THE POSSIBILITY OF DEDUCING IONOSPHERIC AND FIELD-ALIGNED CURRENTS FROM GROUND MAGNETIC PERTURBATIONS

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***Abstract*:** The possibility of separating the effects of ionospheric currents from those of field-aligned currents in high-latitude magnetic perturbation records obtained at the earth's surface is discussed. We note that the ionospheric current consists of two basic elements: toroidal and poloidal components, where the divergence of the toroidal part and the curl of the poloidal part are both zero. Thus, if one can derive from the ground magnetic records the equivalent ionospheric current function which is directly related to the toroidal part of the ionospheric current flowing in a thin spherical shell, the curl of the ionospheric current that is given by the conductivity and electric field, is expressed by the scalar current function. It is shown that the combination of the ground magnetic data from a world network and a simple model of the ionospheric conductivity makes it possible to estimate both the ionospheric fields and currents. By taking the divergence, the field-aligned currents are also calculated.

1. Introduction

Magnetic records observed at the earth's surface represent an integrated effect of a variety of source currents flowing in the ionosphere and magnetosphere as well as within the earth. NISHIDA (1978) has recently discussed in his book entitled "Geomagnetic Diagnosis of the Magnetosphere" the idea that ground magnetic data can be used to sense the dynamics of the magnetosphere. However, for the very reason that the data contain too much information, it has been a serious problem to evaluate the relative importance of these currents in producing particular patterns or modes of global or local magnetic perturbations that we are attempting to analyze.

Figure 1 illustrates a logical loop of electromagnetic processes occurring in the magnetosphere-ionosphere system in a way it is outlined by VASYLIUNAS (1970a). This linkage has recently been followed by HAREL *et al.* (1981) to discuss a self-consistent calculation of the magnetospheric electric field and particle population in the equatorial plane. Since it represents a closed loop, except for a link for ground magnetic effects, one can start the calculation to obtain a solution for the complete chain with drastic simplifications, and also it is possible to break the loop into individual links in order to discuss their observable effects in terms of space and ground-based techniques. The link which is dealt with in the present paper is marked with a dashed box, which indicates the ionosphere. We assume throughout this paper that the ground magnetic perturbations in high latitudes are produced only by the ionospheric currents and field-aligned currents.

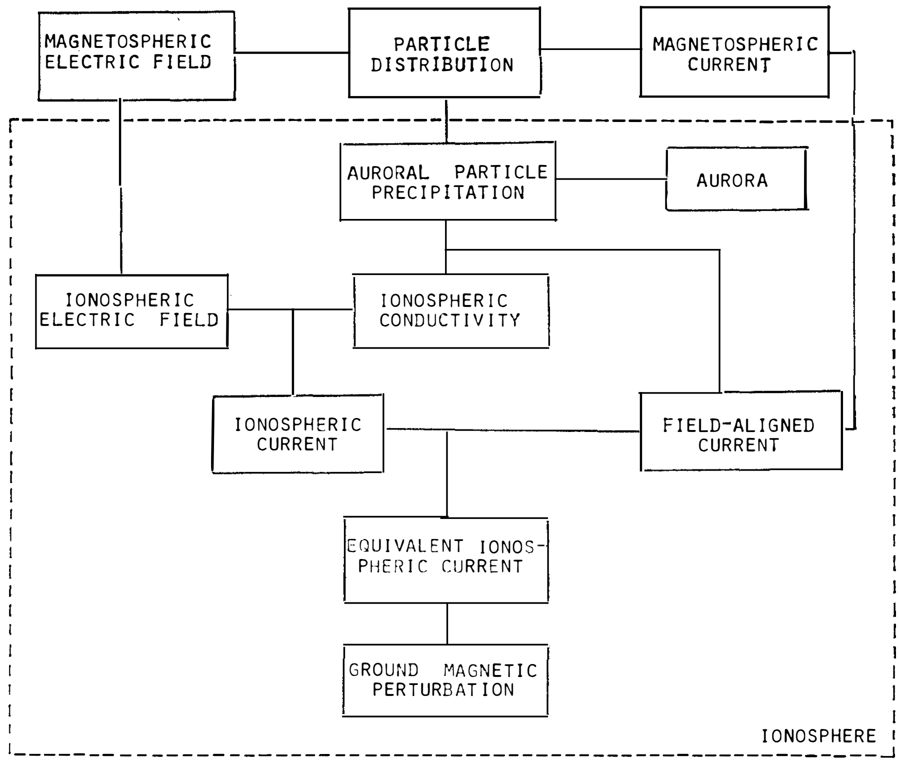

Fig. 1. Flow chart of electromagnetic processes occurring in the ionosphere-magnetosphere system.

If one can assume that the magnetic perturbations under study are nearly static with respect to local time, the associated magnetic potential Φ can be directly related to a steady current distribution in a thin spherical shell, based on standard procedures from the potential theory. The harmonic analysis has extensively been utilized as a powerful tool to obtain accurately the scalar magnetic potential which corresponds to a given set of the ground magnetic data.

One problem that arises in connection with the deduction of the current distribution is that it is, in principle, impossible to determine uniquely the three-dimensional distribution of the external current system solely from magnetic observations made at the earth's surface (CHAPMAN, 1935). It is for this reason that the current distribution in the spherical shell obtained from the magnetic potential has conventionally been designated as the equivalent current distribution. In spite of this difficulty, since the first discovery of field-aligned currents by ZMUDA *et al.* (1966), it has become increasingly of interest to find whether sources and sinks of the auroral ionospheric currents, called the auroral electrojets, can be estimated from surface observations.

KERN (1966) has shown that in the case of the uniform ionospheric conductivity, the ionospheric current $\boldsymbol{i}$ is expressed by a scalar current function J which is related to the divergence-free portion of $\boldsymbol{i}$ as

$$\boldsymbol{i}=\frac{\Sigma_1}{\Sigma_2}\mathrm{grad}\, J+\mathrm{curl}\, J\boldsymbol{r}$$

where Σ_1 and Σ_2 are respectively the height-integrated Pedersen and Hall conductivities, and $\boldsymbol{r}$ is the unit vector in the direction of the outward normal to the earth's surface. The divergence of this current, which gives the field-aligned current $j_{\parallel}$, becomes then, using the field-line inclination χ

$$j_{\parallel} \sin \chi = \operatorname{div} \boldsymbol{i} = \frac{\Sigma_1}{\Sigma_2} \nabla^2 J .$$

It must be noted, however, that the ionospheric conductivity exhibits generally a complicated function of both latitude and longitude, particularly in auroral latitudes where high conductivity gradients are expected to be easily generated by auroral particle bombardments.

The main purpose of this paper is to describe in detail a scheme as well as the physical background in which both the large-scale ionospheric and field-aligned currents can be estimated on the basis of the combination of observed ground magnetic perturbations and a simple model of the ionospheric conductivity.

2. Outline of the Scheme

In order to simplify the problem the following two main assumptions are made: (1) The ionosphere is regarded as a two-dimensional spherical sheet with height-integrated layer conductivity, meaning that we are interested only in large-scale patterns of the electric fields and currents involving spatial scales larger than the thickness of the ionosphere. (2) Only steady state is considered.

The equation of current continuity under these conditions is given by

$$\operatorname{div} \boldsymbol{i} = j_{\parallel} \sin \chi , \tag{1}$$

where $\boldsymbol{i}$ is the ionospheric height-integrated current density, $j_{\parallel}$ is the density of the field-aligned current (positive for a downward current and negative for an upward current), and χ is the inclination angle of a geomagnetic field line with respect to the horizontal in the ionosphere. Ohm's law for the ionospheric current is written as

$$\boldsymbol{i} = \boldsymbol{\sigma} \cdot \boldsymbol{E} = -\boldsymbol{\sigma} \cdot \operatorname{grad} \Phi , \tag{2}$$

where $\boldsymbol{E}$ and Φ are the electric field and potential, respectively, in the frame rotating with the earth and $\boldsymbol{\sigma}$ is the dyadic of the height-integrated ionospheric conductivity.

The two-dimensional ionospheric current can be separated into two elements: first, $\boldsymbol{i}_{SF}$ (source free ionospheric current) which is confined within the ionosphere, and second, $\boldsymbol{i}_C$ (closing current via $j_{\parallel}$) which serves merely to close the field-aligned current (see VASYLIUNAS, 1970a). Thus we have

$$\boldsymbol{i} = \boldsymbol{i}_{SF} + \boldsymbol{i}_C , \tag{3}$$

where, by definition above,

$$\operatorname{div} \boldsymbol{i}_{SF} = 0 \tag{4}$$

and

$$\operatorname{curl} \boldsymbol{i}_C = 0 . \tag{5}$$

These imply that $\boldsymbol{i}_{\mathrm{SF}}$ and $\boldsymbol{i}_{\mathrm{C}}$ can be derived respectively from

$$\boldsymbol{i}_{\mathrm{SF}}=-\mathrm{grad}\ \psi\times\boldsymbol{n}_r \tag{6}$$

and

$$\boldsymbol{i}_{\mathrm{C}}=-\mathrm{grad}\ \tau \tag{7}$$

using the associated scalar functions ψ and τ. ψ is specifically called the equivalent current function. Here, the unit vector in the radial direction from the center of the earth is denoted by $\boldsymbol{n}_r$. The equivalent current system is a toroidal horizontal sheet current, and the remaining current $\boldsymbol{i}_{\mathrm{C}}$ represents a poloidal component. Equation (2) now is rewritten as

$$-\mathrm{grad}\ \tau-\mathrm{grad}\ \psi\times\boldsymbol{n}_r=-\boldsymbol{\sigma}\cdot\mathrm{grad}\ \Phi\ . \tag{8}$$

The current $\boldsymbol{i}_{\mathrm{C}}$ (and also its potential τ) relates to $j_{\parallel}$ in the form

$$\frac{1}{a^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\tau}{\partial\theta}\right)+\frac{1}{a^2\sin^2\theta}\frac{\partial^2\tau}{\partial\lambda^2}=-j_{\parallel}\sin\chi\ . \tag{9}$$

Note that in the absence of conductivity gradients, $\boldsymbol{i}_{\mathrm{C}}$ represents just the Pedersen current and $\boldsymbol{i}_{\mathrm{SF}}$ is the Hall current.

As suggested by FUKUSHIMA (1969, 1976) and VASYLIUNAS (1970b), the ground level magnetic field caused by $j_{\parallel}$ is almost the same as that the caused by $-\boldsymbol{i}_{\mathrm{C}}$. It can be proved that these are rigorously equal when field lines are in the radial direction. The equivalent electric current system estimated from ground observations for the magnetic perturbations produced by $\boldsymbol{i}$ and $j_{\parallel}$ (in other words $\boldsymbol{i}$ and $-\boldsymbol{i}_{\mathrm{C}}$) is identical with $\boldsymbol{i}_{\mathrm{SF}}$, because from eq. (3)

$$\boldsymbol{H}(\boldsymbol{i})+\boldsymbol{H}(j_{\parallel})=\boldsymbol{H}(\boldsymbol{i})+\boldsymbol{H}(-\boldsymbol{i}_{\mathrm{C}})=\boldsymbol{H}(\boldsymbol{i}_{\mathrm{SF}})\ ,$$

where $\boldsymbol{H}(\boldsymbol{I})$ expresses the ground magnetic perturbation produced by the current $\boldsymbol{I}$.

By taking curl of eq. (8), we have

$$\mathrm{curl}\ (\mathrm{grad}\ \psi\times\boldsymbol{n}_r)=\mathrm{curl}\ (\boldsymbol{\sigma}\cdot\mathrm{grad}\ \Phi)\ , \tag{10}$$

because curl grad $\tau=0$. A character of this vector equation, which has only the non-zero $\boldsymbol{r}$ component, is discussed in Appendix.

The left-hand side of eq. (10) is the two-dimensional Laplasian of ψ; namely

$$[\mathrm{curl}\ (\mathrm{grad}\ \psi\times\boldsymbol{n}_r)]_r=\frac{1}{a^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{a^2\sin\theta}\frac{\partial^2\psi}{\partial\lambda^2}\ , \tag{11}$$

where θ and λ denote colatitude and longitude, respectively, and a is the radius of the current sheet (*i.e.*, the earth's radius plus the ionospheric height).

By introducing elements of the height-integrated conductivity dyadic as

$$\boldsymbol{\sigma}=\begin{bmatrix}\Sigma_{\theta\theta} & \Sigma_{\theta\lambda}\\ -\Sigma_{\theta\lambda} & \Sigma_{\lambda\lambda}\end{bmatrix}, \tag{12}$$

the $\boldsymbol{r}$ component of the right-hand side of eq. (10) becomes

$$[\mathrm{curl}\,(\boldsymbol{\sigma}\cdot\mathrm{grad}\,\Phi)]_r$$
$$=\frac{1}{a^2\sin\theta}\left[\sin\theta\Sigma_{\theta\lambda}\frac{\partial^2\Phi}{\partial\theta^2}+\frac{\Sigma_{\theta\lambda}}{\sin\theta}\frac{\partial^2\Phi}{\partial\lambda^2}+\left(\frac{\partial}{\partial\theta}(\sin\theta\Sigma_{\theta\lambda})+\frac{\partial}{\partial\lambda}\Sigma_{\theta\theta}\right)\frac{\partial\Phi}{\partial\theta}\right.$$
$$\left.+\left(\frac{\partial}{\partial\theta}\Sigma_{\lambda\lambda}-\frac{\partial}{\partial\lambda}\left(\frac{\Sigma_{\theta\lambda}}{\sin\theta}\right)\right)\frac{\partial\Phi}{\partial\lambda}+\Sigma_{\lambda\lambda}\frac{\partial^2\Phi}{\partial\theta\partial\lambda}-\Sigma_{\theta\theta}\frac{\partial^2\Phi}{\partial\lambda\partial\theta}\right].$$

It is noted that the last two terms cancel each other out in high latitudes where $\sin\chi\approx 1$, since

$$\Sigma_{\lambda\lambda}\frac{\partial^2\Phi}{\partial\theta\partial\lambda}-\Sigma_{\theta\theta}\frac{\partial^2\Phi}{\partial\lambda\partial\theta}$$
$$\approx\frac{1}{k}[\Sigma_{\theta\lambda}/\sin\chi-\Sigma_{\theta\lambda}\sin\chi]\frac{\partial^2\Phi}{\partial\theta\partial\lambda}\approx 0\ ,$$

where k is a coefficient which specifies the ratio between the Hall and Pedersen conductivities. Hereinafter we use the terms height-integrated Hall and Pedersen conductivities, Σ_H and Σ_P, to simply mean that in high latitudes,

$$\boldsymbol{\sigma}=\begin{bmatrix}\Sigma_P & \Sigma_H\\ -\Sigma_H & \Sigma_P\end{bmatrix}. \tag{13}$$

In the (θ, λ) spherical coordinate system, eq. (10) can be reduced to the form

$$A\frac{\partial^2\Phi}{\partial\theta^2}+B\frac{\partial\Phi}{\partial\theta}+C\frac{\partial^2\Phi}{\partial\lambda^2}+D\frac{\partial\Phi}{\partial\lambda}=F\ , \tag{14}$$

where the coefficients of this second-order differential equation are given by

$$\begin{aligned}A&=\sin\theta\cdot\Sigma_H\\ B&=\frac{\partial}{\partial\theta}(\sin\theta\Sigma_H)+\frac{\partial}{\partial\lambda}\Sigma_P\\ C&=\Sigma_H/\sin\theta\\ D&=\frac{\partial}{\partial\theta}\Sigma_P-\frac{\partial}{\partial\lambda}\left(\frac{\Sigma_H}{\sin\theta}\right)\\ F&=\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{\sin\theta}\frac{\partial^2\psi}{\partial\lambda^2}\ .\end{aligned}$$

Note that if all the conductivity gradients are neglected, eq. (14) is reduced to the Poisson equation for Φ as

$$\frac{\Sigma_H}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right)+\frac{\Sigma_H}{\sin^2\theta}\frac{\partial^2\Phi}{\partial\lambda^2}=\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2\psi}{\partial\lambda^2} \tag{15}$$

indicating that the solution can be given by

$$\Sigma_H\Phi=\psi\ .$$

This simply means that the world pattern of the electric potential is exactly the same as that of the equivalent ionospheric currents which are derived from ground magnetic perturbations and that only the constant Hall conductivity is important in estimating the electric potential from the equivalent current system. This simple relationship has long been employed commonly in many of the works of geomagnetism; see FELDSTEIN (1969) for a review of the derivation of the large-scale electric field from ground magnetic data.

Once the electric potential is calculated by solving eq. (14) numerically for a given conductivity model and for ψ which is obtained from the distribution of ground magnetic vectors, the components of the height-integrated ionospheric current can readily be calculated by

$$\begin{bmatrix} i_\theta \\ i_\lambda \end{bmatrix} = \begin{bmatrix} \Sigma_{\theta\theta} & \Sigma_{\theta\lambda} \\ -\Sigma_{\theta\lambda} & \Sigma_{\lambda\lambda} \end{bmatrix} \begin{bmatrix} E_\theta \\ E_\lambda \end{bmatrix}, \tag{16}$$

where

$$E_\theta = -\frac{\partial\Phi}{a\partial\theta}, \qquad E_\lambda = -\frac{\partial\Phi}{a\sin\theta\partial\lambda}. \tag{17}$$

By inserting the ionospheric current into eq. (1), it is possible to derive the distribution of the field-aligned current density.

3. Discussion

It has been shown in the present paper that the combination of ground magnetic observations and a relatively simple model of the ionospheric conductivity distribution makes it possible to give an approximate method of estimating the intensities of both ionospheric and field-aligned currents in high latitudes. The world distribution of the electric potential can also be determined as one of the by-products.

It is not our intention here to discuss, in quantitative detail, how the ionospheric and field-aligned currents are physically coupled or what the origin of the field-aligned currents are. It must be noted in this connection that the numerical method presented in the present paper is applicable only to high latitudes where $\sin\chi \approx 1$ and the effects of the magnetospheric closing currents, such as the ring current and the tail current, can be neglected relative to those of the ionospheric and field-aligned currents. A detailed discussion is found in FUKUSHIMA and KAMIDE (1973) on how seriously the neglect of the field line curvature influences the resultant distribution of the equivalent ionospheric current vectors.

It may also be necessary to point out that our scheme uses ground magnetic observations only indirectly, in the sense that based on observed magnetic data from the irregular network on the earth's surface, we first calculate the equivalent ionospheric current function whose Laplasian is then utilized as the forcing factor in eq. (14) to solve the electric potential at each grid point of the regular network. As discussed by KAMIDE *et al.* (1976), there are several possible errors in the derivation of the current function on the basis of vector magnetic field, especially in an area where we expect a boundary of the auroral electrojet but there is no magnetic observatory. However, we believe that by following the sophisticated method of RICHMOND *et al.* (1979), the average difference between the observed ground magnetic perturbations and those calculated from the estimated current function is less than 10%.

Finally, we note that the deduced results, *i.e.*, ionospheric and field-aligned currents as well as the electric field, depend linearly on the ionospheric conductivity values. This indicates that if we increase conductivity values by a factor 2 everywhere, the obtained electric fields and currents are simply increased by a factor of 2, meaning that the relative importance of the ionospheric and field-aligned currents in producing the ground magnetic perturbations does not depend upon such a change of the assumed ionospheric conductivity.

The following summarizes the proposed practical steps in an evaluation of the ionospheric and field-aligned currents:

1. The calculation of the current function ψ at a regular network of points on the earth's surface by using ground magnetic records from irregularly distributed observatories. Several methods have been proposed to determine the external part of the responsible current function (*e.g.*, BOSTRÖM, 1971; KAMIDE *et al.*, 1976; RICHMOND *et al.*, 1979) as well as a hand-drawn method for obtaining contour maps of the magnetic potential which is similar to the stream lines of the equivalent ionospheric currents flowing in a thin spherical shell.

2. The computation of the electric potential at each regular point for the following boundary condition:

$$\Phi=0 \qquad \text{at the north pole}$$
$$\partial\Phi/\partial\theta=0 \qquad \text{at the equator .}$$

In solstitial seasons, an asymmetry in the ionospheric conductivity (particularly in the 'background' conductivity produced by the solar radiation) between the northern and southern hemispheres exists, and hence the second boundary condition does not seem appropriate. However, in view of the fact that our method can be applied directly only to the high latitudes where $\sin\chi\approx 1$, this boundary condition at the equator is not important. Calculations must be made for several different models of the ionospheric conductivity in order to see how seriously the estimated potential distribution is modified by the assumed conductivity. In solving the second-order elliptic differential equation in two dimensions to obtain the most probable potential value at each of, for example, 2184 grid points ($=91\times 24$, every 1° in latitude and every 15° in longitude), the AL (accelerated Liebmann) over-relaxation method can be utilized (GARY, 1969). In general 200 to 1000 iterations are required before satisfactory, converged potential values are reached at all the grid points. Accuracy of this process varies between 5×10^{-2} and 10^{-3}, depending upon different cases of quiet and substorm conditions.

3. The calculation of the electric field and the ionospheric current components for the array. The north-south and east-west components of the electric field and current can be calculated on the basis of eqs. (16) and (17).

4. The derivation of the field-aligned currents. From eq. (1) the field-aligned current intensity is calculated by taking the divergence of the ionospheric two-dimensional current vectors.

The application of the present method to some of the average equivalent current systems, such as DP2 and DS, has been published by KAMIDE *et al.* (1981). Efforts to follow the proposed scheme for modeling three-dimensional current systems by using the IMS meridian chain data in high latitudes are being made by, for example, AKASOFU *et al.* (1981), where the present method and the similar method, called the

Forward Method proposed by KISABETH (1979), are compared in terms of their advantages and disadvantages.

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Appendix

One may ask the question as to why the estimated field-aligned current $j_{\|}$ [$=\mathrm{div}\,\boldsymbol{i} =\mathrm{div}\,(\boldsymbol{\sigma}\cdot\mathrm{grad}\,\Phi)$] should have no $\boldsymbol{\sigma}$ dependence. This question comes about because the ionospheric current $\boldsymbol{i}$ can be expressed in terms only of the equivalent current function ψ as

$$\mathrm{curl}\,(\mathrm{grad}\,\psi\times\boldsymbol{n}_r)=\mathrm{curl}\,\boldsymbol{i} \tag{A}$$

implying that the ionospheric current and its divergence might be obtainable from information of only ψ. However, it must be noted that (A) in no way means that $\mathrm{grad}\,\psi\times\boldsymbol{n}_r=\boldsymbol{i}$. By dotting eq. (A) with $\boldsymbol{n}_r$ (since only the r component has meaning for the two-dimensional currents), we have

$$\boldsymbol{n}_r\cdot\mathrm{curl}\,(\mathrm{grad}\,\psi\times\boldsymbol{n}_r)=\boldsymbol{n}_r\cdot\mathrm{curl}\,\boldsymbol{i}\,. \tag{B}$$

We point out that even if ψ is given from worldwide observations of ground magnetic perturbations, eq. (B) cannot be solved for $\boldsymbol{i}$ without further information. This is because eq. (B) is a single equation in which there are two unknown functions i_θ and i_λ.

Our 'further information' is that $\boldsymbol{i}$ can be derived from a scalar function Φ, *i.e.*, we have a second equation

$$\boldsymbol{i}=\boldsymbol{\sigma}\cdot\mathrm{grad}\,\Phi \tag{C}$$

which puts essentially a constraint on $\boldsymbol{i}$. The nature of this constraint appears to depend on the ionospheric conductivity distribution, such that $j_{\|}$ depends on $\boldsymbol{\sigma}$.

Note added in support: The author learned after the present work was submitted for publication that similar methods are proposed by MISHIN and his colleagues, although their numerical scheme appears to be completely different from that suggested in the present paper (V. M. MISHIN and Y. I. FELDSTEIN, personal communication, 1982). In other words, the concept described in eq. (10) is in common. The algorithm developed in U.S.S.R. is most extensively summarized in the following article:

MISHIN, V. M., SHPYNEV, G. V. and BARAZHAPOV, A. D. (1981): Large-scale electric field and currents in the high-latitude ionosphere and magnetosphere as a function of solar wind parameters. Adv. Space Res., **1**, 159–169.