FRAUNHOFER DIFFRACTION BY ATMOSPHERIC ICE CRYSTALS (EXTENDED ABSTRACT)

Yoshihide TAKANO

Upper Atmosphere Research Laboratory, Faculty of Science, Tohoku University, Aoba Aramaki, Sendai 980

1. Introduction

There is a lack of information concerning the forward-scattered light intensity for ice crystals with various shapes at different orientations. The purpose of this study is to establish a foundation for remote sensing of properties of ice crystals suspended in the atmosphere. We shall concentrate on the diffraction phenomenon which is a component of the light scattering by particles with large size parameters. Ice crystals are simulated by hexagonal cylinders and spheroids. Their Fraunhofer diffracted intensities are computed on the basis of Kirchhoff's law.

2. Theory

Fraunhofer diffraction amplitude for an aperture S with an arbitrary shape is, ignoring a constant factor, given (BORN and WOLF, 1975) by

$$A=k^2\iint_S e^{-i(Xx'+Yy')}\,\mathrm{d}x'\,\mathrm{d}y'\,, \tag{1}$$

where

$$\left.\begin{aligned} X&=k\sin\theta\cos\phi\,,\\ Y&=k\sin\theta\sin\phi\,, \end{aligned}\right\} \tag{2}$$

k is the wave number, $i=\sqrt{-1}$, θ is the scattering angle, and ϕ is the azimuth angle. The diffracted intensity is given by the square of the absolute value of the diffracted amplitude.

The xyz coordinate system is taken as follows: the z axis and the x axis are directed respectively along the c axis and the a axis of a hexagonal cylinder and the origin O is placed at the center of the hexagonal cylinder. Let us consider a plane wave incident on a hexagonal cylinder with the zenith angle $\pi/2-\alpha$ and the azimuth angle β. The geometric shadow of the hexagonal cylinder projected onto the plane vertical with the incident direction is expressed by the x, y coordinates (x', y') of its marginal vertices in the frame whose z' axis is taken along the incident direction. The transformation of the $X(xyz)$ coordinate system into the $X'(x'y'z')$ coordinate system is expressed as

$$\boldsymbol{X'}=\boldsymbol{BCDX}, \tag{3}$$

where

$$\boldsymbol{B}=\begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4}$$

$$\boldsymbol{C}=\begin{pmatrix} \sin\alpha & 0 & -\cos\alpha \\ 0 & 1 & 0 \\ \cos\alpha & 0 & \sin\alpha \end{pmatrix}, \tag{5}$$

$$\boldsymbol{D}=\begin{pmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{6}$$

$$\tan\phi = -\tan(\beta+\pi/3)/\sin\alpha\,, \quad (0\leq\phi\leq\pi)\,. \tag{7}$$

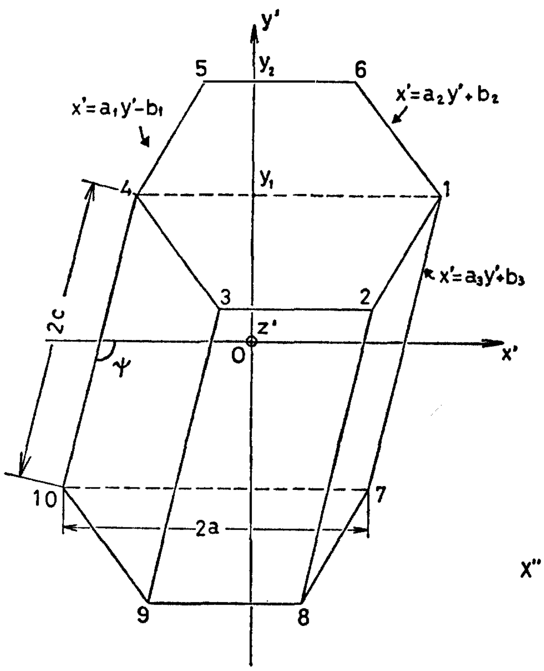

Fig. 1. Geometric shadow of a hexagonal cylinder projected onto the plane normal to the incident direction.

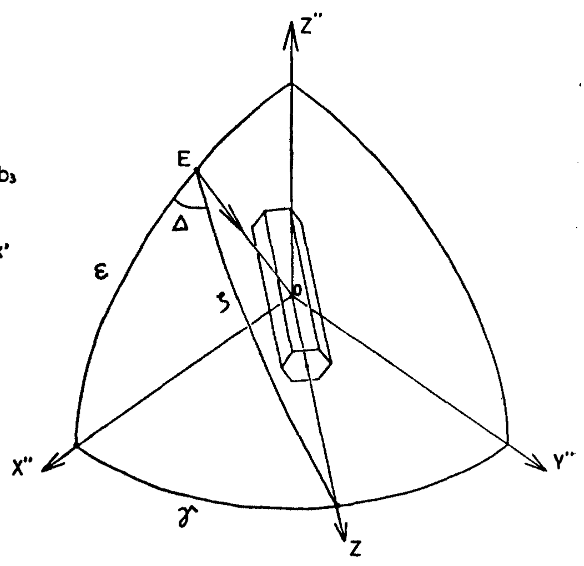

Fig. 2. Geometry for a hexagonal column oriented in the horizontal plane.

Therefore, if the length of the a axis $2a$, the length of the c axis $2c$, and the incident direction α, β are given, a_1, a_2, a_3, b_1, b_2, b_3, y_1, and y_2 in Fig. 1 are easily determined by eq.(3), namely, the geometric shadow of the hexagonal cylinder is decided. Next we apply Babinet's principle (*e.g.*, Van de Hulst, 1957) which states that the diffraction pattern of a particle is identical to that of an aperture of the form and size of the particle's geometric shadow area. Integrating eq.(1) over the boundary shown in Fig. 1, the diffracted amplitude A_{II} in the direction θ, ϕ is given in the form:

$$
\begin{aligned}
&A_H(\theta,\ \phi,\ \alpha,\ \beta,\ ka,\ c/a) \\
&= \frac{4k^2}{X(a_1X+Y)} \sin\left[\frac{1}{2}(y_2-y_1)(a_1X+Y)\right] \\
&\cdot \sin\left\{\frac{1}{2}\left[-a_1(y_1+y_2)+2b_1\right]X-\frac{1}{2}(y_1+y_2)Y\right\} \\
&+ \frac{4k^2}{X(a_2X+Y)} \sin\left[\frac{1}{2}(y_2-y_1)(a_2X+Y)\right] \\
&\cdot \sin\left\{\frac{1}{2}\left[a_2(y_1+y_2)+2b_2\right]X+\frac{1}{2}(y_1+y_2)Y\right\} \\
&+ \frac{4k^2}{X(a_3X+Y)} \sin\left[y_1(a_3X+Y)\right] \sin(b_3X)\ . \qquad (8)
\end{aligned}
$$

Next, let us consider the diffracted intensity distribution for ice crystals whose long axes orient randomly in the horizontal plane as shown in Fig. 2. Here the direction OZ'' expresses the zenith direction. ε is the angle between the incident direction EO and the horizontal plane $X''OY''$. The azimuth angle Φ fixed in space is measured from the $X''OZ''$ plane. Considering that the angle γ takes the value from 0 to 2π with an equal probability and at each γ the hexagonal column rotates about its central axis, the diffracted intensity of hexagonal columns oriented randomly in the horizontal plane is expressed as

$$
\begin{aligned}
&I_{HC}(\theta,\ \Phi,\ \varepsilon,\ ka,\ c/a) \\
&= \int_0^{\pi} \frac{1}{\pi} d\gamma \int_0^{\pi/3} \frac{3}{\pi} d\beta\ I_H(\theta,\ -\psi\underline{\pm\Delta\pm}\Phi,\ \alpha,\ \beta,\ ka,\ c/a)\ . \qquad (9)
\end{aligned}
$$

When hexagonal plates are oriented randomly in the horizontal plane, *i.e.*, the direction of their c axes coincides with the Z'' axis, the diffracted intensity I_{HP} is given by

$$
I_{HP}(\theta,\ \Phi,\ \varepsilon,\ ka,\ c/a) = \int_0^{\pi/3} \frac{3}{\pi} d\beta\ I_H(\theta,\ -\psi\pm\Phi,\ \varepsilon,\ \beta,\ ka,\ c/a)\ . \qquad (10)
$$

3. Computed Result

In Fig. 3, the contours of the diffracted intensity for ice crystals oriented randomly in the horizontal plane are portrayed. To smooth out the fluctuations of the diffracted intensity, an integration over particle size weighted by the modified gamma function (HANSEN and TRAVIS, 1974) is performed. Since the diffracted intensity pattern is symmetrical with respect to the azimuth directions of $\Phi=0°$ and 180°, and $\Phi=90°$ and 270°, only the diffraction pattern in one quadrant is drawn. The aspect ratio of the particle SP is 4, and the average geometrical cross section $\tilde{G}$ by k^2 is $\pi(300)^2$. The contours of the relative value of diffracted intensity are drawn at every half order of magnitude. For oblate spheroids, the distribution pattern of the diffracted intensity is elliptical. For prolate spheroids, the contours slightly deviate from an elliptical pattern. At very low sun elevations, the geometric shadows of hexagonal plates nearly take rectangular forms whose long sides

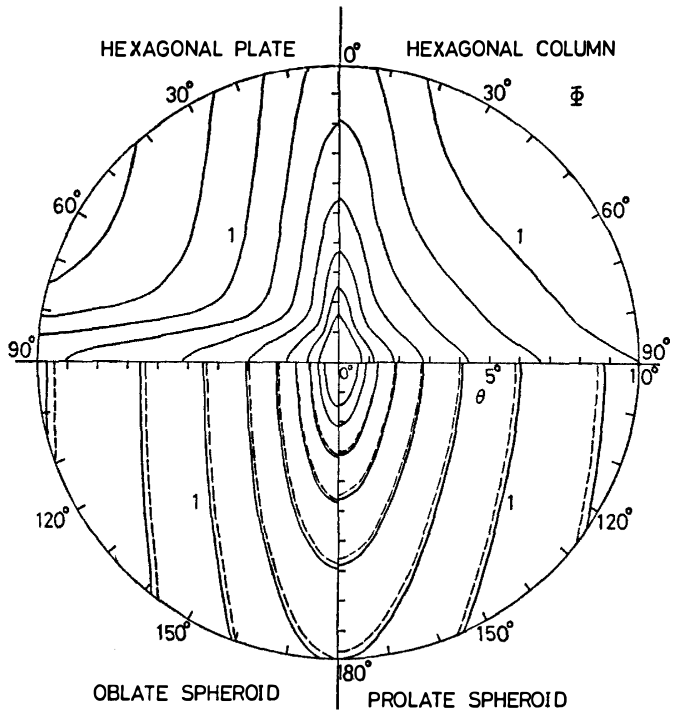

Fig. 3. Diffracted intensity distribution for ice crystals randomly oriented in the horizontal plane. $\varepsilon=10°$, $SP=4$, $k^2\tilde{G}=\pi(300)^2$. The solid line and the broken line correspond to the cases of the variance $v=0.1$ and $v=0.2$, respectively.

are horizontal. The diffracted intensity pattern of a rectangular aperture has minima in its diagonal directions. Thus, for hexagonal plates, the diffraction pattern at a low sun elevation becomes concave in intermediate Φ directions, and it looks like a cross. On the other hand, for hexagonal columns, the diffraction pattern looks like an asteroid or a rhombus. This is because many different diffraction patterns are superposed for the case of hexagonal columns.

Thus, the diffraction pattern is useful to infer shapes and fall attitudes of ice crystals in the atmosphere.

References

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