Benford's Law as Applied to Ordinary-Chondrite Mass Distributions Among Observed Falls, NWA Finds and Antarctic Finds

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If a string is generated of a million randomly distributed digits of the numbers 1 through 9, each digit will appear 1/9 (11.11%) of the time, but there are numerous data sets where the digits are not uniformly distributed. In many instances, there is a systematic decrease in the frequency of appearance of the numbers 1 through 9 as the first digit in a set of data: if the digit 0 is omitted, the number 1 occurs as the first digit ~30.1% of the time, 2 occurs ~17.6% of the time, 3 occurs ~12.5% of the time, and so on through 9, which occurs ~4.6% of the time. In these data sets, the frequency of the digits 1 through 9 occurring as the first digit follows a logarithmic series wherein the probability of the first digit being d is given by: $P(d) = \log_{10}(1+(1/d))$. This distribution, known as "Benford's Law", is both scale and base invariant, the only probability distribution with these properties. For data sets that conform to Benford's Law, it does not matter if a set of lengths is listed in meters, miles, parsecs or light years. It does not matter if a set of logarithms is in base 51, base 10 or base 2. If a set of financial data follows Benford's Law when the units are in dollars, it will also follow Benford's Law if the units are converted to euros, pesos, yen, yuan or rubles: in each case, the digit 1 will appear as the first digit with a frequency of ~30%. Samples with numeric values of 1000000, 1953, 17.814, 1.2 and 0.0016 are identical in having the number 1 as the first significant digit.

Benford's Law has been shown to apply to data sets of sufficient dynamic range that involve societal phenomena (e.g., baseball statistics, street addresses, numbers appearing in magazine articles, stock market figures, voting patterns, census data, tax returns, crime statistics, on-line social networks, on-line auctions, sports matches, digital signal processing, and duration of notes in classical music compositions) as well as natural phenomena (e.g., streamflow data, surface areas of islands and rivers, durations of volcanic eruptions, reversals of the geomagnetic field, speeds of seismic P waves in the upper mantle, extrasolar gamma-ray source fluxes, and distances between galaxies). Deviations from Benford's Law have been widely used to detect tampering and fraud in accounting data, election results, sports matches and "deep fake" digital images. Benford's Law nonconformity in data sets of natural phenomena may indicate these sets are unrepresentative of the total population or that the sample distribution has been modified. Such modification could be induced by natural physical alteration of the underlying samples, in this case, by terrestrial weathering causing disintegration of ordinary-chondrite (OC) finds.

The numerical data used here are for formally approved meteorites from the 8 August 2020 version of the online Meteoritical Bulletin Database. I list mass distributions for three OC data sets: observed falls, NWA finds and Antarctic finds (Table 1), arbitrarily placed into bins 0.5 log-units wide. As shown in Fig. 1, observed falls extend to higher mass ranges than either set of finds. Antarctic finds exhibit a relatively broad peak extending to much lower mass ranges than the other two data sets. NWA finds have a more truncated distribution, with few specimens occurring within very high or very low mass ranges.

Table 1. Numbers of ordinary chondrites with recorded masses (g) that fit into different ranges (log scale).

$\log_{10}$(mass)	OC falls	NWA OC finds	Antarctic OC finds
6.5 – 7.0	1	0	0
6.0 – 6.5	2	1	0
5.5 – 6.0	4	0	1
5.0 – 5.5	39	16	1
4.5 – 5.0	62	34	2
4.0 – 4.5	125	132	35
3.5 – 4.0	204	299	157
3.0 – 3.5	198	664	516
2.5 – 3.0	148	1305	1377
2.0 – 2.5	73	1634	3179
1.5 – 2.0	32	1089	6018
1.0 – 1.5	21	381	9096
0.5 – 1.0	7	62	8261
0.0 – 0.5	1	5	5708
-0.5 – 0.0	2	1	2249
-1.0 – -0.5	2	0	364
-1.5 – -1.0	0	0	19
-2.0 – -1.5	0	0	4
total	921	5623	36987

Among ordinary chondrites, Antarctic finds conform more closely to Benford's Law than observed falls or NWA finds (Tables 2, 3). In the case of OC mass distributions, Benford's Law appears to apply to the set of individual objects that survive intact on the Earth's surface after disruption of meteoroids in the atmosphere. Deviations from Benford's Law can result from tampering with data sets. The set of observed OC falls reflects tampering with the original Benford distribution (produced by meteoroid disruption) by the deliberate aggregation of paired individual samples under single names. For example, ~14,000 separate stones have collectively been labeled Pultusk (H5); ~180,000 separate stones have been labeled Holbrook (L/LL6). NWA OC finds have been affected by natural "tampering" of the original Benford distribution by weathering-induced sample disintegration. Antarctic OC finds conform most closely to Benford's Law because these samples tend to be relatively unweathered (and intact) and have not been aggregated as pairs under collective meteorite names (in contrast to observed falls).

Table 2. Percentages of recorded OC masses (g) in each set compared to the values expected from Benford's Law.

first digit	OC falls	NWA OC finds	Antarctic OC finds	Benford's Law
1	29.6	28.9	30.7	30.1
2	17.7	18.7	17.8	17.6
3	13.1	11.7	12.0	12.5
4	10.2	9.9	9.3	9.7
5	7.9	8.4	7.7	7.9
6	7.3	6.5	6.9	6.7
7	5.6	6.5	5.7	5.8
8	4.8	5.4	5.4	5.1
9	3.7	4.1	4.6	4.6
total	99.9	100.1	100.1	100.0
no. of samples	921	5623	36987	

Table 3. Percent deviations from Benford's Law for the percentages of recorded OC masses.

first digit	OC falls	NWA OC finds	Antarctic OC finds
1	1.66	3.99	1.99
2	0.57	6.25	1.14
3	4.80	6.40	4.00
4	5.15	2.06	4.12
5	0	6.33	2.53
6	8.96	2.99	2.99
7	3.45	12.07	1.72
8	5.88	5.88	5.88
9	19.57	10.87	0
mean percent deviation	5.6±6.0	6.3±3.3	2.7±1.8

Observed falls are the most representative set of the OC flux to the Earth; the H:L:LL intergroup ratios of falls are 1:1.17:0.27 (Table 4). H chondrites weather more readily than L or LL because H chondrites have higher modal abundances of metallic Fe-Ni (which is highly susceptible to oxidation). Terrestrial weathering can affect the H:L:LL intergroup ratios. The H:L:LL intergroup ratios among Antarctic finds are 1:0.87:0.36; those of NWA finds are 1:1.32:0.62. Among NWA OC finds, the proportion of L chondrites increases by 15% relative and the proportion of LL chondrites increases by 31% relative compared to Antarctic OC finds as a result of more-severe weathering of ordinary chondrites in Northwest Africa.

Table 4. Numbers and percentages of major OC groups.

	OC falls	NWA OC finds	Antarctic OC finds
H	382 (41%)	1919 (34%)	16693 (45%)
L	439 (48%)	2566 (45%)	14564 (39%)
LL	106 (11%)	1185 (21%)	5753 (16%)
total number of specimens	927	5670	37010
H:L:LL	1:1.17:0.27	1:1.32:0.62	1:0.87:0.36
L:LL	1:0.23	1:0.47	1:0.41

Falls include four H/L and 11 L/LL; NWA finds include 29 H/L and 67 L/LL; Antarctic finds include 10 H/L and 25 L/LL.

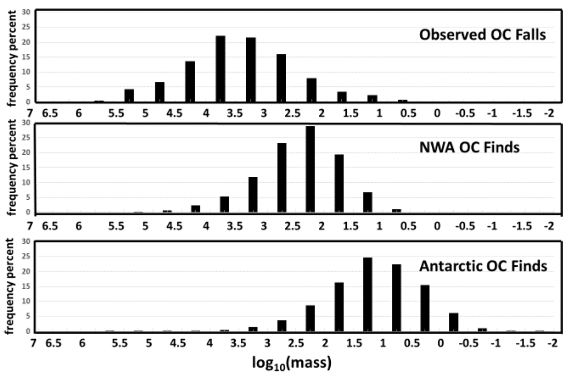

Fig. 1. Diagrams of frequency percent vs. $\log_{10}$(mass) (in grams) of observed OC falls, NWA OC finds and Antarctic OC finds. The mass ranges are arbitrarily confined into bins 0.5 log-units wide. All distributions are approximately log normal. Observed falls extend to higher mass ranges than either set of finds. Antarctic finds extend to the lowest mass ranges.