ANALYTICAL SOLUTIONS OF KATABATIC WIND AT MIZUHO AND SYOWA STATIONS, ANTARCTICA

Takashi ADACHI

Research Institute of Japan Weather Association,
Kaiji Center Bldg., 5, Koji-machi 4-chome, Chiyoda-ku, Tokyo 102

Abstract: A new analytical solution of governing equations for stationary katabatic winds is derived under the assumptions that the two horizontal orthogonal components of geostrophic winds vary linearly with height and the effect of the cold sloping surface decreases exponentially with increasing height. Eddy diffusivity is assumed constant.

Calculated wind profiles agree well with observations at Mizuho and Syowa Stations, Antarctica except in the surface layer.

1. Introduction

The Ekman Spiral in the atmospheric boundary layer is a well-known classical solution under many assumptions, *i.e.*, steady state horizontal homogeneity and barotropic atmosphere with constant eddy diffusivity.

MENDENHALL (1967) analyzed data over five years for various stations and found the effects of thermal wind in the observed wind spirals.

Analytical solutions under the assumption of the baroclinicity have been obtained by some investigators. VENKATESH and CSANADY (1974) and WIIN-NIELSEN (1974) presented analytical solutions of boundary layer equations under the assumption that the two horizontal orthogonal components of the geostrophic wind vary linearly with height. MAHRT and SCHWERDTFEGER (1970) and MACKAY (1971) solved the boundary layer equations with exponential thermal wind.

This paper presents a new analytical solution with more realistic vertical profile of pressure gradient than previous solutions, and simulation of katabatic wind profiles at Mizuho Station (70°41′53″S, 44°19′54″E, 2230 m a.s.l) and Syowa Station (69°00′S, 39°35′E, 14.5 m a.s.l), Antarctica. Locations of these two stations are shown in Fig. 1 (after SATOW *et al.*, 1983).

Mizuho Station is located on the antarctic coastal ice slope which is horizontally uniform. Syowa Station is located on East Ongul Island near the Antarctic coast. Numerical simulations of strong katabatic winds at both stations were reported by ADACHI (1979, 1983).

The discussion in this paper is restricted to analytical solutions which are practical for weather forecasting and other applications from the point of view of computing time.

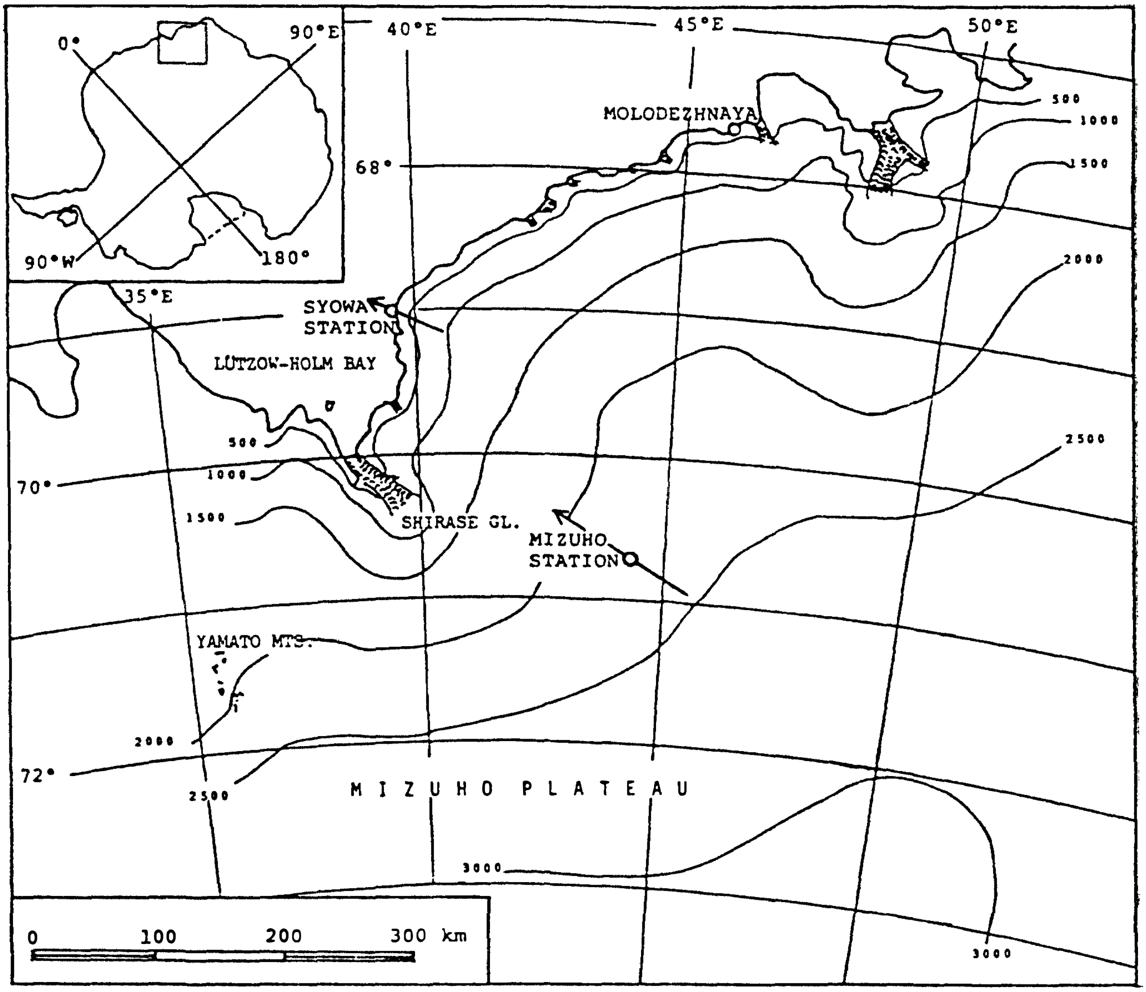

Fig. 1. Locations of Mizuho and Syowa Stations. (Arrows indicate slope direction near the two stations. This map is a partially modified version of the map of SATOW *et al. (1983)).*

2. Governing Equations and Boundary Conditions

The coordinate system used in this paper is illustrated in Fig. 2. The boundary layer equations adopted for the katabatic wind are (ADACHI, 1984);

$$-f(v-v_{gk})=K\frac{d^2u}{dz^2}, \quad (1)$$

$$f(u-u_{gk})=K\frac{d^2v}{dz^2}, \quad (2)$$

$$u_{gk}=u_g+u_k, \quad (3)$$

$$v_{gk}=v_g+v_k, \quad (4)$$

$$u_k=0, \quad (5)$$

$$v_k=v_{k0}\exp(-bz), \quad (6)$$

$$u_g=u_{g0}(1-z/h_u), \quad (7)$$

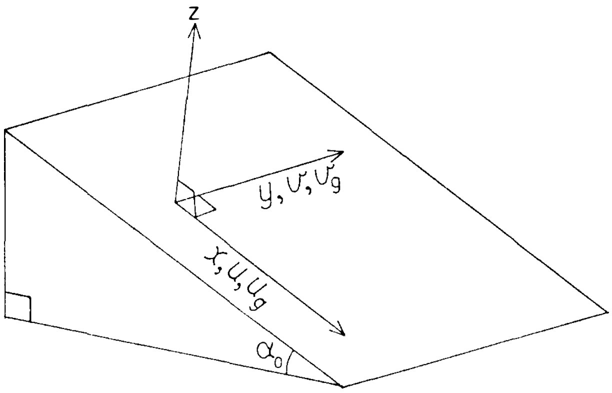

Fig. 2. Coordinate system for governing equations. (u, v: Wind components, u_g, v_g: geostrophic wind components, α_0: inclination of surface slope)

$$v_g = v_{g0}(1 - z/h_v). \tag{8}$$

K, f, v_{k0}, b, u_{g0}, v_{g0}, h_u and h_v are constants which must be specified. The unknowns are u and v. The only independent variable is z since a steady and horizontally uniform solution is sought.

The effect of a cold sloping surface (v_k) is represented by (ADACHI, 1983);

$$v_k = -(g/f)\sin\alpha \cdot (\Theta - \theta)/\bar{\Theta}. \tag{9}$$

The vertical profile of v_k from eq. (9) with observed values of Θ, $\bar{\Theta}$, θ and α was approximated by eq. (6).

Equation (6) is an empirical formula. v_k is the exponential thermal wind on the ice slope but does not include geostrophic wind shear due to large-scale motion. Therefore, v_k is the meteorological effect of the cold sloping surface which has the same units as wind speed.

Geostrophic wind components (u_g and v_g) are assumed to be linear with height (z) as shown in eqs. (7) and (8). It is evident that h_u is the height where u_g is zero and h_v is the height where v_g is zero.

Symbols

x: Along slope coordinate (see Fig. 2),
y: Rectangular to x (see Fig. 2),
z: Coordinate normal to x-y plane (see Fig. 2),
u: Wind component (x-axis),
v: Wind component (y-axis),
K: Eddy diffusivity,
f: Coriolis parameter,
g: Gravity acceleration,
Θ: Potential temperature of undisturbed atmosphere,
θ: Potential temperature,
$\bar{\Theta}$: Mean potential temperature,

α: Inclination of flow lines over ice slope,
α_0: Inclination of ice slope,
u_g: Geostrophic wind component (x-axis),
v_g: Geostrophic wind component (y-axis),
v_k: Effect of cold sloping surface.

The boundary conditions are:

$$u=u_1, \quad v=v_1, \quad u_g=u_{g1} \quad \text{and} \quad v_g=v_{g1}, \quad \text{at } z=z_1, \tag{10}$$

$$u=u_g \quad \text{and} \quad v=v_g, \quad \text{at } z\to\infty. \tag{11}$$

If $z_1=0$, $u_1=v_1=0$. This is the no-slip condition at the ground surface.
If $z_1 \doteqdot 10$ m, u_1 and v_1 become the surface wind components ordinarily observed at a meteorological observatory.

3. Analytical Solution

Noting that the coriolis parameter is negative in the Southern Hemisphere, the solutions of eqs. (1) to (8) in the Southern Hemisphere under boundary conditions (10) and (11) are:

$$u=u_g+\{(u_1-u_{g1})\cos A-(v_1-v_{g1})\sin A\}\exp(-A)-\frac{2a^2 v_{k0}\exp(-bz)}{4a^4+b^4}b^2$$
$$+\frac{2a^2 v_{k0}\exp(-bz_1-A)}{4a^4+b^4}(b^2\cos A+2a^2\sin A), \tag{12}$$

$$v=v_g+\{(v_1-v_{g1})\cos A+(u_1-u_{g1})\sin A\}\exp(-A)+\frac{2a^2 v_{k0}\exp(-bz)}{4a^4+b^4}2a^2$$
$$-\frac{2a^2 v_{k0}\exp(-bz_1-A)}{4a^4+b^4}(2a^2\cos A-b^2\sin A). \tag{13}$$

Where:

$$A=a(z-z_1), \tag{14}$$

$$a=(-f/2K)^{1/2}. \tag{15}$$

If $u_{g0}=v_{g0}=0$, $z_1=0$ and $u_1=v_1=0$, eqs. (12) and (13) are equivalent to the solutions of MAHRT and SCHWERDTFEGER (1970) under the assumptions of nogeostrophic wind, no-slip condition at the ground surface and exponential thermal wind.

If $1/h_u=1/h_v=0$, $z_1=0$ and $u_1=v_1=0$, eqs. (12) and (13) are equivalent to the solutions of MACKAY (1971) under the same assumptions as MAHRT and SCHWERDTFEGER (1970) but for constant geostrophic wind.

If $v_{k0}=0$, $z_1=0$ and $u_1=v_1=0$, eqs. (12) and (13) are equivalent to the solutions of VENKATESH and CSANADY (1974) under the assumptions of linear geostrophic wind component and no-slip condition at the ground surface.

If $v_{k0}=0$, eqs. (12) and (13) are equivalent to the first step of the solutions of WIIN-NIELSEN (1974) under the same assumptions as VENKATESH and CSANADY (1974) but for the surface layer boundary coundition.

It is clear that eqs. (12) and (13) are more universal than analytical solutions of previous investigators.

4. Comparison with Observation

4.1. Mizuho Station

The constants in eqs. (6) to (15) are (ADACHI and KAWAGUCHI, 1984):

$z_1=0$	(m)	$b=-0.014$	(m^{-1})
$u_1=v_1=0$	(m/s)	$u_{g0}=u_{g1}=8.0$	(m/s)
$g=9.8$	(m/s^2)	$v_{g0}=v_{g1}=5.1$	(m/s)
$f=-1.4\times10^{-4}$	(s^{-1})	$h_u=488$	(m)
$\alpha_0=3\times10^{-3}$	(rad)	$h_v=780$	(m)
$v_{k0}=12.7$	(m/s)	$K=0.03,\ 0.1,\ 0.2$	(m^2/s).

Vertical profiles of u_g and v_g are estimated from observed upper layer wind components. The vertical profile of v_k is estimated from observed potential temperatures and inclination of the ice slope near Mizuho Station. As the atmosphere is very stable, the values of eddy diffusivities are assumed to be very small (ADACHI, 1983).

Figure 3 shows calculated vertical profiles of the two horizontal wind components (u, v) with three different values of eddy diffusivity, and observed profiles which are averages of 26 observations (ADACHI and KAWAGUCHI, 1984). Light solid lines are for $K=0.2\ m^2/s$. Heavy solid lines are for $K=0.1\ m^2/s$. Broken lines are for $K=0.03\ m^2/s$.

Dot-and-dash lines show components of estimated geostrophic wind (u_g, v_g). The dashed line is v_{gk} which is the sum of v_g and the estimated effect of the cold sloping surface (v_k),

Calculated wind components for $K=0.1\ m^2/s$ agree well with observed components except in the surface layer as in Fig. 3. This discrepancy is due to the assumption of constant eddy diffusivity. The actual eddy diffusivity is proportional to height above ground in the surface layer.

Figure 4 shows calculated vertical profiles of wind velocity ($\sqrt{u^2+v^2}$) for three different values of eddy diffusivity and the observed profile which are composed from the components in Fig. 3. The calculated wind velocity for $K=0.1\ m^2/s$ agrees well with the observed velocity except in the surface layer.

Figure 5 shows the calculated wind spiral for $K=0.1\ m^2/s$ and the observed spiral. Both spirals are similar except in detail.

4.2. Syowa Station

Constants in eqs. (6) to (15) are:

$z_1=0$	(m)	$u_{g0}=u_{g1}=3$	(m/s)
$u_1=v_1=0$	(m/s)	$v_{g0}=v_{g1}=16$	(m/s)
$g=9.8$	(m/s^2)	$h_u=\infty$	(m)
$f=-1.4\times10^{-4}$	(s^{-1})	$h_v=1280$	(m)
$v_{k0}=0$	(m/s)	$K=1,\ 2$	(m^2/s).

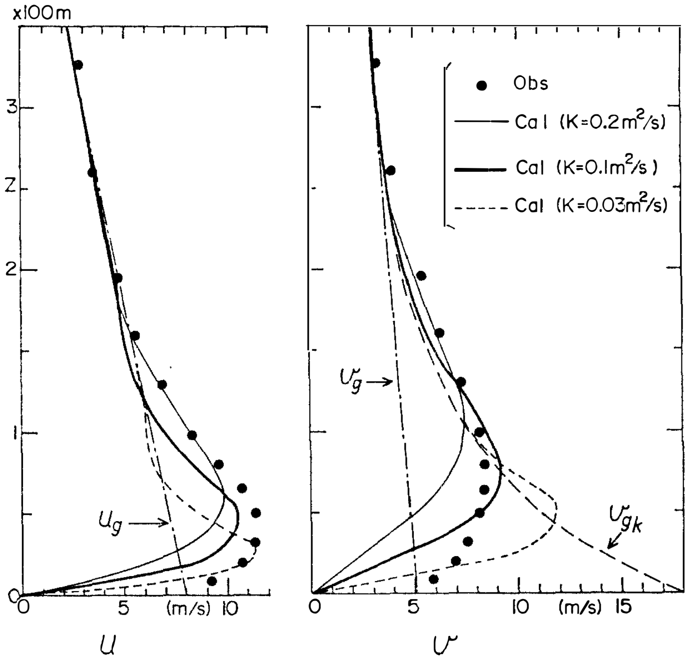

Fig. 3. Calculations for three different values of eddy diffusivity vs. observed wind components at Mizuho Station.
u_g, v_g: Estimated geostrophic wind components, v_k: effect of cold sloping surface, $v_{gk}=v_g+v_k$.

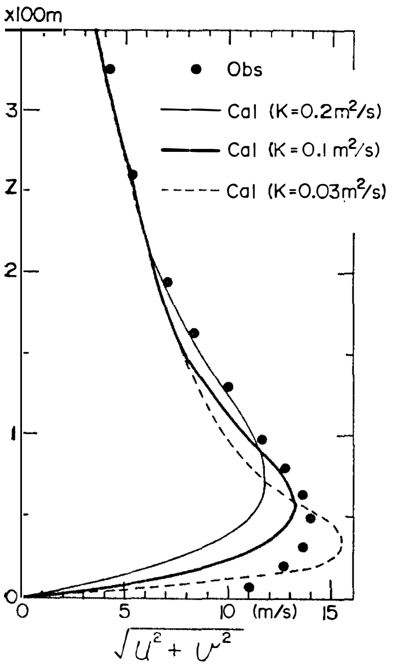

Fig. 4. Vertical profiles of calculated wind velocity vs. observations at Mizuho Station.

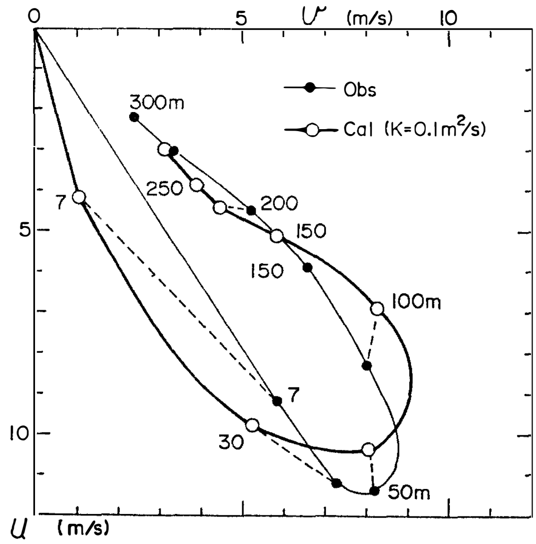

Fig. 5. Calculated wind spiral vs. observations at Mizuho Station.

Syowa Station is about 6 km from the Antarctic coast in the down-wind direction. However, strong katabatic winds reach Syowa Station and the thickness of the katabatic wind may not be uniform. The effective value of slope inclination at Syowa Station and the effect of the cold sloping surface cannot be exactly estimated. However, the slope direction is estimated to be from ESE to WNW as shown in Fig. 1.

Therefore, the resultant (v_{gk}) of the geostrophic wind component (v_g) and the effects of the cold sloping surface (v_k) are estimated by fitting to the y-component of the observed wind from 400 to 1000 m height and extrapolating linearly to the surface.

Geostrophic wind component (u_g) is approximated by fitting to the x-component of the observed wind from 1000 to 1500 m height and extrapolating linearly to the surface. The u_g do not include the effects of the cold sloping surface. Therefore, u_g should be fitted at a higher level than v_{gk}. Figure 6 shows calculated vertical profiles of horizontal wind components (u, v) for two different values of eddy diffusivity and the observed profiles which are averages of 14 observations (ADACHI, 1983).

Calculated wind components agree well with the observed profiles except in the surface layer and the y-component above 1000 m. These discrepancies can be expected because of the assumptions of the constant eddy diffusivity and the linear profile of v_{gk} which is not fitted to the y-component of observed wind above 1000 m.

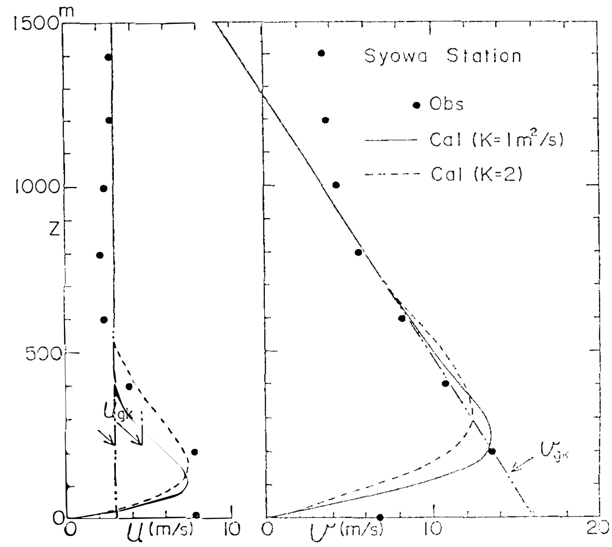

Fig. 6. Calculations for two values of eddy diffusivity vs. observations at Syowa Station.

5. Concluding Remarks

A new analytical solution for stationary katabatic winds on the antarctic coastal slope are presented and shown to agree well with observations except in the surface layer. This analytical solution is more universal than analytical solutions of previous investigators.

It is a future problem to derive analytical solutions with variable eddy diffusivity and realistic vertical profile of pressure gradient.

Acknowledgments

The author is very grateful to Prof. J. KONDO and Associate Prof. N. YASUDA, Faculty of Science, Tohoku University, for valuable advice and suggestions.

The author would like to express his gratitude to Dr. S. OHTA, Dr. S. YOSHIDA, Dr. J. KOBAYASHI, Dr. S. MATSUMOTO, Mr. M. SHINYA and Mr. K. KUSANO, Japan Weather Association, for their continuing encouragement.

The author is much indebted to Dr. O. CHIBA, Faculty of Science, Kochi University, for his constant advice and kindness.

It is pleasure to acknowledge the hospitality and encouragement of the staff of the National Institute of Polar Research.

The author wishes to thank Mrs. T. KOYAMA for drawing the figures in this paper.

References

ADACHI, T. (1979): Numerical simulation of katabatic wind profile at Syowa Station, Antarctica. Nankyoku Shiryô (Antarct. Rec.), **67**, 64–74.

ADACHI, T. (1983): Numerical simulation of strong katabatic winds at Syowa and Mizuho Stations, Antarctica. Mem. Natl Inst. Polar Res., Spec. Issue, **29**, 50–60.

ADACHI, T. (1984): Shamen-fû no kenkyû (Research of the slope wind). Kyokuchi Kishô Yosoku Puroguramu no Shisutemu-ka no Kaihatsu Kenkyû Hôkokusho (Report of Research and Development of Forecasting Systems of Local Weather). Tokyo, Nihon Kishô Kyôkai (Japan Weather Assoc.), 6–30.

ADACHI, T. and KAWAGUCHI, S. (1984): Numerical simulation of katabatic wind at Mizuho Station, East Antarctica. Mem. Natl Inst. Polar Res., Spec. Issue, **34**, 37–53.

MACKAY, K. P. (1971): Steady state hodographs in a baroclinic boundary layer. Boundary-Layer Meteorol., **2**, 161–168.

MAHRT, L. J. and SCHWERDTFEGER, W. (1970): Ekman spirals for exponential thermal wind. Boundary-Layer Meteorol., **1**, 137–145.

MENDENHALL, B. R. (1967): A statistical study of frictional wind veering in the planetary boundary layer. Degree of Master of Science, Colorado State University.

SATOW, K., NISHIMURA, H. and INOUE, J. (1983): Glaciological data collected by the Japanese Antarctic Research Expedition in 1981. JARE Data Rep., **82** (Glaciol. 9), 81 p.

VENKATESH, S. and CSANADY, G. T. (1974): A baroclinic planetary boundary-layer model, and its application to the Wangara data. Boundary-Layer Meteorol., **5**, 459–473.

WIIN-NIELSEN, A. (1974): Vorticity, divergence and vertical velocity in a baroclinic boundary layer with a linear variation of the geostrophic wind. Boundary-Layer Meteorol., **6**, 459–476.

(Received April 18, 1984; Revised manuscript received July 18, 1984)