Full Stokes ice sheet model Elmer/Ice, and its application to regional drainage systems in Greenland and Antarctica

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For decades, approximations to the full Stokes equations – the set of partial differential equations describing ice dynamics – have been the standard in numerical glaciology in applications to ice sheets. In particular, the shallow ice approximation (SIA) for grounded ice sheets and the shallow shelf approximation (SSA) for the floating ice shelves have been deployed in many applications (e.g., Greve and Blatter 2009). Based on the assumption of shallowness of the geometry, these approximations lead to simplifications of the Stokes equations that are numerically very efficient, i.e., easy to solve and economical in memory consumption. Induced by these simplifications, the SIA and SSA are not valid in particular at places of pronounced interest (see Fig. 1): ice domes, ice streams and marine ice sheets (transition from grounded to floating ice).

Ice flow is governed by the conservation laws (aka balance equations) of mass, linear momentum and energy. Under the assumption of incompressibility (mass density ρ = const), conservation of mass is equivalent to conservation of volume, expressed by a vanishing divergence of the velocity field,

$$\mathrm{div}\,\mathbf{u} = 0.$$

Conservation of linear momentum, which, due to the low Froude number, reduces to a balance between the Cauchy stress tensor, $\boldsymbol{\sigma}$ (which usually is split into its deviatoric part, $\boldsymbol{\tau}$, and the isotropic pressure, p) and the acceleration due to gravity, $\mathbf{g}$, yields the actual Stokes equation,

$$\mathrm{div}\,\boldsymbol{\tau} - \mathrm{grad}\,p + \rho\mathbf{g} = 0.$$

Besides the, compared to the SIA and SSA, increased size of the problem, the major difficulty is introduced by the closure relation that expresses the deviatoric stress components in terms of the velocities. The standard approach in ice sheet modeling is to use the isotropic Norton-Hoff law for a shear-thinning fluid, in glaciology also known as Glen's flow law. In particular at ice domes, where slow velocities and vertical compression prevail, anisotropic effects of the ice fabric (i.e., the arrangement of crystal axes in grains) need to be taken into account.

Based on the finite element method (FEM), Elmer/Ice is able to deploy local mesh refinement where needed, as shown for the example of the Greenland ice sheet in Fig. 2. This enables us to keep classical SIA resolutions in (by overall size dominating) regions of slow, vertical-shear-dominated areas. We use observations of velocities to control the horizontal rearrangement of mesh points using the software YAMS (Frey 2001).

Simply by the size of the equation system, full-Stokes ice sheet codes must utilize high performance computing (HPC) techniques. Elmer/Ice is based on Elmer, an open-source multi-physics FEM code maintained by CSC – IT Center for Science, Finland (Gagliardini et al. 2013, http://elmerice.elmerfem.org). Elmer's parallel capabilities are constantly improved and ported to new HPC platforms, which in turn improves applicability of Elmer/Ice to increasingly larger problems. Despite the deployment of HPC techniques, simulations exceeding centennial timescales are not yet feasible with Elmer/Ice, which induces a problem concerning the spin-up of prognostic simulations, which usually takes at minimum a glacial cycle to obtain a correct initial temperature field. If the option of assuming steady-state temperature conditions is not feasible, the only remaining alternative is to obtain the initial temperature field using a SIA code, as it was done by Gillet-Chaulet et al. (2012) and Seddik et al. (2012) where the results of a SIA run obtained with SICOPOLIS (http://sicopolis.greveweb.net/) were imposed on the full-Stokes simulations as initial conditions. A second issue related to initialization is to obtain a correct distribution of basal sliding coefficients – which is especially crucial in fast outlet systems. Recently, data assimilation methods have been developed to infer the drag acting at the bedrock from observations of surface velocities (Gillet-Chaulet et al. 2012, Schäfer et al. 2012).

Currently ongoing work within the JSPS project "Simulations of the Evolution and Dynamics of the Antarctic Ice Sheet in Past and Future Climates" (Grant-in-Aid for Scientific Research (A), FY2010-2013, PI R. Greve) concerns the application of Elmer/Ice to the Shirase drainage basin that connects Dome Fuji Station to the Lützow-Holm Bay region of Queen Maud Land, East Antarctica. The basin is characterized by the convergence of the ice flow towards Shirase Glacier, one of the fastest-flowing glaciers in Antarctica. Shirase Glacier flows at a speed of more than 2 km a^{-1} at the grounding line and drains about 10 Gt a^{-1} of ice through a narrow outlet into Lützow-Holm Bay. With nearly 90% of the total ice discharge from the basin being calved by the glacier, the fast-flowing nature of Shirase Glacier is important for the investigation of the ice sheet mass budget in this region. Data for the present geometry are obtained from the BEDMAP2 data set, and a mesh of the computational domain is created using an initial footprint that contains elements varying from 15 km to 500 m horizontal resolution. We apply Elmer/Ice both in its normal, full Stokes mode and also in its SIA mode to the drainage system in order to quantify the difference between the two approaches. A series of future climate experiments that encompass responses to direct climatic and ice-dynamical forcing will be run, following largely the set-up provided by the SeaRISE community effort (Bindschadler et al. 2013).

For the future, we envisage to install Elmer/Ice on either JAMSTEC's Earth Simulator or, depending on feasibility, another supercomputer accessible to the Japanese ice sheet modeling community. In order to create a link to the ongoing Green Network of Excellence (GRENE) Arctic Climate Change Research Project (FY2011-2016), we plan to carry out detailed modeling studies for the Qaanaaq drainage basin situated in the northwestern part of the Greenland ice sheet, which is also the focus of field activities in Greenland conducted within the GRENE project.

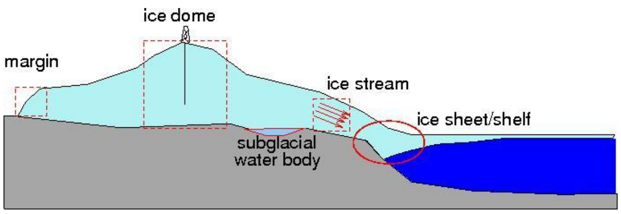

Figure 1. Areas of failure of the common approximations (SIA, SSA) of the full Stokes equations.

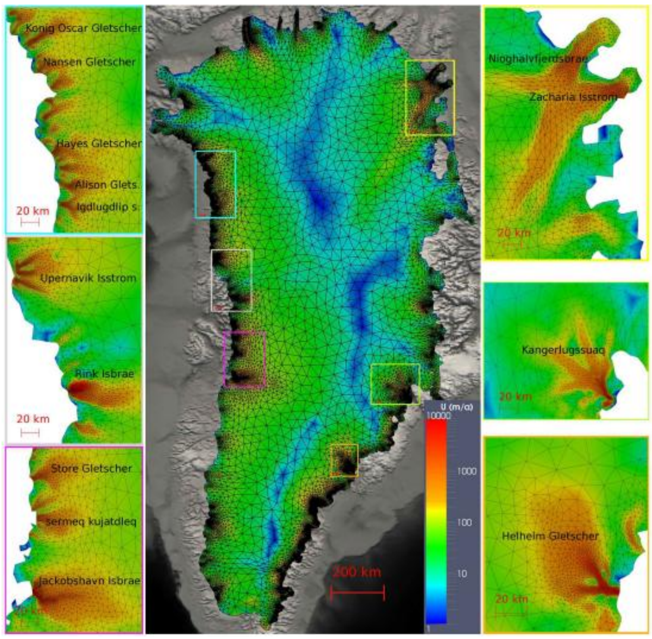

Figure 2. Locally refined mesh for the Greenland ice sheet (Gillet-Chaulet et al. 2012).

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