A MAGNETIC CLASSIFICATION OF ANTARCTIC STONY METEORITES ON COMPUTER

Zhijun YU

Beijing Petroleum College, No. 12-1215, Beijing, China

Abstract: This paper is a study of classification of Antarctic meteorites based on magnetic parameters using a discriminant analysis method on computer. The results of computation are almost the same as NAGATA's classification except two samples.

1. Introduction

NAGATA has successfully established a magnetic classification of Antarctic stony meteorites in his papers (NAGATA, 1979a,b). In these papers a data set of magnetic parameters of meteorites is given and two inequalities of thermomagnetic measurements of E, H, L, LL and C chondrites are established as follows:

$$\left\{\frac{I_s(\alpha)}{I_s}\right\}_{\mathrm{E}}>\left\{\frac{I_s(\alpha)}{I_s}\right\}_{\mathrm{H}}>\left\{\frac{I_s(\alpha)}{I_s}\right\}_{\mathrm{L}}>\left\{\frac{I_s(\alpha)}{I_s}\right\}_{\mathrm{LL}}$$

and

$$\left\{\frac{I_s(\alpha)}{I_s}\right\}_{\mathrm{LL}}>\left\{\frac{I_s(\alpha)}{I_s}\right\}_{\mathrm{C}}.$$

In the diagram with I_s versus $I_s(\alpha)/I_s$ for 40 stony meteorites, E, H, L, LL and C chondrites and achondrite groups are separated from each other.

Based on the data set of magnetic parameters of the said papers, a new classification of Antarctic stony meteorites is established by means of multivariate stepwise discriminant analysis on computer. The comupter analysis gives almost the same results as NAGATA's classification.

2. Method of Multivariate Stepwise Discriminant Analysis

The data matrix is $[x_{ijk}]$, where i ($i=1, 2, \cdots, G$) is the number of groups, $j=1, 2, \cdots, n_i$ ($i=1, 2, \cdots, G$) is the number of samples in each group, and k ($k=1, 2, \cdots, V$) is the number of variables. So the element x_{ijk} is the kth variable of jth sample of ith group.

Assuming that the sample of each group is independent normal random vectors, from the original data matrix may give a set of discriminant functions as follows:

$$F_i(X)=\ln q_i+\sum_{k=1}^{V} C_{ki}x_k+Co_i \qquad (1)$$
$$i=1, 2, \cdots, G,$$

where

F_i: discriminant function for ith group,
q_i: pretest probability for ith group which is usually replaced by frequency of samples: $q_i=(n_i/N)$,
C_{ki}: coefficient of kth variable of ith group,
Co_i: constant of ith group,
x_k: variable selected into discriminant function.

For determining which group the sample $X=[x_1, x_2, \cdots, x_V]'$ belongs to, we substitute the parameters of the sample into eq. (1) to obtain a value set of discriminant function: $F_1, F_2, \cdots, F_G$.

If

$$F_i=\max_{1\le g\le G}[F_g(X)]\,,$$

we can ascertain that sample X belongs to ith group. And the distribution of probabilities for each group i will be

$$P_i=\frac{\exp\,[F_i(X)-\max_{1\le g\le G}\{F_g(X)\}]}{\sum_{P=1}^{G}\exp\,[F_p(X)-\max_{1\le g\le G}\{F_g(X)\}]} \qquad (2)$$
$$i=1, 2, \cdots, G\,.$$

3. Stepwise Discriminant Computation

We define variance within, among and total as:

$$W=[w_{kl}]$$
$$B=[b_{kl}]$$
$$T=[t_{kl}]=W+B\,,$$

where

$$w_{kl}=\sum_{i=1}^{G}\sum_{j=1}^{n_i}(x_{ijk}-\bar{x}_{i\cdot k})(x_{ijl}-\bar{x}_{i\cdot l})$$
$$b_{kl}=\sum_{i=1}^{G}n_i(\bar{x}_{i\cdot k}-\bar{x}_k)(\bar{x}_{i\cdot l}-\bar{x}_l)$$

$$t_{kl}=\sum_{i=1}^{G}\sum_{j=1}^{n_i}(x_{ijk}-\bar{x}_k)(x_{ijl}-\bar{x}_l)$$

$$\bar{x}_{i\cdot k}=\frac{1}{n_i}\sum_{j=1}^{n_i}x_{ijk}\ , \qquad \bar{x}_k=\frac{1}{N}\sum_{i=1}^{G}\sum_{j=1}^{n_i}x_{ijk}\ ,$$

and

$$N=\sum_{i=1}^{G}n_i\ .$$

Wilks Λ statistic may be calculated by

$$U=\frac{|W|}{|T|}\ .$$

First of all we calculate matrix W and T, with

$$W=[w_{kl}]$$
$$T=[t_{kl}]$$
$$k=1, 2, \cdots, V$$
$$l=1, 2, \cdots, V\ .$$

For example we give the computation of pth step. Assuming that we have completed pth step of computation and selected m variables into discriminant function (no selection for the first step), then we calculate the $(p+1)$th step as follows:

(1) Determine the test statistic of discriminant effect of each variable

For non-selected variables x_k we calculate,

$$U_k=\frac{W_{kk}^{(p)}}{t_{kk}^{(p)}}\ .$$

For selected variables x_k we calculate,

$$U_k=\frac{t_{kk}^{(p)}}{W_{kk}^{(p)}}\ .$$

(2) Eliminate or select variable

First we consider a variable which may be eliminated from the discriminant function, then we find the largest of $U_k{}^{(p-1)}$. Assuming rth variable x_r corresponds with $U_r=\max_{k\in m}\{U_k{}^{(p-1)}\}$ we take F test and calculate

$$F_2=\frac{1-U_r}{U_r}\cdot\frac{N-G-(m-1)}{G-1}$$
$$=\frac{W_{rr}^{(p-1)}-t_{rr}^{(p-1)}}{t_{rr}^{(p-1)}}\cdot\frac{N-G-(m-1)}{G-1}\ .$$

Table 1. Magnetic parameters of Antarctic stony meteorites.

Group	Sample		$I_s\left(\frac{\text{emu}}{\text{g}}\right)$	$\frac{I_s(\alpha)}{I_s}$	$\frac{I_s(\alpha+\gamma)}{I_s}$	$\frac{I_s(\gamma)}{I_s}$	$\frac{I_s(Mt)}{I_s}$	Meteorite	Classification	
1	1	1	48.000	97.000	0.000	3.000	0.000	Y-691 (a)	E	E chondrite
	1	2	32.300	94.000	6.000	0.000	0.000	Y-694 (d)	H	
	2	3	(15.500	85.000	10.000	5.000	0.000)	Y-7301 (j)	H	
	3	4	33.500	95.000	5.000	0.000	0.000	Y-74371	H5	
	4	5	27.900	94.000	6.000	0.000	0.000	Y-74647	H5	
2	5	6	34.400	95.000	5.000	0.000	0.000	Kesen	H4	H chondrite
	6	7	24.200	87.000	13.000	0.000	0.000	Yonozu	H4, 5	
	7	8	24.300	94.000	6.000	0.000	0.000	Seminole	H4	
	8	9	27.400	88.000	10.000	2.000	0.000	Mt. Baldr b	H6	
	9	10	40.000	90.000	5.000	5.000	0.000	Mt. Brown	H6	
	1	11	14.300	38.000	0.000	0.000	0.000	Y-7305 (K)	L	
	2	12	16.600	90.000	0.000	10.000	0.000	Y-7304 (m)	L5	
	3	13	6.800	79.000	21.000	0.000	0.000	Y-74191	L3	
	4	14	8.100	81.000	19.000	0.000	0.000	Y-74362	L6	
	5	15	22.900	82.000	18.000	0.000	0.000	Fukutomi	L5	
3	6	16	11.000	80.000	20.000	0.000	0.000	Mino	L	L chondrite
	7	17	8.400	65.000	35.000	0.000	0.000	Allan Hills-769	L6	
	8	18	9.700	85.000	14.000	0.000	0.000	Dalgety Down	L4	
	9	19	13.000	85.000	10.000	0.000	0.000	Bjurböle	L4	
	10	20	12.000	80.000	15.000	3.000	0.000	Barratta	L4	
	11	21	10.000	80.000	15.000	5.000	0.000	Homestead	L5	
	1	22	6.000	45.000	35.000	20.000	0.000	Y-74442	LL4	
4	2	23	3.200	19.000	7.000	74.000	0.000	Y-74646	LL5, 6	LL chondrite
	3	24	4.700	45.000	55.000	0.000	0.000	St. Severin	LL6	

5		1	25	11.900	0.000	0.000	0.000	100.000	Orgueil	C1	C chondrite
		2	26	11.200	0.000	0.000	0.000	100.000	Ivuna	C1	
		3	27	10.800	0.000	0.000	0.000	100.000	Y-693 (c)	C3	
		4	28	0.830	0.000	0.000	0.000	10.000	Y-74662	C2	
		5	29	10.300	6.000	0.000	0.000	94.000	Leoville	C3	
		6	30	0.610	0.000	0.000	0.000	5.000	Allende	C3	
		7	31	7.800	0.000	0.000	0.000	100.000	Karoonda	C4	
		8	32	8.000	0.000	0.000	0.000	100.000	Makoia	C2	
6	6	1	33	0.190	81.000	0.000	0.000	0.000	Y-692 (b)	Diogenite	Achondrite
		2	34	0.170	56.000	0.000	0.000	0.000	Y-74013	〃	
		3	35	0.320	100.000	0.000	0.000	0.000	Y-74037	〃	
		4	36	0.320	100.000	0.000	0.000	0.000	Y-74097	〃	
		5	37	0.200	100.000	0.000	0.000	0.000	Y-74648	〃	
		6	38	0.042	100.000	0.000	0.000	0.000	Y-75032	〃	
	7	1	39	0.061	100.000	0.000	0.000	0.000	Y-74159	Eucrite	
		2	40	0.022	100.000	0.000	0.000	0.000	Y-74450	〃	
		3	41	0.076	100.000	0.000	0.000	0.000	Allan Hills-765	〃	
	8	1	42	0.530	100.000	0.000	0.000	0.000	Y-7308 (l)	Howardite	

If $F_2 \leq F_\alpha$ (α is the level of significance), *i.e.* discriminability of x_r is not significant, it will be eliminated from the function, then computation goes to (3). If $F_2 > F_\alpha$, *i.e.* it is significant though the discriminability of x_r is smaller, it will not be eliminated and a non-selected variable with smallest $U_k^{(p)}$ may be selected into the function. We assume this variable is the sth variable x_s, that is

$$U_s = \min_{k \notin m} \{U_k^{(p)}\} .$$

We take F test and calculate

$$F_1 = \frac{1-U_s}{U_s} \cdot \frac{N-G-m}{G-1} = \frac{t_{ss}^{(p)} - w_{ss}^{(p)}}{w_{ss}^{(p)}} \cdot \frac{N-G-m}{G-1} .$$

If $F_1 > F_\alpha$, *i.e.* discriminability of x_s is significant, it will be selected into the function, and then computation goes to (3). If $F_1 \leq F_\alpha$, selection of variables is finished and computation goes to the next step.

(3) Eliminate for W and T

Elimination of $(p+1)$th step for W and T may be calculated by formulas.

$$w_{kl}^{(p+1)} = \begin{cases} w_{rl}^{(p)}/w_{rr}^{(p)} & (k=r,\ l \neq r) \\ w_{kl}^{(p)} - w_{kr}^{(p)} \cdot w_{rl}^{(p)}/w_{rr}^{(p)} & (k \neq r,\ l \neq r) \\ 1/w_{rr}^{(p)} & (k=r,\ l=r) \\ -w_{kr}^{(p)}/w_{rr}^{(p)} & (k \neq r,\ l=r) \end{cases}$$

$$t_{kl}^{(p+1)} = \begin{cases} t_{rl}^{(p)}/t_{rr}^{(p)} & (k=r,\ l \neq r) \\ t_{kl}^{(p)} - t_{kr}^{(p)} \cdot t_{rl}^{(p)}/t_{rr}^{(p)} & (k \neq r,\ l \neq r) \\ 1/t_{rr}^{(p)} & (k=r,\ l=r) \\ -t_{kr}^{(p)}/t_{rr}^{(p)} & (k \neq r,\ l=r). \end{cases}$$

Then computation returns to (1).

(4) Calculate coefficients and constants of discriminant function

Assuming m variables have been selected into discriminant functions, we may calculate coefficients and constants of the functions by the following formulas:

$$C_{ki} = (N-G) \sum_{k \in m} W_{kl}^{(p)} \bar{x}_{i \cdot l}$$

$$Co_i = -\frac{1}{2} \sum_{k \in m} C_{ki} \bar{x}_{i \cdot k} , \qquad (i=1, 2, \cdots, G)$$

where p is the final step of selection.

(5) Discrimination

Supposing the discriminated sample is $X=(x_1, x_2, \cdots, x_m)'$, then the discriminant functions of ith group may be calculated by formulas:

Table 2. Results of computer analysis.

No.	SORT	F1	F2	F3	F4	F5	F6	F7	F8
1	1	98.251	90.496	65.789	17.034	−9.425	25.702	23.074	23.458
2	2	55.174	63.914	53.068	3.833	−9.418	26.407	24.023	23.974
3	3	10.171	35.901	40.122	−1.594	−9.414	24.126	21.633	21.158
4	2	58.309	65.881	53.901	3.367	−9.417	27.031	24.711	24.686
5	2	42.915	56.383	49.287	−1.875	−9.414	27.690	25.507	25.323
6	2	60.817	67.421	54.674	4.535	−9.418	26.769	24.408	24.411
7	2	34.064	50.662	47.494	7.481	−9.420	21.952	19.107	18.893
8	2	32.884	50.222	46.194	−6.546	−9.411	28.739	26.721	26.427
9	2	42.744	56.033	49.929	8.877	−9.421	22.627	19.831	19.696
10	2	77.391	77.397	60.184	20.079	−9.427	21.852	18.835	19.050
11	3	−39.414	−10.439	−1.207	−49.107	−9.453	−11.409	−18.035	−18.605
12	3	12.124	37.303	39.785	−12.110	−9.406	30.259	28.502	27.984
13	3	−12.749	21.580	34.126	1.086	−9.417	19.234	16.234	15.582
14	3	−9.544	23.630	34.847	−1.272	−9.415	20.803	17.980	17.345
15	2	31.484	48.873	47.367	15.906	−9.426	17.462	14.083	13.888
16	3	−1.256	28.681	37.537	4.512	−9.419	18.983	15.910	15.375
17	3	−5.375	25.540	38.273	31.474	−9.436	5.134	0.399	−0.038
18	3	−6.824	25.238	34.626	−9.590	−9.411	24.417	22.021	21.377
19	3	−1.249	27.762	34.242	−14.524	−9.412	24.194	21.751	21.167
20	3	−0.321	28.803	36.612	0.103	−9.417	20.013	17.060	16.525
21	3	−4.112	26.924	36.386	1.383	−9.416	20.860	18.025	17.441
22	4	−8.175	22.998	39.003	61.477	−9.451	−7.303	−13.531	−13.877
23	4	−11.316	19.989	38.590	90.636	−9.460	−14.688	−21.792	−22.112
24	4	−11.517	20.953	39.054	67.120	−9.459	−13.264	−20.203	−20.516
25	5	−101.298	−58.789	−35.520	−76.258	11.062	−38.413	−48.087	−48.976
26	5	−103.248	−59.987	−36.121	−77.166	11.062	−38.208	−47.851	−48.761
27	5	−104.363	−60.671	−36.465	−77.685	11.063	−38.092	−47.716	−48.639
28	5	−106.286	−61.649	−36.758	−78.613	−7.422	−37.230	−46.743	−47.694
29	5	−99.853	−56.291	−32.702	−75.844	9.836	−33.350	−42.415	−43.344
30	5	−105.463	−61.132	−36.487	−78.232	−8.450	−37.280	−46.802	−47.744
31	5	−112.722	−65.806	−39.043	−81.577	11.065	−37.217	−46.705	−47.719
32	5	−112.164	−65.464	−38.871	−81.317	11.065	−37.276	−46.772	−47.780
33	6	−48.776	−4.750	12.760	−55.307	−9.402	26.624	24.647	23.576
34	6	−66.246	−22.132	−2.426	−62.370	−9.425	6.909	2.604	1.574
35	6	−35.179	8.657	24.400	−49.789	−9.384	41.574	41.361	40.263
36	6	−35.179	8.657	24.400	−49.789	−9.384	41.574	41.361	40.263
37	6	−35.514	8.451	24.297	−49.945	−9.384	41.609	41.401	40.300
38	6	−35.954	8.181	24.161	−50.150	−9.384	41.655	41.455	40.348
39	6	−35.901	8.213	24.178	−50.125	−9.384	41.649	41.448	40.343
40	6	−36.010	8.146	24.144	−50.176	−9.384	41.661	41.461	40.355
41	6	−35.859	8.239	24.191	−50.106	−9.384	41.645	41.443	40.338
42	6	−34.594	9.016	24.581	−49.517	−9.384	41.512	41.290	40.199

$$F_i(X)=\ln q_i+\sum_{k\in m} x_k C_{ki}+Co_i\,. \qquad (i=1, 2, \cdots, G)$$

If

$$F_i(X)=\max_{1\leq g\leq G}\{F_g(X)\}\,,$$

then sample X belongs to i group with probability calculated by eq. (2).

4. Discriminant Functions and Magnetic Classification of Antarctic Stony Meteorites

Table 1 is the magnetic parameters of Antarctic stony meteorites. The data matrix includes 42 samples of 8 groups and 5 variables.

The results of computer analysis are shown as in Table 2.

The discriminant order of variables is shown in Table 3.

Table 3.

NO=2	$I_s(\alpha)/I_s$
NO=1	I_s
NO=3	$I_s(\alpha+\gamma)/I_s$
NO=4	$I_s(\gamma)/I_s$
NO=5	$I_s(Mt)/I_s$

5. Conclusion

1) Table 2 shows that groups of diogenite, eucrite, and howardite can not be separated; they may belong to a sixth group, *i.e.* achondrite group.

2) Table 2 agrees with Table 1 in good conformity of 40/42=95.238%.

3) From Table 1, in the magnetic parameters sample No. 3 is close to sample No. 21 and far away from group 2 (see Table 4), so it does not belong to group 2.

And sample No. 15 is near No. 7 (see Table 5), far away from group 3, so it may belong to group 2, where the determinant function of sample No. 15 for the second group (F2=48.873) approaches to the third group (F3=47.367). It would be significant to mention that No. 15 is far away form the centre of group 3 in the five dimension space.

Table 4.

Meteorite	I_s	$I_s(\alpha)/I_s$	$I_s(\alpha+\gamma)/I_s$	$I_s(\gamma)/I_s$	$I_s(Mt)/I_s$
No. 3	15.5	85	10	5	0
No. 21	10.0	80	15	5	0

Table 5.

Meteorite	I_s	$I_s(\alpha)/I_s$	$I_s(\alpha+\gamma)/I_s$	$I_s(\gamma)/I_s$	$I_s(Mt)/I_s$
No. 15	22.9	82	18	0	0
No. 7	24.2	87	13	0	0

4) From Table 3, the discriminant effects of variables $I_s(\alpha)/I_s$ and I_s are significant and larger than others, that is in close agreement with Nagata's conclusion.

5) Our discriminant results are unique, which is an advantage of this classification.

References

NAGATA, T. (1979a): Magnetic classification of Antarctic stony meteorites (III). Mem. Natl Inst. Polar Res., Spec. Issue, **12**, 223–237.

NAGATA, T. (1979b): Magnetic classification of Antarctic stony meteorites (IV). Mem. Natl Inst. Polar Res., Spec. Issue, **15**, 273–279.

XU Daoyi, LIU Chengzuo, ZHANG Juming, XIAO Yiyue, ZHANG Yanpo, ZHANG Qire and SUN Huiwen (1977): Introduction to Geomathematics. Beijing, Geological Publishing House.

(Received June 28, 1981; Revised manuscript received August 31, 1981)