A STUDY OF DIFFUSED WHISTLERS

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Abstract: Whistler data were analyzed for making a study of irregularity of the ionosphere, especially field-aligned ducts. A pure dipole geomagnetic field model and the electron number density distribution data were used for estimation of whistler duct width. Estimated duct width is not linearly increased with increasing diffuseness and its relation curve becomes suddenly steep near the 40° latitude. The duct width estimated by diffuseness (dt) varies with geomagnetic latitude, for instance, the duct width becomes 1000–2000 km at 30° and 500–700 km at 40° for the same diffuseness ($dt=15$ ms).

1. Introduction

It has been theoretically shown that the whistlers energy may be trapped in the ducts, the field-aligned columns of enhanced ionization, extending between the northern and the southern hemispheres.

On theoretical grounds by Smith *et al.* (1960), it has been shown that VLF waves can propagate along the field-aligned columns of enhanced ionization. But there are no clear experimental evidences, especially the correlation between the duct formation and the enhanced whistler activity at low latitude.

Somayajulu and Tantry (1968) calculated the effective width of VLF ducts from the corresponding diffuseness of the recorded whistler waves at 5 kHz. They found that the width varied from 15 to 25 km for the quiet day ($\Sigma Kp<30$) and 40 to 180 km for the disturbed day ($\Sigma Kp>30$).

Tanaka and Hayakawa (1973) also calculated the spread in travel time assuming the snake-like propagation in a single field-aligned duct with varying duct width, enhancement factor of electron density gradient and initial wave normal angles to the magnetic field above the F2 maximum. Their results show that the maximum initial wave normal angle trapped inside the duct decreases with increasing duct width, and the maximum time difference between the minimum and maximum initial wave normal angles decreases. These time differences are themselves probably upper limits because of the smallness of the transmission cone at the whistler exit point. Therefore, they suggest that the most probable interpretation of whistler diffuseness is that it is due to the difference in travel time for whistlers propagating along elemental ducts lying on the inner and outer field lines through a duct region.

By this reasoning, they deduced the effective duct width as a time difference between inner and outer field lines through a duct region with a simple model of a constant electron density and constant geomagnetic field intensity.

The precise computation with an actual electron density distribution and a magnetic field model will be necessary for more precise estimation of the effective duct width.

In the following chapter, the method of duct width estimation and its results are also discussed. The pure dipole magnetic field model and the electron distribution data by MAEDA and KIMURA (1970) are used in this paper.

2. Estimation of Duct Width

According to STOREY (1953), propagation time (t) for a particular frequency (f) is given by

$$t=\frac{1}{2c}\int_{\text{path}}\frac{f_P f_H}{f^{1/2}(f_H-f)^{3/2}}\,\mathrm{d}s=\int_{\text{path}}\frac{\mathrm{d}s}{v_g}\,, \tag{1}$$

where

f_P: plasma frequency,
f_H: electron gyro-frequency,
c: the velocity of light in free space,
$\mathrm{d}s$: the path length,
v_g: group velocity.

Differentiating eq. (1), we get

$$\mathrm{d}t=\frac{f_P f_H}{2cf^{1/2}(f_H-f)^{3/2}}\,\mathrm{d}s=\frac{\mathrm{d}s}{v_g}\,. \tag{2}$$

The diffuseness (dt) at 5 kHz is obtained as a result of vertical line traces on sona-

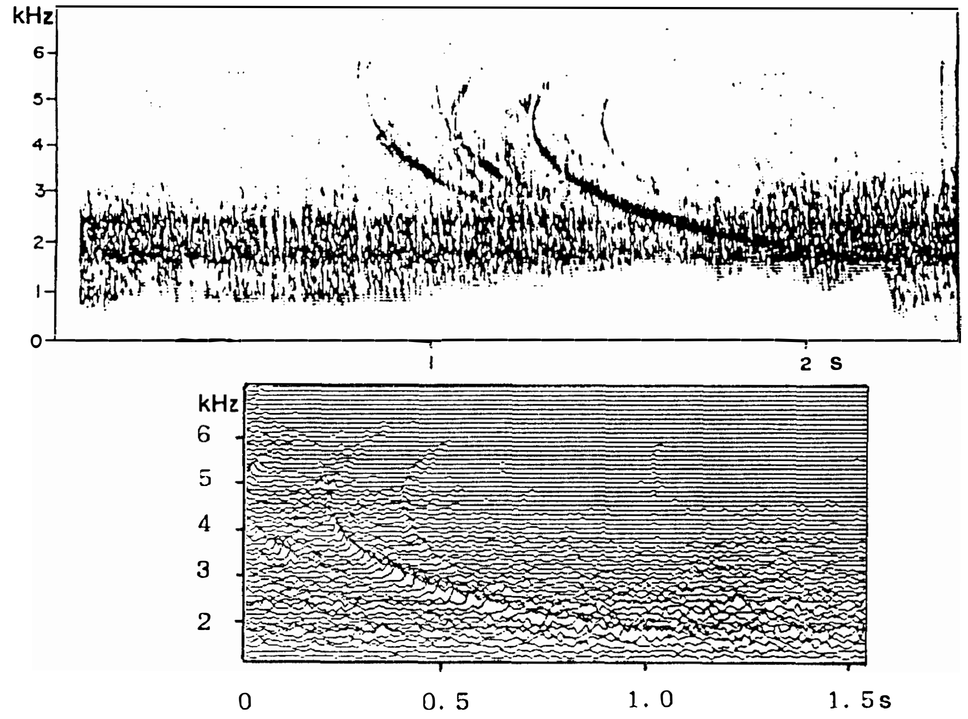
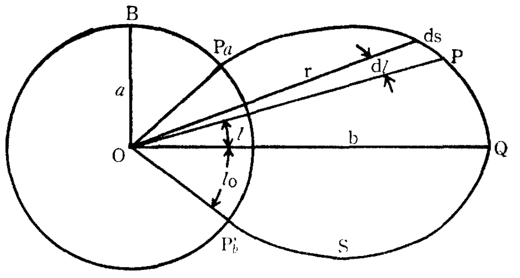


Fig. 1. An example of dynamic spectrogram of noise whistlers.



Symbols a ; mean earth radius (6371.229kM)
r ; geomagnetic radius
b ; geomagnetic equator radius
l_0 ; geomagnetic latitude of field line at a (P$_b$)
l ; geomagnetic latitude
S ; arc-length along field line between geomagnetic equator and $l_0=\overset{\frown}{QP}$

Fig. 2. A schematic illustration of the geomagnetic configuration.

grams as shown in Fig. 1. The increment in path length (ds) is calculated for a whistler energy of 5 kHz and it is possible to calculate the effective differential height or the duct width as a function of latitude by employing the calculated value of (ds) as shown in Fig. 2. The scale of magnetosphere is expanded because of intensification in Fig. 2.

According to CHAPMAN and SUGIURA (1956), the increment in path length (ds) is calculated by

$$ds=b\sqrt{1+3\sin^2 l}\cos l\, dl\,. \qquad (3)$$

The calculated values of whistler propagation time *versus* geomagnetic latitude

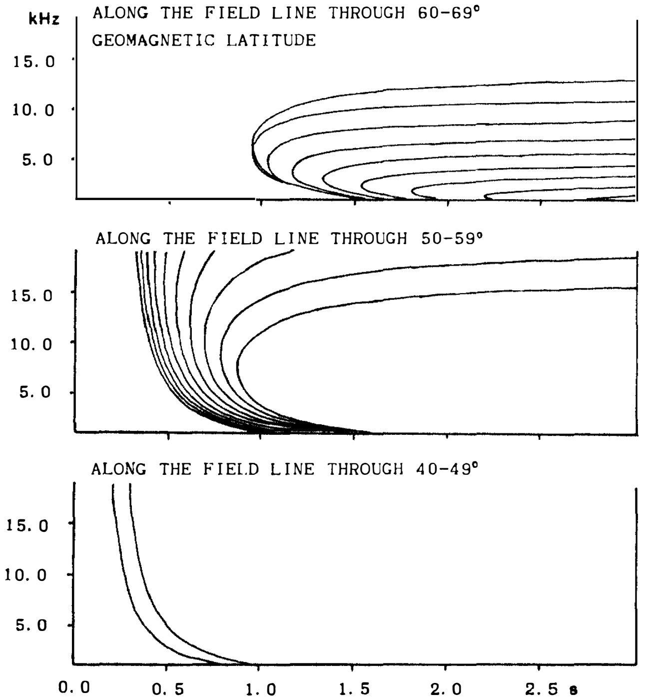


Fig. 3. Propagation times along the field line through 40–69° of geomagnetic latitude.

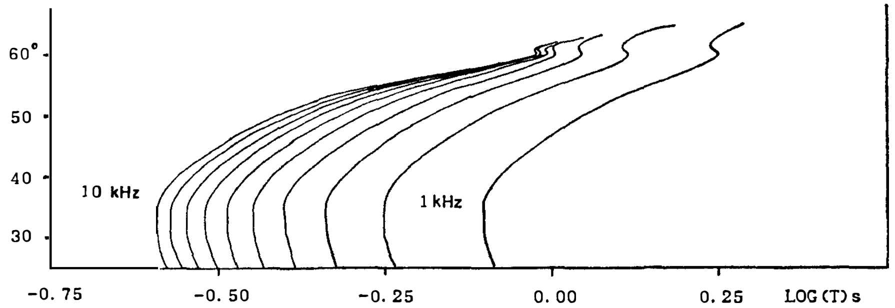


Fig. 4. Propagation times versus geomagnetic latitude (wave frequency 1–10 kHz).

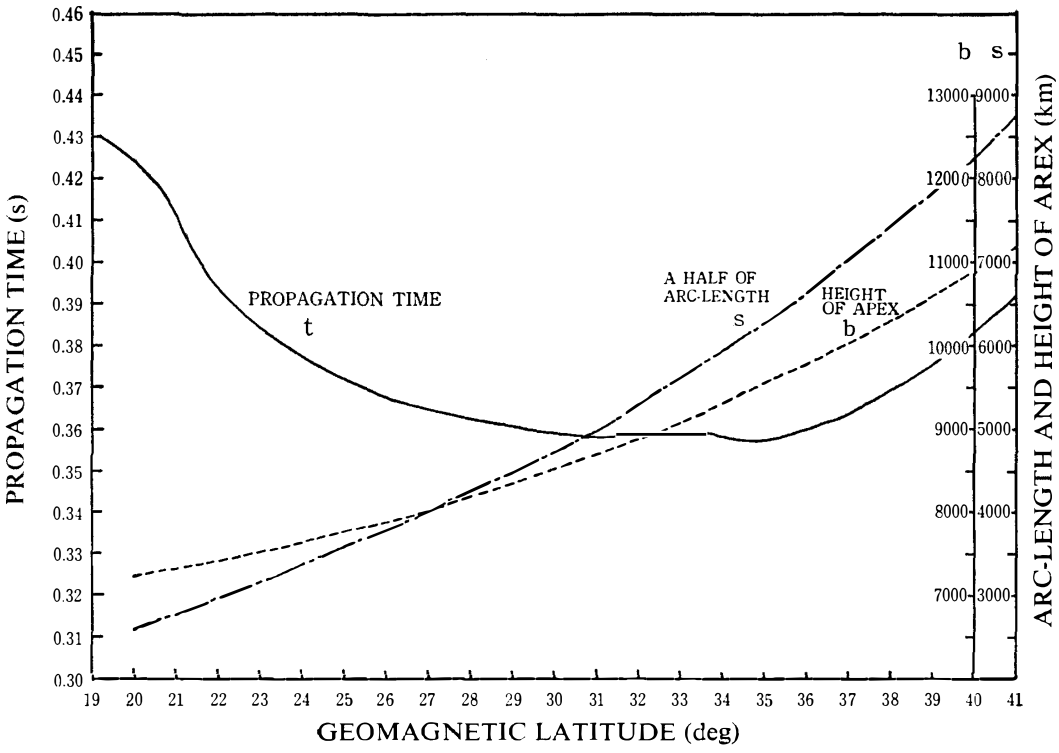


Fig. 5. Propagation times versus geomagnetic latitude (geomagnetic latitude 19–41°).

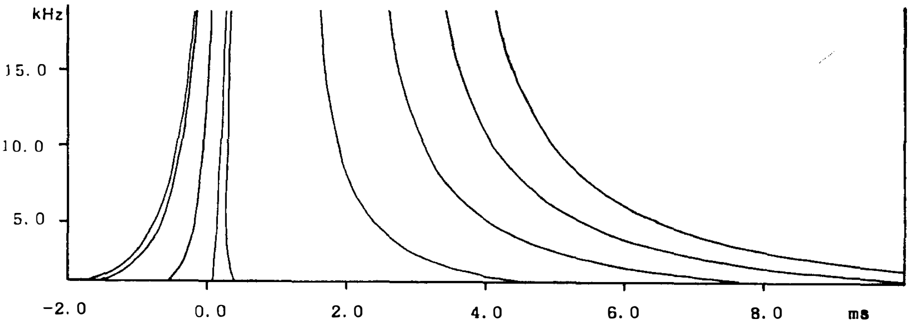
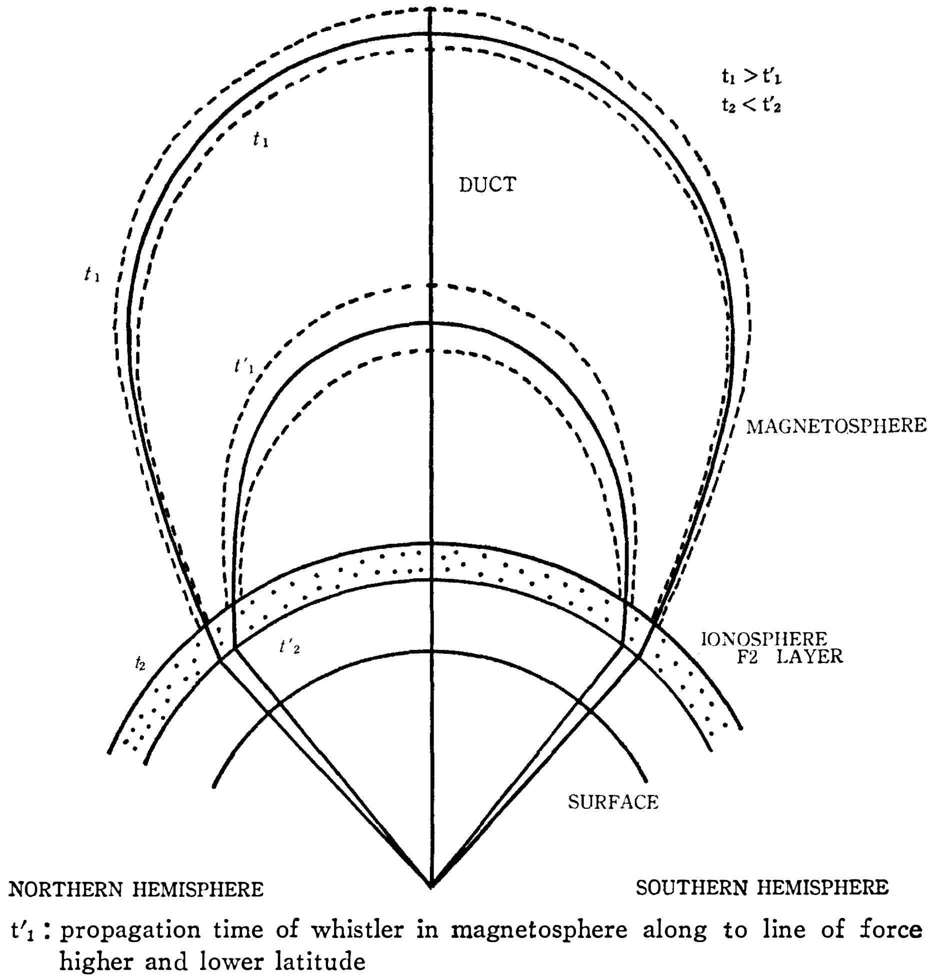


Fig. 6. Time difference between adjacent ducts for whistler propagation (30–39°).

are shown in Figs. 3, 4, 5, and time difference between ajacent lines of force which are different from each other only one degree in geomagnetic latitude is shown in Fig. 6.

We may now obtain values of (d*s*) for diffuseness (d*t*) at 5 kHz and then, from the field line geometry, deduce the width of the duct region at the apex of the field line. The propagation time difference between geomagnetic latitude 29° and 23.5° becomes 15 ms and its differential height is about 500 km at the apex of geomagnetic line of force so that the effective duct width corresponding to the diffuseness ($dt=15$ ms) becomes 500 km as shown in Fig. 5. These results are very different from the results of SOMAYAJULU and TANTRY (1968) and TANAKA and HAYAKAWA (1973).

The group velocity v_g of whistler becomes minimum in the F2 layer because of maximum density N. In higher latitude, the total path length becomes longer than in lower latitude, but the cross path length of the F2 layer becomes shorter because of the field line configuration. In the lower latitude, the total path length becomes shorter but the cross path length of the F2 layer becomes longer, and the total propagation time differences cancel each other and are not much changed as shown in Fig. 7. The scale of the magnetosphere is expanded because of intensification in Fig. 7.



t_1, t'_1 : propagation time of whistler in magnetosphere along to line of force at higher and lower latitude

t_2, t'_2 : propagation time of whistler in ionosphere along to line of force at higher and lower latitude

Fig. 7. A schematic illustration of the propagation path of whistler waves.

According to the above-mentioned reasoning, the curve of time difference *versus* path length becomes less sharp, almost flat, and the duct width for 15 ms diffuseness markedly increases to a value of about 1000–2000 km at geomagnetic latitude 29° as shown in Fig. 5. The effective duct width does not have the same value for the same diffuseness (dt) and varies with geomagnetic latitude, for instance, the effective duct width becomes 1000–2000 km at 30°, but 500–700 km at 40° for the same diffuseness ($dt=15$ ms).

3. Discussion

The relationship between the diffuseness and the effective duct width is not so simple. It is complicated with geomagnetic latitude because of the effect of electron density distribution and path length.

In this paper, we have deduced the effective apparent duct width from the whistler data which were received at Hirosaki by using a computer with electron number density distribution data. From the diffuseness of sonagrams of the whistlers the effective width of the VLF duct for the wave of 5 kHz has been calculated. The estimated width of the ducts is very wide and conflicts with pre-estimated value by SOMAYAJULU and TANTRY (1968) and TANAKA and HAYAKAWA (1973). The method of duct width estimation is almost the same as that of TANAKA and HAYAKAWA (1973) except for the different electron gyro-frequency and electron number density models. The effective duct widths were so affected by whistler diffuseness that the identification was important. The signal intensity of whistler affects the results and also time duration of lightning discharge affects the duct width.

In the future, other methods of duct width estimation, such as active sensing by spacecraft or other approaches for the duct formation model, will be necessary and more precise computer calculation will be necessary including the whistler ray path.

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