_n)/\partial g/\partial k_n\}\,|_{k_n=k_j}, \tag{A-9}$$

and $I_1(k_j)$ is a factor representing localization of the incident wave:

$$I_1(k_j)=(1/\delta B_0 V_2)(r_0)^{-1/2}\phi_i(k_j). \tag{A-10}$$

Thus, there are the geometrical damping factor, $r^{-1/2}$, and the dissipative damping factor, $\exp(-k_{ji}r)$, where k_{ji} is the imaginary part of k_j.

(b) Longitudinally elongating injection

The Fourier transform is convenient. Expression of s^* does not change since the boundary conditions do not change in both cases of (a) and (b). Thus, we can obtain the following form instead of eq. (A-1):

$$S(x)=\int_{-\infty}^{+\infty}\{s^*(k_x)/g(k_x)\}\,\phi_i(k_x)\exp(ik_x x)\mathrm{d}k_x. \tag{A-11}$$

where x and k_x are, respectively, a latitudinal distance from the injection latitude and the x-component of a wave number. To proceed further analysis, let eq. (A-11) transform into a form as below:

$$S(x)=\int_{-\infty}^{+\infty}\Phi_i(x')F_s(x')\mathrm{d}x', \tag{A-12}$$

hence, we can obtain the following equation similar to eq. (A-8),

$$S(r)=\delta B_0 I_2(k_j)T(k_j)\exp(ik_j r), \tag{A-13}$$

where

$$I_2(k_j)=(1/\delta B_0 V_2)\phi_i(k_j). \tag{A-14}$$

Therefore, there is only the dissipative damping factor.