HF DOPPLER MEASUREMENT IN THE AURORAL IONOSPHERE

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Abstract: HF doppler shift measurements were made at Rankin Inlet, N. W. T. in Canada for one winter month in 1980. It became clear that the trace features of the HF doppler frequency shift observed at Rankin Inlet are quite different from those observed in the middle latitudes. The shift is characterized by its diffuse appearance in an *f-t* diagram, and also by the existence of discrete line structures in the diffuse band. The lines are generally inclined to the time axis.

An ionospheric model, which can explain this feature of the HF doppler shift at high latitudes, is presented in this paper.

According to the model, the discrete line structure is thought to be a manifestation due to the signal being reflected from a moving irregularity in the ionosphere. Velocities were computed with parameters consistent with the data. The result shows that the velocities of irregularity range from 1 to 50 m/s. These values of velocity could possibly be thought as a drift motion due to the electric field in the ionosphere. The corresponding electric field intensity is estimated at a few mV/m at the auroral ionospheric height, which is consistent with values obtained from the IS radar at Chatanika.

1. Introduction

The HF doppler measurements were made at Rankin Inlet (62.8°N, 92.3°W in geographic coordinates), N. W. T. in Canada during February 1980, of the reception of the WWV standard frequencies of 5, 10, and 15 MHz. The distance from the transmitter location (Fort Collins, Colorado, the United States) to the receiving point (Rankin Inlet) is about 2500 km, which would be near the limit of the one hop propagation ray path from the transmitter. The reflection point of the signals in the ionosphere, hence, is located near the southern border of the auroral zone as shown in Fig. 1.

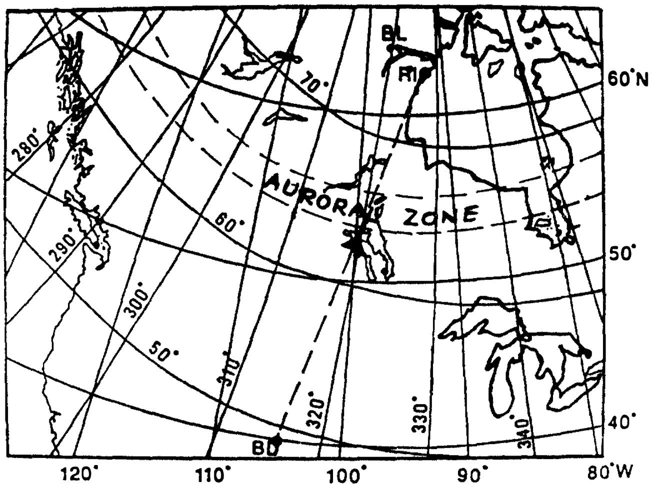

Fig. 1. Location of Rankin Inlet (Receiver, RI) and Boulder (Transmitter BD, Fort Collins near Boulder). The reflection point is marked with a cross.

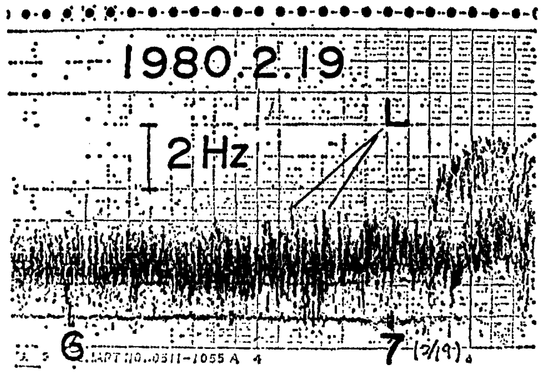

Fig. 2. An example of doppler trace observed at Rankin Inlet. Note the line structures (denoted by L) in the diffuse band at the right.

The doppler traces observed at Rankin Inlet generally show a diffuse character in an f-t diagram (Fig. 2), which is quite different from the ones observed in the middle latitudes. The most significant feature is that there appears often a discrete line structure in the diffuse band (L in Fig. 2). Furthermore, the important feature of the discrete line is that it generally is inclined to the time axis, that is, the doppler frequency shifts to lower or higher frequencies with time.

The purpose of this paper is to present an ionospheric model which can explain the character of these discrete lines. The present model suggests that an inclined discrete line is due to motion of an irregularity in the ionosphere. The electric field in the auroral ionosphere during a quiet condition which may cause the irregularity to be in motion is estimated as small as a few mV/m in the east-west direction.

2. Model and Basic Equations

When a signal is reflected from a horizontally stratified layer in the ionosphere, the doppler shifted frequency Δf is given by (see eq. (A-16) in the Appendix),

$$\Delta f(t)=(f_0/C)(1/T(t))\mathrm{d}T(t)/\mathrm{d}t(D\cot\theta_0(t)\cos\theta_0(t)$$
$$-\int_0^{z_0(t)}\sqrt{\cos\theta_0(t)-\gamma N(z,t)}\mathrm{d}z)-(2f_0/C)\sin\theta_0(t)\mathrm{d}D/\mathrm{d}t\,, \qquad (1)$$

where f_0 is the transmitted frequency; $2D$, the distance between a transmitter and a receiver; $z_0(t)$, the height of the reflecting layer; θ_0, the angle incident to the ionosphere. $T(t)$ is the time dependent part of the electron density $N(z, t)$, assuming the function form of $N(z, t)=n(z)\cdot T(t)$. C is the velocity of light and γ is a constant given in the Appendix.

As seen in eq. (1), there are two terms which contribute to the observed doppler frequency shift. One is that due to the time change of electron density in the ionosphere, and another is that due to the time change of the distance $2D$. In almost all cases, the distance $2D$ is fixed so that the second term in eq. (1) is then equal zero.

In the active ionosphere, for example the auroral ionosphere, there may exist several disturbances such as the gravity waves, accoustic waves, and irregularities which are drifting due to the electric field in the ionosphere. In such cases the reflection of an incident signal may occur, not at a horizontally stratified layer but at slightly tilted surface as drawn in Fig. 3. This is the case with which we are dealing in the present investigation.

In the following, we shall present a model for a signal reflecting at a stratified layer which is inclined slightly to the horizontal and we shall estimate the resulting doppler frequency shift of the received signal.

We should note that in the present model, the mathematical formulation described for the horizontal layer model (see the Appendix) may be approximately valid when the tilt (in Fig. 3) of the layer is not very large.

Assuming a stationary state in the ionosphere ($\mathrm{d}T(t)/\mathrm{d}t=0$), the first term in eq. (1) vanishes and eq. (1) becomes

$$\Delta f(t)=-(2f_0/C)\sin\theta_0(t)\mathrm{d}D/\mathrm{d}t\,. \qquad (2)$$

Note that in eq. (2), the distance "$2D$" is used for the distance SR, not TR (Fig. 3).

Assuming that the irregularity moves with velocity V, so that the reflection point shifts from P to P′ (Fig. 3), then a doppler shift due to the different path length between SPR and S′P′R would be observed when a signal from T is received at R.

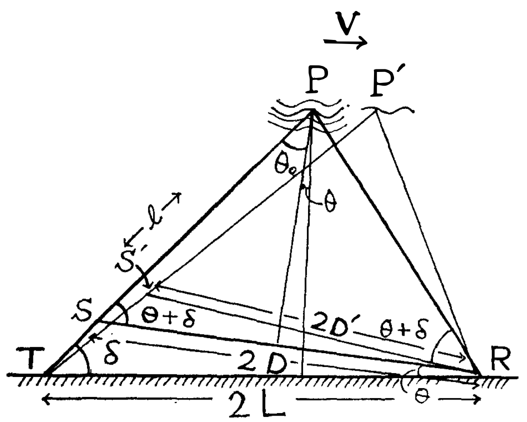

Fig. 3. Inclined reflection point. Reflection point moves with a velocity V. A signal suffers a doppler shift due to the difference of the paths SPR and S′P′R equal to 2dD/dt. l is the distance TP.

This shift of frequency is due to the difference between $2D$ and $2D'$, that is $\mathrm{d}D/\mathrm{d}t$, which is approximately equal to Δf of eq. (2).

Eliminating θ_0 from eq. (2), Δf is expressed as

$$\Delta f(t)=-(2f_0/C)\cos(\theta+\delta)\mathrm{d}D/\mathrm{d}t\,, \tag{3}$$

where θ and δ are the angles shown in Fig. 3. Since $2D$ is expressed as

$$2D=2L\cdot\sin\delta/\sin(\theta+\delta)\,,$$

where $2L$ is the distance between T and R, the time derivative D is given by

$$\dot{D}=L\cdot\frac{\sin\theta\cdot\dot{\delta}-\sin\delta\cdot\cos(\theta+\delta)\cdot\dot{\theta}}{\sin^2(\theta+\delta)}\,. \tag{4}$$

$\dot{\delta}$ in eq. (4) is written in terms of the drift velocity ($=V$) of the reflection point (see the Appendix), as,

$$\dot{\delta}=(V/z_0)\sin^2\delta\,, \tag{5}$$

where z_0 is the reflection height of the signal (Fig. 3).

The quantities θ and $\dot{\theta}$ are also written as (see the Appendix),

$$\theta=0.5\cdot\tan^{-1}\left(\frac{2z_0-D\tan\delta}{D+z_0(\tan\delta-\cot\delta)}\right), \tag{6}$$

and

$$\dot{\theta}=\frac{\cos^2 2\theta}{2}\left\{-\frac{D\cdot(\dot{\delta}/\cos^2\delta)\cdot[D+z_0(\tan\delta-\cot\delta)]}{[D+z_0(\tan\delta-\cot\delta)]^2}\right.$$
$$\left.-\frac{(2z_0-D\tan\delta)z_0\cdot(1/\cos^2\delta+\dot{\delta}/\sin^2\delta)}{[D+z_0(\tan\delta-\cot\delta)]^2}\right\}. \tag{7}$$

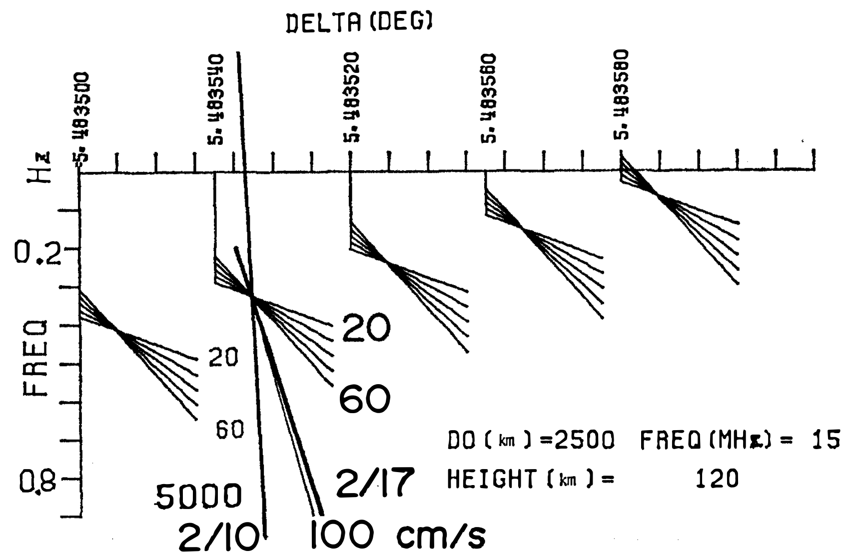

Fig. 4. Curves of $\Delta f(t)$ calculated from eq. (2). In the figure, calculation of $\Delta f(t)$ (t=0–60 s) was made by taking values 20, 30, 40 and 60 m/s as drift velocities V, and values 5.483500–5.483580° for delta. 2/17 and 2/10 refer to the dates of the event shown in Figs. 5 and 6. A steep line near 2/10 shows the gradient of the line structure determined from Fig. 6. 5000 refers to 5000 cm/s, the value determined from the best fitting of the observed and calculated curves. For 2/17 100 cm/s is used corresponding to Fig. 5.

From these eqs. (4) to (7), Δf of eq. (3) is calculated using values of V and δ as parameters.

Figure 4 is an example of such a calculated Δf. In the figure, values of $D=2500$ km, $f_0=15$ MHz, and $z_0=120$ km are used. Sets of curves are shown for the angles $\delta=5.483500$ to $5.483580°$. For example, if V is 20 m/s, then Δf starts from -0.38 Hz at $t=0$ when $\delta=5.483500°$. The frequency, then shifts along the curve marked (20) in the figure, and finally it reaches a frequency of -0.48 Hz at $t=60$ s.

As seen in the figure, Δf has a steeper gradient for larger values of V. When the initial frequency ($t=0$), the final frequency ($t=t_0$) and the gradient of the line structure (df/dt) are given by the observed data, then V can be estimated by selecting the best fitting curve from the figure. In Table 1, values of V determined in this way are listed for several cases including those for the examples shown in Figs. 5 and 6.

Table 1. Table of events analyzed in the present investigation. The resulting drift velocity and the corresponding electric field intensity are shown in columns V and E.

Date (1980)	Hour (local)	Kp	f_0 (MHz)	V (m/s)	E (mV/m)
2/5	21	1	10	50	2.5
2/10	06	1−	5	50	2.5
2/17	18	1−	15	1	0.05
2/17	20	1+	10	20	1.0
2/19	08	0+	15	2	0.25
2/19	07	0+	5	2	0.25

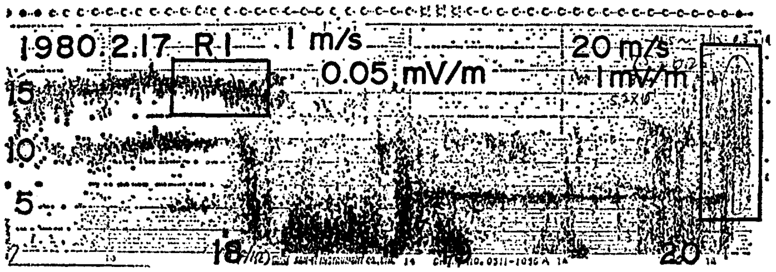

Fig. 5. Example (February 17, 1980). In the figure, 15, 10 and 5 refer to WWV frequencies of 15, 10 and 5 MHz. Local time is used. Events analyzed are shown in the enclosures. The falling tone is clearly seen in the event on the left. The values of drift velocity and the corresponding electric field intensity are 1 m/s and 0.25 mV/m, respectively. On the right the values are 20 m/s and 1 mV/m.

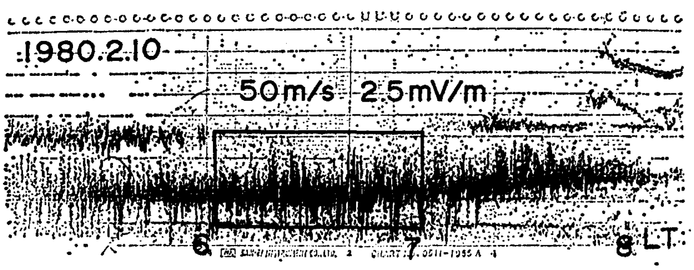

Fig. 6. Example (February 10, 1980).

The values of drift velocity shown in Table 1, seem to be one or two orders smaller than those found for the velocity of the gravity waves. Accordingly, the motion of tilted reflection surface in the present model would, not be due to a gravity wave but, to something else. In the auroral ionosphere, the most plausible candidate, which would cause the irregularities in the ionosphere to be in drift motion, would be the

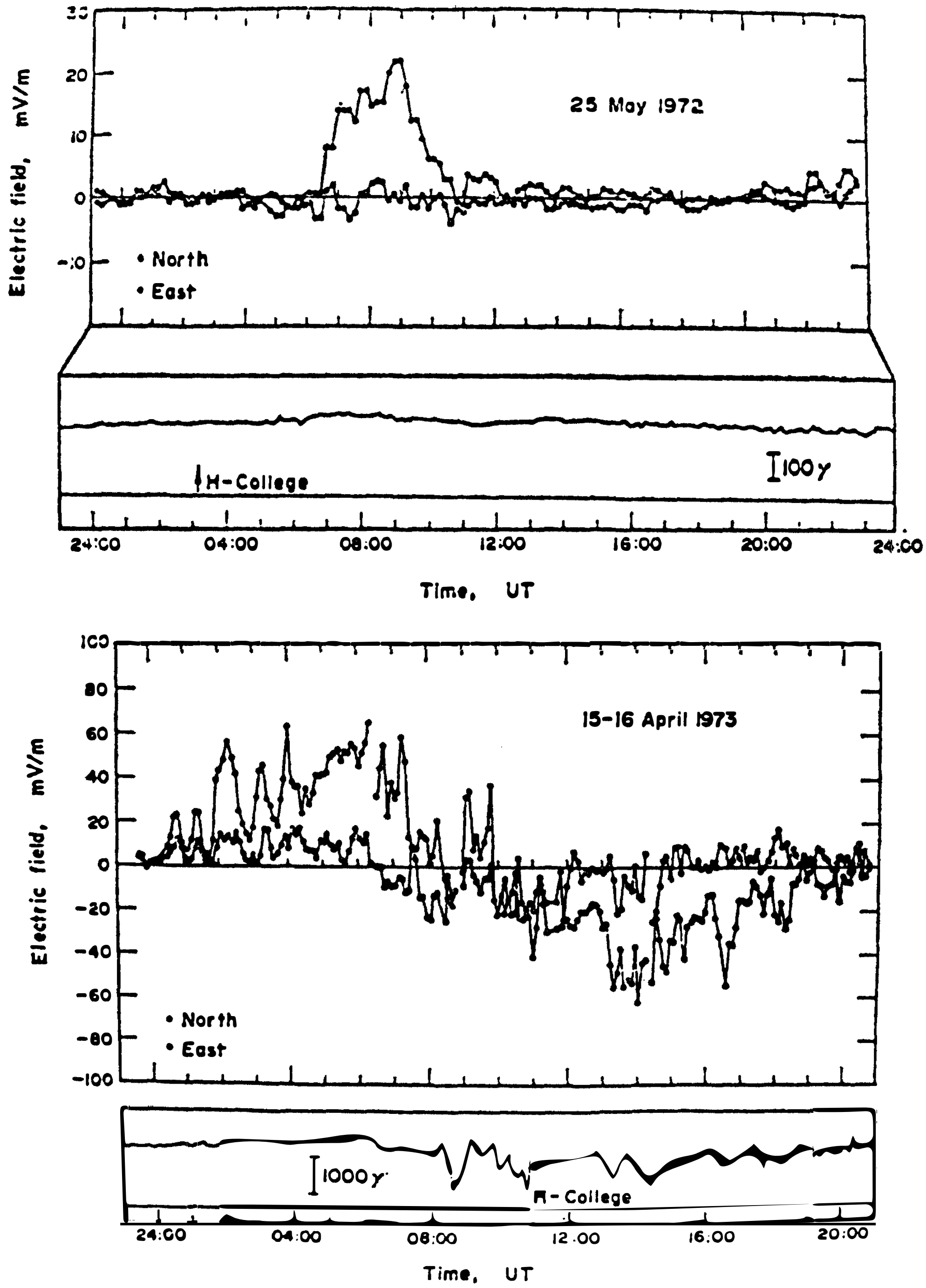

Fig. 7. Electric fields for quiet (upper panel) and disturbed (lower panel) conditions deduced from the IS radar at Chatanika, Alaska (BANKS and DOUPNIK, 1975). Compare the electric field intensities (E–W component) in the figure with those from the present investigation.

electric field in the ionosphere. In the present paper, therefore, if we assume that the drift motion of irregularity is due to the electric field, because we are dealing with the N-S component of drift motion (see Fig. 1), we estimate the intensity of the E-W component of the electric field. The results thus obtained are shown in column E in Table 1.

3. Comparison

Since the reflection point is located near the southern border of the auroral zone (Fig. 1), it would be best to compare the present results with those deduced from the IS radar at Chatanika, Alaska.

In Fig. 7 we show an example of the electric field during both quiet and disturbed conditions (BANKS and DOUPNIK, 1975). During a quiet condition, the intensity of E-W component of electric field falls to values of only a few mV/m, whereas during a disturbed condition, the E-W component of electric field reaches as much as 20 mV/m.

These values of E-W component of electric field during a quiet condition deduced from Chatanika IS radar observations agree well with those obtained from the present investigation (Table 1). Consequently, the assumption that the drift motion of reflection point is due to the electric field in the auroral ionosphere, may be correct.

The present investigation is restricted in that, 1) the model assumes a time-independent reflection height z_0, and 2) because of instrumental reasons, the present HF doppler records were available only when the magnetic activity was not very high (see Kp in Table 1). These restrictions will be overcome in a future investigation.

Acknowledgment

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Appendix: Derivation of Basic Equation

Dealing with the case of a plane wave obliquely incident from below, on the horizontally stratified ionosphere which does not vary in the x and y directions, the differential equation for the electric field intensity of the "horizontally polarized wave", Ey is given by (BUDDEN, 1961)

$$\frac{d^2Ey}{dz^2}+k_0^2q^2Ey=0\,, \tag{A-1}$$

where z-axis is taken as a vertical direction, k_0 the wave number in free space and $q^2(z, t)=\mu^2(z, t)-\sin^2\theta_0(t)$. μ is the refractive index and θ_0 is a reflection angle at the height of reflection ($z=z_0$) in the ionosphere.

A WKB solution of eq. (A-1) is expressed as, assuming the form $\exp(i(k_x\cdot x-\omega\cdot t))$, where k_x is the x-component of k, and ω is the angular frequency

$$Ey(x, z, t)=\frac{A}{\sqrt{q(z, t)}}\exp+\left(k_0\int_0^z q(z, t)dz+k_0\sin\theta_0(t)\cdot x-\omega\cdot t\right)$$
$$+\frac{B}{\sqrt{q(z, t)}}\exp-\left(k_0\int_0^z q(z, t)dz+k_0\sin\theta_0(t)\cdot x-\omega\cdot t\right), \tag{A-2}$$

where A and B are constants, the first and second terms in eq. (A-2) stand for up-going and down-going waves, respectively.

Now let R be the reflection coefficient at the ionosphere, then the reflected (down-going) wave intensity $Ey^-(x, z, t)$, will be represented by

$$Ey^-(x, z, t)=\frac{A}{\sqrt{q(z, t)}}R\exp-i\left(k_0\int_0^z q(z, t)dz+k_0\sin\theta_0(t)x-\omega t\right). \tag{A-3}$$

At the reflection height, $z=z_0$, in the ionosphere, $Ey^-(z_0)=Ey^+(z_0)$, so that R should be

$$R=\exp\left(2ik_0\int_0^{z_0(t)} q(z, t)dz\right). \tag{A-4}$$

From eq. (A-4), the reflected wave is written as

$$Ey^-(x=2D, z=0, t)=\frac{A}{\sqrt{q(z, t)}}\exp-i\left(2k_0\int_0^{z_0(t)} q(z, t)dz+2Dk_0\sin\theta_0(t)-\omega t\right), \tag{A-5}$$

where $2D$ is the distance between the transmitter and the receiver points.

Now received frequency, ω_D, is given by

$$\omega_D=-\frac{d}{dt}\left(2k_0\int_0^{z_0(t)} q(z, t)dz+2Dk_0\sin\theta_0(t)-\omega t\right), \tag{A-6}$$

so that the doppler frequency, Δf, is finally expressed as

$$f=(\omega_D-\omega)/2\pi=-\frac{2f_0}{C}\cdot\frac{d}{dt}\left(\int_0^{z_0(t)} q(z, t)dz+D\sin\theta_0(t)\right), \tag{A-7}$$

where C is the light velocity.

Equation (A-7) gives the general expression of the doppler shift frequency Δf.

Let us now estimate the eq. (A-7) in the present case.

Neglecting the geomagnetic field, we have

$$\mu^2(z, t)=1-\omega_p^2(z, t)/\omega^2=1-\gamma\cdot N(z, t)\,, \tag{A-8}$$

where $\omega_p(=(e^2/m\varepsilon_0)\,N$ in MKSA; e, electron charge; m, electron mass; ε_0, permeability in free space; N, electron number density) is the plasma frequency, and $\gamma = e^2/m\varepsilon_0\omega^2$.

The first term of the right-hand side in eq. (A-7) is given by, without the coefficient $-2f_0/C$,

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_0^{z_0(t)} q(z, t)\,\mathrm{d}z\right) = q(z_0(t), t)\cdot\frac{\mathrm{d}z_0(t)}{\mathrm{d}t} + \int_0^{z_0(t)} \frac{\partial}{\partial t} q(z, t)\mathrm{d}z\,. \qquad \text{(A-9)}$$

The first term in the above equation vanishes, because $\mu(z_0, t) = \sin\theta_0(t)$ at the reflection height $z = z_0$. From eq. (A-8), eq. (A-9) is thus given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_0^{z_0(t)} q(z, t)\mathrm{d}z\right) = \int_0^{z_0(t)} \frac{\partial}{\partial t}\sqrt{\cos^2\theta_0(t) - \gamma\cdot N(z, t)}\,\mathrm{d}z\,. \qquad \text{(A-10)}$$

Assuming a variable separation form $N(z, t) = n(z)\cdot T(t)$ in eq. (A-10), the equation becomes,

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_0^{z_0(t)} q(z, t)\mathrm{d}z = \int_0^{z_0(t)}\left[\frac{(\mathrm{d}/\mathrm{d}t)\cos^2\theta_0(t) - (1/T(t))\cdot(\mathrm{d}T(t)/\mathrm{d}t)\cdot\cos^2\theta_0(t)}{2\sqrt{\cos^2\theta_0(t) - \gamma\cdot N(z, t)}}\right.$$
$$\left. + \frac{1}{2}\cdot\frac{1}{T(t)}\cdot\frac{\mathrm{d}T(t)}{\mathrm{d}t}\cdot\sqrt{\cos^2\theta_0(t) - \gamma\cdot N(z, t)}\right]\mathrm{d}z\,. \qquad \text{(A-11)}$$

Going back to the eq. (A-7), the second term is written as

$$\frac{\mathrm{d}}{\mathrm{d}t}(D\sin\theta_0(t)) = D\cos\theta_0(t)\frac{\mathrm{d}\theta_0(t)}{\mathrm{d}t} + \sin\theta_0(t)\cdot\frac{\mathrm{d}D}{\mathrm{d}t}\,. \qquad \text{(A-12)}$$

Considering that $\mathrm{d}x = \tan\theta(z, t)\mathrm{d}z$, where θ is the angle between wave normal and vertical within the ionosphere,

$$D = \int_0^{z_0(t)} \tan\theta(z, t)\mathrm{d}z\,. \qquad \text{(A-13)}$$

Since $\mu(z, t) = \sin\theta_0(t)/\sin\theta(t)$ (Snell's law), $\tan\theta$ is represented as

$$\tan\theta(z, t) = \sin\theta_0(t)/\sqrt{\mu^2(z, t) - \sin^2\theta_0(t)}\,. \qquad \text{(A-14)}$$

Hence eq. (A-13) becomes

$$D = \sin\theta_0(t)\int_0^{z_0(t)} \frac{\mathrm{d}z}{\sqrt{\cos^2\theta_0(t) - \gamma\cdot N(z, t)}}\,. \qquad \text{(A-15)}$$

Using the eqs. (A-11), (A-12) and (A-15), Δf of eq. (A-7) is given in the following final form.

$$\Delta f(t) = \frac{f_0}{C}\cdot\frac{1}{T(t)}\cdot\frac{\mathrm{d}T(t)}{\mathrm{d}t}\cdot\left(D\cot\theta_0(t)\cos\theta_0(t) - \int_0^{z_0(t)}\sqrt{\cos^2\theta_0(t) - \gamma\cdot N(z, t)}\,\mathrm{d}z\right)$$
$$-\frac{2f_0}{C}\sin\theta_0(t)\cdot\frac{\mathrm{d}D}{\mathrm{d}t}\,. \qquad \text{(A-16)}$$

This is the basic eq. (1) in Section 2 of the present paper.

Assuming a horizontal motion $V(V, 0, 0)$ of the waves/irregularities, $\mathrm{d}x$, is written as (Fig. 3) $\mathrm{d}x^2 = l^2\cdot\Delta\delta^2 + \Delta l^2$, where $\Delta\delta$ and Δl are differential quantities of δ and l. This equation is rewritten as

$$V^2=l^2\dot{\delta}^2+\dot{l}^2 ,$$

since $z_0=l\sin\delta$, $dl/dt=-l\cot\delta\cdot\dot{\delta}$, where $dz_0/dt=0$ is assumed.

From these equations, we obtain the following equation

$$\dot{\delta}^2=V^2\sin^4\delta/z_0^2 . \qquad \text{(A-17)}$$

This is the eq. (5) in Section 2.

The eq. (6) in Section 2 derived as follows. In Fig. 3, we have a relation as,

$$\tan(\delta+2\theta)=z_0/(L-x) .$$

Hence tan 2θ is given by

$$\tan 2\theta=\frac{z_0-(D-x)\tan\delta}{(D-x)+z_0\tan\delta} ,$$

θ is expressed as, since $x=z_0/\tan\delta$,

$$\theta=0.5\tan^{-1}\frac{2z_0-D\tan\delta}{D+z_0(\tan\delta-\cot\delta)} . \qquad \text{(A-18)}$$

This is the eq. (6) in Section 2.