DIRECT CONTRIBUTION OF OBLIQUE FIELD-ALIGNED CURRENTS TO MAGNETIC FIELD VARIATIONS ON THE GROUND

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Abstract: Attempt has been made to evaluate a direct contribution of the horizontal component of oblique field-aligned currents to surface magnetic variations in the auroral region associated with a localized electric potential distribution on the horizontal plane, in comparison with that of the ionospheric eddy current. This direct effect is proportional to a product of two ratios, the Pedersen to Hall conductivities and the horizontal to vertical components of the local dipole magnetic intensity. In addition to these, the shorter the horizontal scale-length in the east-west direction compared with that in the north-south direction, the larger the direct effect of field-aligned currents. For the localized perturbation with an isotropic horizontal structure at 60° geomagnetic latitude, it attains to about 80% of the magnetic contribution of the Hall current at the earth's surface. It is also shown that only the vertical component of oblique field-aligned currents connects with the irrotational part of ionospheric currents, and that the effective conductance in the magnetosphere for three-dimensional current systems is reduced to a smaller value than that for the case of vertical field lines.

1. Introduction

Electromagnetic interaction between the magnetosphere and the ionosphere through field-aligned currents (FAC) associated with localized EM-disturbances in high latitudes, particularly effects of induced currents in the anisotropic conducting ionosphere to ground magnetic field signature, has been developed with the assumption of vertical magnetic field lines. TAMAO (1964) showed that only the induced ionospheric Hall current can give rise to ground magnetic variations at the time of incidence of localized shear Alfvén wave propagating downward along a vertical field line into the plane ionosphere. Making use of intuitive consideration, FUKUSHIMA (1969) obtained a generalized theorem of no ground magnetic effect of vertical field-aligned currents connected with Pedersen currents in the uniform conductivity ionosphere.

Recently, detailed quantitative studies of ionospheric effects on localized EM-disturbances have been made in association with horizontal distributions of geomagnetic pulsations on the ground (*cf.* HUGHES and SOUTHWOOD, 1976; WALKER *et al.*, 1979; OGUTI and HAYASHI, 1984). GLASSMEIER (1983, 1984) has performed quantitative model calculations of the influence of a horizontally non-uniform conducting ionosphere on localized HM-waves. Joint observations of ground magnetic and ionospheric electric fields in the auroral zone have made it possible to examine a modelling of ionospheric effects on surface magnetic variations (BAUMJOHANN *et al.*, 1981). Another attempt in obtaining more large-scale distributions of three-dimensional current systems and associated electric potential in high latitudes, based on the ground magnetic field data together with assumed conductivity distributions, has yielded reasonable results in comparison with those observed more directly by radar and satellite (*cf.* KAMIDE *et al.*, 1981; AKASOFU *et al.*, 1981).

All of such studies referred above are on the basis of vertical magnetic field lines. General formulation in describing EM-interaction between the ionosphere and the magnetosphere for the spherical surface ionosphere with nonuniform conductivities under the dipole magnetic field line configuration has been obtained (TAMAO, 1983). In the present paper we shall apply the method to a simple model of localized pattern of potential distribution to demonstrate the direct contribution of oblique field-aligned currents to ground magnetic field within the limit of a plane ionosphere.

2. Model and Formulation

Let us now describe briefly a model and formulation to be used in the following numerical calculations. We assume a two-region model separated by an interface (the ionosphere) with height-integrated electrical conductivities, $\sum_P$ (Pedersen) and $\sum_H$ (Hall). Region 1 is representing the neutral atmosphere below the ionosphere wherein both electric and magnetic field perturbations can be represented in terms of electric and magnetic potentials, V and U, respectively, because the time-scale is so slow as to neglect the propagation time of EM-waves with the speed of light. For the same reason the toroidal type magnetic field is negligible in comparison with the poloidal one, and thus we have

$$\boldsymbol{E}_1 = -\nabla V \qquad \text{and} \qquad \boldsymbol{B}_1 = \boldsymbol{B}_1^{\mathrm{P}} = -\nabla U\,,$$

where the superscript P means the poloidal type. In region 2 we employ a cold magnetized plasma under the ambient magnetic field $\boldsymbol{B}_0$ with a local unit vector along a field line, $\hat{e} = \boldsymbol{B}_0/B_0$. Under the condition of $\omega R_{\mathrm{I}} \sum_{\mathrm{P,H}} \ll 1$ where ω and R_{I} are the angular frequency of perturbations and the radius of ionospheric height, we can neglect the inductive electric field associated with the fast magnetosonic wave (TAMAO, 1984). For typical values of $R_{\mathrm{I}} \simeq 6 \times 10^{8}$ cm and $\sum_{\mathrm{P,H}} \simeq 10^{-8}$ emu the above condition yields $\omega \ll 1/6$ rad/s. In region 2 we have also assumed that the field-aligned electric field component is negligible compared with the perpendicular one, and so $\boldsymbol{E}_2 = -\nabla_{\perp}\Phi$ and $\boldsymbol{B}_2 = \boldsymbol{B}_2^{\mathrm{P}} + \boldsymbol{B}_2^{\mathrm{FAC}}$, where the first term represents the poloidal magnetic field due to the ionospheric eddy currents and the second term comes from the field aligned currents (refer to Fig. 1).

Defining the local outward unit vector perpendicular to the interface, $\hat{N}$, the height-integrated ionospheric current is given by

$$\boldsymbol{J} = \sum\nolimits_{\mathrm{P}} \hat{N} \times (\boldsymbol{E} \times \hat{N}) + \sum\nolimits_{\mathrm{H}} \boldsymbol{E} \times \hat{N}\,. \tag{1}$$

Boundary conditions at the interface, $\hat{N} \times (\boldsymbol{E}_2 - \boldsymbol{E}_1) = 0$ and $4\pi \boldsymbol{J} = \hat{N} \times (\boldsymbol{B}_2 - \boldsymbol{B}_1)$, are re-

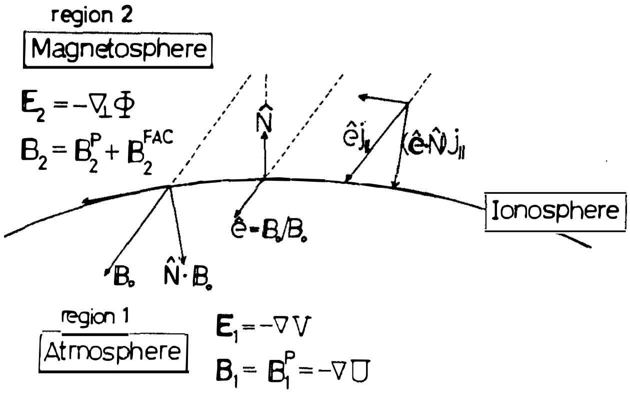

Fig. 1. A two-region model consisting of the cold magnetosphere and the neutral atmosphere separated by the ionospheric interface with anisotropic conductivities. The direction of ambient magnetic field lines is obliquely crossing the interface. Superscripts on the magnetic field, P and FAC, mean the poloidal type and the field due to field-aligned currents, respectively.

duced to the following scalar relations;

$$\operatorname{div}\{\hat{N}\times[(\boldsymbol{E}_2-\boldsymbol{E}_1)\times\hat{N}]\}=0\ , \tag{2}$$

$$\hat{N}\cdot(\boldsymbol{B}_2{}^{\mathrm{P}}-\boldsymbol{B}_1{}^{\mathrm{P}})=-\hat{N}\cdot\boldsymbol{B}_2{}^{\mathrm{FAC}}\ , \tag{3}$$

$$2\operatorname{div}[\hat{N}(\hat{N}\cdot\boldsymbol{B}_2{}^{\mathrm{P}})]=-4\pi\hat{N}\cdot\operatorname{rot}\boldsymbol{J}\ , \tag{4}$$

and

$$\operatorname{div}\boldsymbol{J}=-(\hat{N}\cdot\hat{e})j_\parallel\ . \tag{5}$$

In derivation of eq. (4), we have made use of $\operatorname{div}\{\hat{N}[\hat{N}\cdot(\boldsymbol{B}_1{}^{\mathrm{PI}}+\boldsymbol{B}_2{}^{\mathrm{P}})]\}=0$ where $\boldsymbol{B}_1{}^{\mathrm{P}}=\boldsymbol{B}_1{}^{\mathrm{PI}}+\boldsymbol{B}_1{}^{\mathrm{PM}}$, the superscripts PI and PM stand for poloidal field due to ionospheric and magnetospheric currents, respectively. Equation (3) is exhibiting that magnetic flux of FAC leaks to the region below the ionosphere if the FAC yields the vertical magnetic component to the interface, while the vertical FAC has no direct contribution to ground magnetic fields. Equations (4) and (5) are indicating that only the rotational part of the ionospheric currents produces the poloidal type magnetic fields in both regions above and below the ionosphere whereas the irrotational part connects to the vertical component of FAC. Consequently, efficiency of discharge of ionospheric space charges through field-aligned currents for the case of oblique magnetic field lines is less than that for vertical field lines.

Now we employ local Cartesian coordinates, x (southward), y (eastward) and z (vertically upward), for a model of the plane ionosphere ($z=0$). EM-field perturbations in region 2 ($z>0$) are represented by

$$\boldsymbol{E}_2=-\nabla\Phi+(\hat{e}\cdot\nabla\Phi)\hat{e} \tag{6}$$

and

$$\boldsymbol{B}_2{}^{\mathrm{P}}=-\nabla W\ ,\quad \boldsymbol{B}_2{}^{\mathrm{FAC}}=\operatorname{rot}(A_\parallel\hat{e})\ , \tag{7}$$

under the circumstances mentioned before. Making use of the Fourier integral method in representing horizontal structure of the perturbations, *e.g.*,

$$U(x, y, z)=\int \tilde{U}(k_x, k_y) \exp [kz+i(k_x x+k_y y)]\mathrm{d}k_x \,\mathrm{d}k_y \ ,$$

where $\tilde{U}$ is the Fourier transform of U, and applying the boundary conditions, eqs. (3)–(5), together with the expression of $j_{\parallel}$ in terms of $A_{\parallel}$, we obtain

$$k(\tilde{U}+\tilde{W})=ik_y e_\theta \tilde{A}_{\parallel} \ , \tag{8}$$

$$4\pi \hat{z}\cdot \mathrm{rot}\,\tilde{J}=2k^2 \tilde{W} \ , \tag{9}$$

$$\begin{aligned} 4\pi \,\mathrm{div}\,\tilde{J}= & -e_R\,[k^2+(e_\theta/e_R)^2 k_x^2]\,\tilde{A}_{\parallel} \\ & +i\,[(k_x/R_\mathrm{I})\,\partial \ln (\sin\theta/e_R)/\partial\theta]\tilde{A}_{\parallel} \ . \end{aligned} \tag{10}$$

In the above, e_R and e_θ are the R- and θ-components of the unit vector $\hat{e}$ in spherical coordinates (R, θ, ϕ), and $k=(k_x^2+k_y^2)^{1/2}$. For relatively small-scale localized perturbations with $kR_\mathrm{I}\gg 1$, eqs. (8)–(10) yield the following forms of the magnetic potentials given as

$$\begin{aligned} k^2\tilde{U}= & -2\pi\hat{z}\cdot \mathrm{rot}\,\tilde{\boldsymbol{J}} \\ & -4\pi i(e_\theta k_y/e_R k)[1+(e_\theta k_x/e_R k)^2]^{-1}\,\mathrm{div}\,\tilde{\boldsymbol{J}} \ , \end{aligned} \tag{11}$$

and

$$k^2\tilde{W}=2\pi\hat{z}\cdot \mathrm{rot}\,\tilde{\boldsymbol{J}} \ . \tag{12}$$

In eq. (11), the first and second terms stand for contributions of the ionospheric eddy currents and oblique FAC to ground magnetic field, respectively, the latter vanishes for the case of vertical field line ($e_\theta=0$), as well as the two-dimensional case ($k_y=0$). We introduce an equivalent ionospheric current function for ground magnetic field given by eq. (11) which is composed of two parts, the ionospheric eddy current and the magnetospheric FAC, $\tilde{J}=\tilde{J}_\mathrm{I}+\tilde{J}_\mathrm{M}$, where

$$\tilde{J}_\mathrm{I}=-(2\pi/k^2)\hat{z}\cdot \mathrm{rot}\,\tilde{\boldsymbol{J}} \ , \tag{13}$$

$$\tilde{J}_\mathrm{M}=4\pi i\varepsilon k_y k^{-3}[1+\varepsilon^2(k_x/k)^2]^{-1}\,\mathrm{div}\,\tilde{\boldsymbol{J}} \ , \tag{14}$$

and $\varepsilon=B_\theta/B_R$ is a ratio of the horizontal to vertical component of the dipole magnetic field at the ionospheric level. Thus, we have

$$\tilde{U}=(\tilde{J}_\mathrm{I}+\tilde{J}_\mathrm{M}) \quad \text{and} \quad \tilde{W}=-\tilde{J}_\mathrm{I} \ , \tag{15}$$

respectively.

3. Numerical Results and Discussion

As an application of the formulation developed in the previous section, we demonstrate a few of quantitative results exhibiting a relative importance of the direct contribution of oblique FAC to ground magnetic fields by employing a simple isolated from of the ionospheric electric potential distribution given by

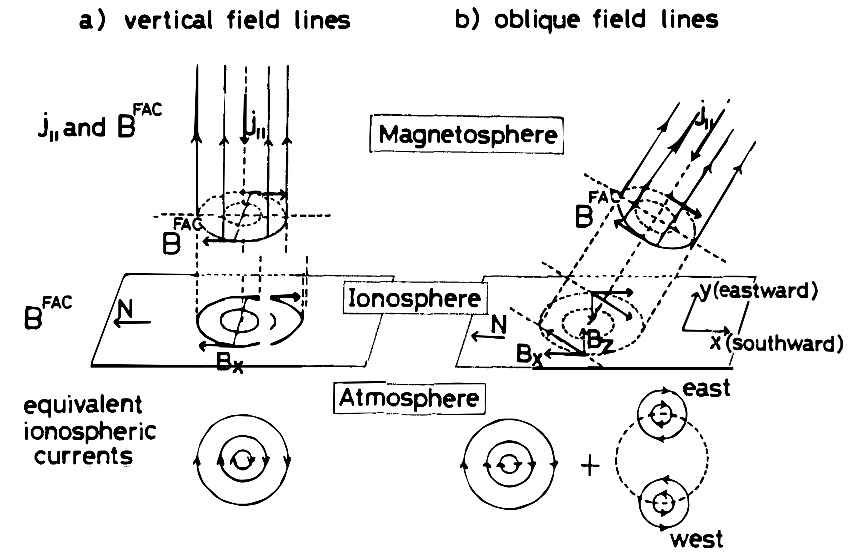

Fig. 2. Schematic diagrams giving intuitive understanding for the different situation between vertical (left) and oblique (right) field-line cases. Equivalent ionospheric current systems for ground magnetic fields in each case are shown (botton), and twin vortices with opposite polarities represent the direct contribution of oblique field-aligned currents.

$$\Phi(x, y, 0)=\Phi_0 \exp[-(x/a)^2-(y/b)^2] .$$

In Fig. 2, schematic diagrams illustrating intuitively a distinction in ground magnetic fields between the vertical (left panel) and oblique (right panel) field line cases for the localized perturbation given above. In such a situation, the direct effect of oblique FAC to the ground magnetic field is represented in terms of twin equivalent ionospheric current vortices with opposite polarity aligned in the east-west direction, while there

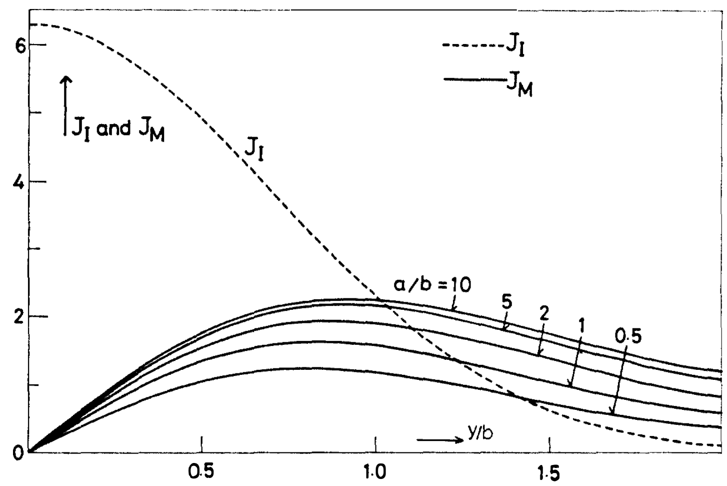

Fig. 3. Intensity variations of equivalent ionospheric current functions normalized in $\Sigma_P\Phi_0$ for the ground magnetic field, J_I (ionospheric origin) and J_M (magnetospheric origin), versus distances along the east-west direction for various values of a ratio of a/b. a and b are scale-lengths of the horizontally localized electric potential in the north-south and east-west directions, respectively.

is a single eddy current corresponding to the isolated electric potential distribution in the case of vertical filed lines.

We have performed numerical integrations of inverse Fourier transform to obtain the ionospheric current function and magnetic potential for the above form of the electric potential distribution. Shown in Fig. 3 are intensity variations of the equivalent ionospheric current functions along the y-axis, $J_I(0, y, 0)$ and $J_M(0, y, 0)$, normalized in $\sum_P \Phi_0$ for various values of a ratio of a/b, $\sum_P = \sum_H$, and θ (colatitude)$=$30°. As was expected in Fig. 2, peaks of J_M are taking place at $y \simeq \pm b$, and its magnitude decrease with decreasing a/b. Plots of the y-component of ground magnetic variations, B_y (broken curves), $B_y{}^I$ (ionospheric effect; dotted curves), and $B_y{}^M$ (direct effect of FAC; full curves) normalized in $\sum_P \Phi_0/b$, *versus* y/b are given in Fig. 4 for three

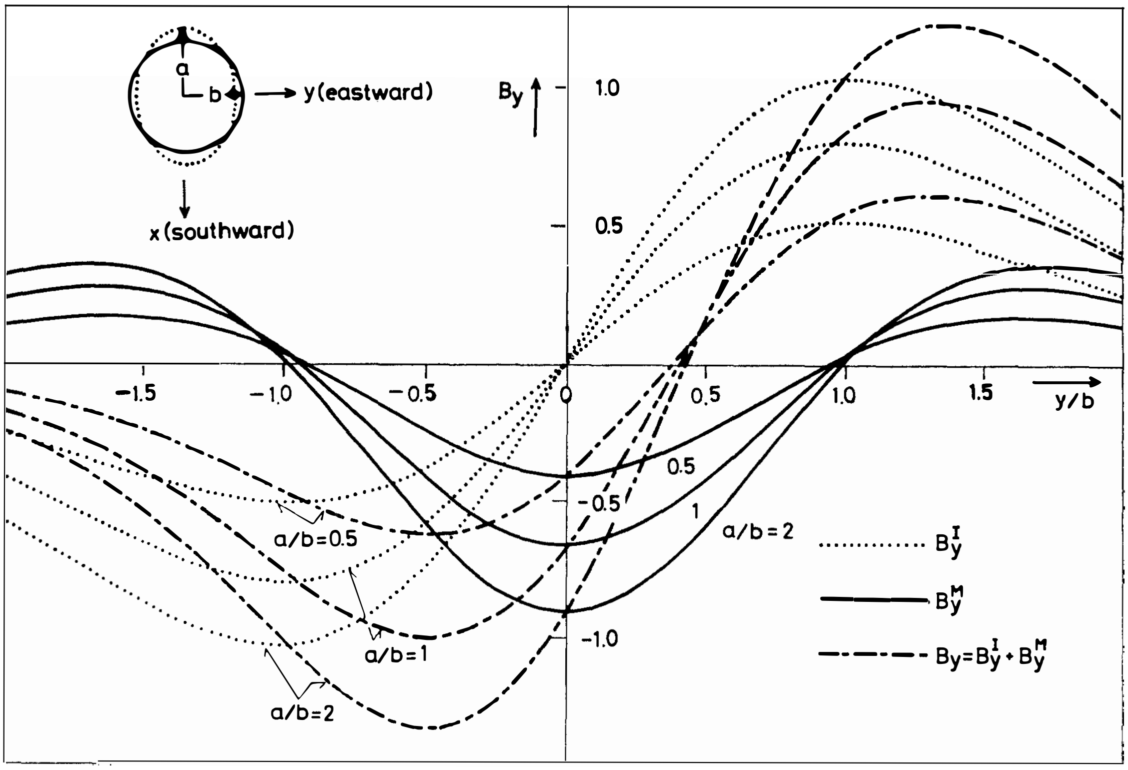

Fig. 4. Variations of the total intensity of the east-west component of ground magnetic field B_y (broken curves) consisting of two parts, the ionospheric origin $B_y{}^I$ (dotted curves) and the direct contribution of oblique field-aligned currents $B_y{}^M$ (full curves), for three different cases of the anisotropy in horizontal scale-lengths of the localized electric potential, $a/b=0.5$, 1, and 2.

different values of a/b in the case of $\sum_P = \sum_H$, $\theta = 30°$, and $z = -b$. These plots show $\pi/2$ phase lag in spatial distributions along the east-west direction between the two contributions to B_y. The maximum value of $B_y{}^M$ for the isotropic horizontal structure ($a=b$) is about 80% of the magnetic contribution due to the ionospheric eddy current, and thus we must be careful in evaluation of ionospheric currents by using a network observation of ground magnetic field variations even in high latitudes. Another difference in the case of oblique magnetic field lines compared with that in the vertical case is the decreased efficiency in discharge process through FAC in a three-dimensional current system. For instance, the induced ionospheric potential for the incidence of the localized shear Alfvén wave with the electrostatic potential Φ_i becomes

$$\Phi_r=[(\Sigma_W-\Sigma_P)/(\Sigma_W+\Sigma_P)]\,\Phi_i\,,$$

where $\Sigma_W=|e_R|/4\pi V_A$ is the effective wave conductance in the cold magnetosphere with the Alfvén speed of V_A.

4. Summary

Formulation to describe EM-coupling between the magnetosphere and the ionosphere through oblique field-aligned currents has been performed. Numerical calculation for the estimation of the direct contribution of horizontally localized oblique FAC to ground magnetic fields, based on the plane ionosphere model with uniform anisotropic conductivity distribution, has made it possible to obtain the following summary results.

1) The horizontal component of oblique FAC can give rise to the direct contribution to magnetic field variations in the region below the ionosphere, while only the induced ionospheric eddy current is the origin of ground magnetic field in the case of vertical magnetic field lines.

2) A relative importance of such direct effect to that of ionospheric eddy currents is proportional to both ratios, the Pedersen to Hall conductivities and the horizontal to vertical component of the dipole magnetic field. It also strongly depends on the anisotropy in horizontal structure of localized disturbances, *i.e.*, the shorter the horizontal scale-length in the longitudinal direction compared with that in latitudinal one, the larger the direct effect of oblique FAC.

3) The direct effect can be represented in terms of the equivalent ionospheric current function. For the simple isolated pattern of electric potential on the horizontal plane yielding a single current vortex in the ionosphere, the direct contribution is equivalent to twin current vortices with opposite polarities and their centers are aligned in the east-west direction. Thus, their maximum contribution to the ground magnetic field occurs at the position below the center of the real Hall current vortex in the ionosphere.

4) In the simple case mentioned above, the maximum contribution of oblique FAC to the east-west component of the ground magnetic field amounts to 80% of that from the real ionospheric eddy current induced by the electric potential with the isotropic horizontal structure even at 60° geomagnetic latitude.

5) Because only the vertical component of oblique FAC connects to the irrotational part of ionospheric currents, the coupling efficiency decreases with a decrease in latitude although angle of dipole field lines to the vertical direction increases.

We have presented a few of numerical examples in the simplest case to demonstrate the direct magnetic effect of oblique FAC. Other cases to be studied along the same line are such perturbations that are localized in latitudes and propagating in the longitudinal direction, and also a relationship between large-scale current vortices in high latitudes and the associated equatorial zonal current.

References

AKASOFU, S.-I., KAMIDE, Y. and KISABETH, J. (1981): Comparison of two modeling for three-dimensional current systems. J. Geophys. Res., **86**, 3389–3396.

BAUMJOHANN, W., PELLINEN, R. J., OPGENNORTH, H. J. and NIELSEN, E. (1981): Joint two-dimensional observations of ground magnetic and ionospheric electric fields associated with auroral zone currents; Current systems associated with local auroral break-up. Planet. Space Sci., **29**, 431–447.

FUKUSHIMA, N. (1969): Equivalence in ground geomagnetic effect of Chapman-Vestine's and Birkeland-Alfvén's electric current systems for polar magnetic storms. Rep. Ionos. Space Res. Jpn., **23**, 219–227.

GLASSMEIER, K.-H. (1983): Reflection of MHD-waves in the Pc 4–5 period range at ionospheres with non-uniform conductivity distributions. Geophys. Res. Lett., **10**, 678–681.

GLASSMEIER, K.-H. (1984): On the influence of ionospheres with non-uniform conductivity distribution on hydromagnetic waves. J. Geophys., **54**, 125–137.

HUGHES, W. J. and SOUTHWOOD, D. J. (1976): The screening of micropulsation signals by the atmosphere and ionosphere. J. Geophys. Res., **81**, 3234–3240.

KAMIDE, Y., RICHMOND, A. D. and MATSUSHITA, S. (1981): Estimation of ionospheric electric fields, ionospheric currents, and field-aligned currents from ground magnetic records. J. Geophys. Res., **86**, 801–813.

OGUTI, T. and HAYASHI, K. (1984): Multiple correlation between auroral and magnetic pulsations, 2. Determination of electric currents and electric fields around patch. J. Geophys. Res., **89**, 7467–7481.

TAMAO, T. (1964): The structure of three-dimensional hydromagnetic waves in a uniform cold plasma. J. Geomagn. Geoelectr., **16**, 89–114.

TAMAO, T. (1983): Contributions of ionospheric currents and oblique field-aligned currents to localized magnetic variations on the ground. IAGA Session on Separation of the Observed Magnetic Field into Main Ionospheric and Magnetospheric Contributions. Abstracts, Hamburg, 512.

TAMAO, T. (1984): Magnetosphere-ionosphere interaction through hydromagnetic waves. Proc. of Achievements of IMS, Graz, Austria, ESA Pub. Branch, **SP-217**, 427–435.

WALKER, A. D. M., GREENWALD, R. A., STUART, W. F. and GREEN, C. A. (1979): Stare auroral radar observations of Pc 5 geomagnetic pulsations. J. Geophys. Res., **84**, 3373–3388.

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