MARINE GRAVIMETRY IN RELATION TO THE ANTARCTIC REGION

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***Abstract*:** Recent techniques of marine gravimetry and gravimeters available are reviewed and some requirements for gravity measurements in the Antarctic region are mentioned in the first part. In the second part the gravity data so far obtained in the Antarctic region including both surface ship and satellite data are shown, and gravity anomalies there are discussed.

1. Gravity Measurement at Sea

Marine gravity can be measured from the air, sea surface and bottom. In this section the surface ship gravity measurement will be described, which is now regarded as having attained a high accuracy level and has been made most extensively. In order to measure gravity on board a surface ship a gravimeter installed on the ship must be (1) kept strictly vertical so that gravimeter's measuring axis may align the gravitational force, (2) disturbing accelerations due to ship's heave, surge and sway must be reduced to as small as 1 mgal, and (3) correction must be made for Eötvös effect, *i. e.*, vertical component of Coriolis force caused by the ship's velocity. These

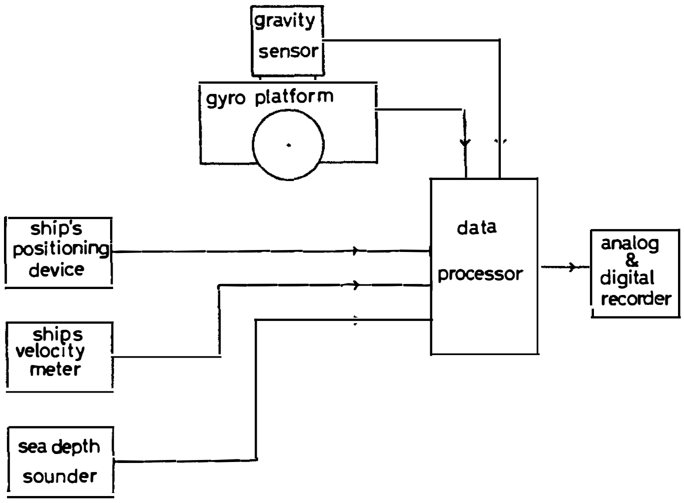

Fig. 1. A general assembly of the surface ship gravimeter system.

requirements are concerned with the hard ware of a gravimeter itself, but, in order to fulfill gravity measurement completely, navigational information such as ship's position and sea depth is very important. Fig. 1 shows a general assembly of a surface ship gravimeter system. There are several surface ship gravimeters available, among which LaCoste and Romberg (USA) and Askania (West Germany) gravimeters are most popular. The Tokyo Surface Ship gravimeter (T.S.S.G.) is the only Japanese meter that has been used since 1963 by the Ocean Research Institute and the Hydrographic Office of Japan.

1.1. Gyro-stabilized platform

In the early stage of sea gravimeter development a gravity sensor was mounted on a damped gimbal which is free to swing in horizontal directions. Since a free gimbal was unsatisfactory as a gravimeter platform, it has been replaced by a gyro-stabilized platform. Although the effectiveness of gyros as a platform stabilizer has been known since long before, the relatively short lifetime prevented their use as a gravimeter component. Recent advance in gyro technology has lengthened the lifetime significantly and made its application to marine gravimeter possible. A typical assembly of a gyro-stabilized platform is shown in Fig. 2.

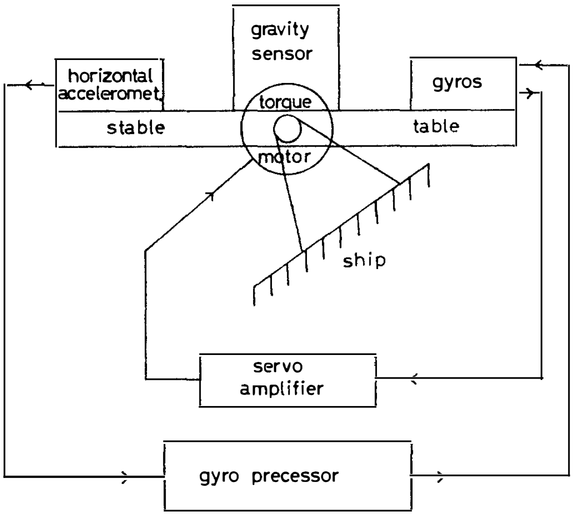

Fig. 2. A typical assembly of gyro-stabilized platform.

There are several methods of stabilization: One of them is the usual vertical gyro type stabilization which is subject, to some extent, to long period disturbing acceleration. Another is the Schuler-tuned stabilization which is the most desirable stabilization but has some difficulty in avoiding oscillatory unstability.

A gyro-stabilized platform consists of a gimbal, a pair of gyroscopes (rate-integrating gyros), a pair of horizontal accelerometers, a pair of torque motors and feedback electronics. When the horizontal accelerometers, which are installed in two horizontal directions perpendicular to each other, detect tilts of the platform, they send signals to gyroscopes so that the gyros may precess. The gyros, in turn, send signals proportional to the precession angle to torque motors which revolve the gimbal so that its tilt is corrected. The equation of motion for a gyro-stabilized platform follows Euler's eq. (1) for a rigid body.

$$\frac{d\bar{L}}{dt}+\bar{\omega}\times\bar{L}=\bar{N} \tag{1}$$

where $\bar{L}$, $\bar{\omega}$ and $\bar{N}$ are angular momentum, vector of rotation and moment of external force, respectively. If the spin angular momentum of the gyro H is assumed constant and treated separately from the angular momentum of the platform, the vector $\bar{\omega}$ is expressed as follows.

$$\begin{gathered}\bar{\omega}=\omega_x\bar{i}+\omega_y\bar{j}+\omega_z\bar{k}\\ \omega_x=\omega'_x,\ \omega_y=\omega'_y,\ \omega_z=\omega'_z+H\end{gathered} \tag{2}$$

where ω_x, ω_y, ω'_x and ω'_y are horizontal components and ω_z and ω'_z are vertical components parallel to the spin axis of the gyro. In this case ω'_x, ω'_y and ω'_z are the rotation rate of the platform. Putting eq. (2) into eq. (1) and expressing in terms of vector components we get the following equations.

$$\begin{gathered}I_x\dot{\omega}'_x+\omega'_y\omega'_z(I_z-I_y)+\omega'_yH=N_x\\ I_y\dot{\omega}'_y+\omega'_z\omega'_x(I_x-I_z)-\omega'_xH=N_y\end{gathered} \tag{3}$$

where I_x, I_y, N_x and N_y are moments of inertia and force in the x and y directions, respectively. In eq. (3), z axis relation is abbreviated. Since a gyroscope is made so that its spin angular momentum is particularly large as compared with the other terms, eq. (3) can be approximated by simpler eq. (4), that is,

$$\begin{gathered}\omega'_yH=N_x\\ \omega'_xH=-N_y.\end{gathered} \tag{4}$$

If angles of tilt of the platform in the x and y directions are expressed by θ_x and θ_y, respectively, then eq. (4) become

$$\begin{gathered}Ht\,\dot{\theta}_y=N_x\\ H\dot{\theta}_x=-N_y.\end{gathered} \tag{5}$$

This is the equation of motion usually applied to a gyro-stabilized platform, and the characteristics of the platform differ according to the relation between the torques N_x and N_y and the platform tilts and horizontal accelerations. An important nature of eq. (5) is that, in a gyro-stabilized platform, the angular velocity is proportional to the torques applied. This implies that the tilt of the platform is 90° out of phase with the applied torque. If the output of the horizontal accelerometers which are set in the x and y directions are denoted by a_x and a_y, respectively, then we get

$$\begin{aligned} a_x &= -k_x\left(\theta_x+\frac{\ddot{x}}{g}\right) \\ a_y &= -k_y\left(\theta_y+\frac{\ddot{y}}{g}\right) \end{aligned} \tag{6}$$

where $\ddot{x}$, $\ddot{y}$, k_x and k_y represent horizontal accelerations and constants of proportion for the x and y directions, respectively. As the level of a gyro-platform is controlled through the output from the horizontal accelerometers, the torques N_x and N_y are the function of a_y and a_x, respectively. That is,

$$\begin{aligned} N_x(a_y) &= N_x\left[-k_y\left(\theta_y+\frac{\ddot{y}}{g}\right)\right] \\ N_y(a_x) &= N_y\left[-k_x\left(\theta_x+\frac{\ddot{x}}{g}\right)\right]. \end{aligned} \tag{7}$$

Putting eq. (7) to eq. (5), we get

$$\begin{aligned} H\dot{\theta}_x &= -N_y\left[-k_x\left(\theta_x+\frac{\ddot{x}}{g}\right)\right] \\ H\dot{\theta}_y &= N_x\left[-k_y\left(\theta_y+\frac{\ddot{y}}{g}\right)\right]. \end{aligned} \tag{8}$$

The Tokyo Surface Ship gravimeter has a feedback control loop characterized by

$$\begin{aligned} N_x &= -k_y\left(\theta_y+\frac{\ddot{y}}{g}\right) \\ N_y &= k_x\left(\theta_x+\frac{\ddot{x}}{g}\right). \end{aligned} \tag{9}$$

This method has a difficulty in following the earth rotation. The LaCoste and Romberg Sea gravimeter uses a platform which has a nature of long period pendulum, where the function of torque is expressed by

$$\begin{aligned} N_x &= -k_y\left(\theta_y+\frac{\ddot{y}}{g}\right)-k'_y\int\left(\theta_y+\frac{\ddot{y}}{g}\right)dt \\ N_y &= k_x\left(\theta_x+\frac{\ddot{x}}{g}\right)+k'_x\int\left(\theta_x+\frac{\ddot{x}}{g}\right)dt. \end{aligned} \tag{10}$$

A Schuler-tuned platform, which seems to be the best platform available, has a control loop characterized by

$$\begin{aligned} N_x &= -k_y\int\left[\theta_y+\frac{\ddot{y}}{g}+\frac{1}{R}\int\int\ddot{y}dt\right]dt \\ N_y &= k_x\int\left[\theta_x+\frac{\ddot{x}}{g}+\frac{1}{R}\int\int\ddot{x}dt\right]dt \end{aligned} \tag{11}$$

where R is the radius of the earth. Although eq. (11) shows the simplest case of the feedback for a Schuler type platform, this method, if the constants k_x and k_y are properly selected, allows the platform to behave like a pendulum with a period of about 84 min and to be free from any horizontal accelerations. Platforms of this type are used for the inertial navigation. The present state-of-the-art of the gyro platform used on the ship shows the level of accuracy better than 1 min of arc in verticality. A modified meridian gyro platform named Model Mark 19 of the Sperry Corporation shows an accuracy of 10 seconds of arc in the case of surface ship use. The platforms of the Tokyo Surface Ship gravimeter and the LaCoste and Romberg gravimeter have the accuracy better than 1 to 2 min of arc. The effect of tilts of the platform on the accuracy of gravity measurement can be summarized as follows.

1) Off-leveling error $-g\theta^2/2$ where θ is a tilt of the platform.

2) Periodic stabilization error $-\frac{1}{2}\ddot{x}_0\theta_0 \cos\phi$ where $\ddot{x}_0$, θ_0 and ϕ are the amplitude of horizontal acceleration, the amplitude of the tilt of the platform and the phase difference between horizontal acceleration and the tilt of the platform.

3) Cross coupling error $\varepsilon\ddot{x}_0\alpha_0 \cos\phi$ where α_0 is the amplitude of deflection of the beam which composes a pendulous gravity sensor, and ε is a constant. The cross coupling error is associated with a gravimeter that has a pendulous sensor like that of the LaCoste or the Askania KSS 5 meter (its old name is Askania Gss 2).

1.2. Removal of vertical disturbing accelerations

Vertical accelerations due to ship's heave show a comparatively narrow frequency distribution, and it is possible to separate gravity signals from disturbing accelerations. The heave acceleration varies with the size of a ship. A ship of a thousand gross tonnage, for example, generates heave accelerations which predominate in the frequency of about 0.1 Hz. Spectra in the lower frequency range decrease to a negligible degree (<1 mgal), making it possible to measure gravity whose period of variation is longer than 10 seconds. If a ship cruises with a velocity of 10 knot (18.5 km/hr), it proceeds about 300 m in one minute. So, by rule of thumb, it can be said that marine gravity with the wavelength longer than 300 m can be measured by surface ship measurement. Since the maximum amplitude of the ship's heave accelerations is 100 to 200 gals, this acceleration has to be reduced to as small as 10^{-5}, or -100 db, if gravity is to be measured with the accuracy of 1 mgal. In addition, gravity measurements require electronics with high and long-term stability in order to avoid a long-term error of measurement or a bias error. For this reason, an analog filter used conventionally in most gravimeters has been replaced by digital filters. Digital filtering is conducted by taking a convolution with measured accelerations and a weighting function, as follows.

$$G(\tau)=\int_{-\infty}^{\infty} g(\tau-t)\phi(t)dt \tag{12}$$

where g, ϕ, and G represent output from the gravity sensor, weighting function and filtered gravity, respectively. Fourier transform of $\phi(t)$ gives frequency and phase characteristics of the filter. If the weight function $\phi(t)$ is a symmetric function with respect to $t=0$, then the filtered-out signal is not subjected to phase-lag. This is the largest advantage of digital filtering.

1.3. Eötvös effect

The equation of motion which holds in rotating co-ordinates with a vector of rotation rate $\bar{\omega}$ is

$$\bar{a}=\ddot{\bar{\gamma}}+2\bar{\omega}\times\dot{\bar{\gamma}}+\dot{\bar{\omega}}\times\bar{\gamma}+\bar{\omega}\times(\bar{\omega}\times\bar{\gamma})\,, \tag{13}$$

where

$\bar{a}$: acceleration vector
$\bar{\gamma}$: position vector
$\bar{\omega}$: rotation rate vector of the co-ordinates.
Time derivatives here are made with respect to the rotating co-ordinates.

As for the actual earth, $\bar{\omega}$ is directed north and has a magnitude of $\Omega=0.00007292\ \mathrm{s}^{-1}$. Eötvös effect is a vertical component of Coriolis force $2\bar{\omega}\times\bar{\gamma}$. Eq. (14) shows three components of Coriolis force at latitude φ in the directions east, north and vertical.

$$\begin{aligned} E_E&=-2\Omega V_N \sin\varphi \\ E_N&=2\Omega V_E \sin\varphi \\ E_V&=-2\Omega V_E \cos\varphi, \end{aligned} \tag{14}$$

where E and V are Coriolis force and ship's velocity, and suffixes E, N and V show eastward, northward and vertical directions. The vertical component E_V is related only to the east-west component of the ship's velocity, showing a magnitude of about ± 80 mgal at the Equator when a ship's east-west velocity is 10 knot. At the latitude of 60° this decreases to half the magnitude at the Equator. When E_V is differentiated with respect to V_E, we have

$$\begin{aligned} \varDelta E_V&=-2\Omega\cos\varphi\cdot\varDelta V_E \\ &=-1.46\times10^{-4}\cos\varphi\cdot\varDelta V_E. \end{aligned}$$

If

$$\begin{aligned} \varDelta V_E&=0.1\ \text{knot} \\ &=5.1\ \text{cm/s}, \end{aligned}$$

then

$$\varDelta E_V=-0.74\cos\varphi\quad\text{mgal}.$$

If the Eötvös effect is to be corrected for with the accuracy of 1 mgal, ship's velocity has to be measured with the accuracy of ±0.13 knot at the Equator, and ±0.26 knot at latitude of 60°.

2. Marine Gravimeters

Three types of gravimeters for marine use are described. They are LaCoste and Romberg gravimeter Model S, Askania seagravimeter KSS 30 and Tokyo Surface Ship gravimeter.

1) LaCoste and Romberg gravimeter Model S (LaCoste, 1967): This gravimeter was developed in the late 1950's and has been used most extensively. It keeps about a 1 mgal accuracy in rough sea condition. This meter consists of a sensor of LaCoste suspension, a gyro-stabilized platform and a data processor, and it is characterized by a very stable sensing unit. The sensor of LaCoste suspension is an astatic vertical pendulum constructed by means of a zero-length spring. In contrast to the meter for land use, in which the displacement of the beam of pendulum is read for the measurement, the sensor for the sea meter is heavily damped so that velocity of the beam may be proportional to vertical acceleration. This makes the time-lag of the measurement negligibly small, which would otherwise be very large in the LaCoste type meter. Fig. 3 shows the whole assembly of the LaCoste and Romberg meter Model S.

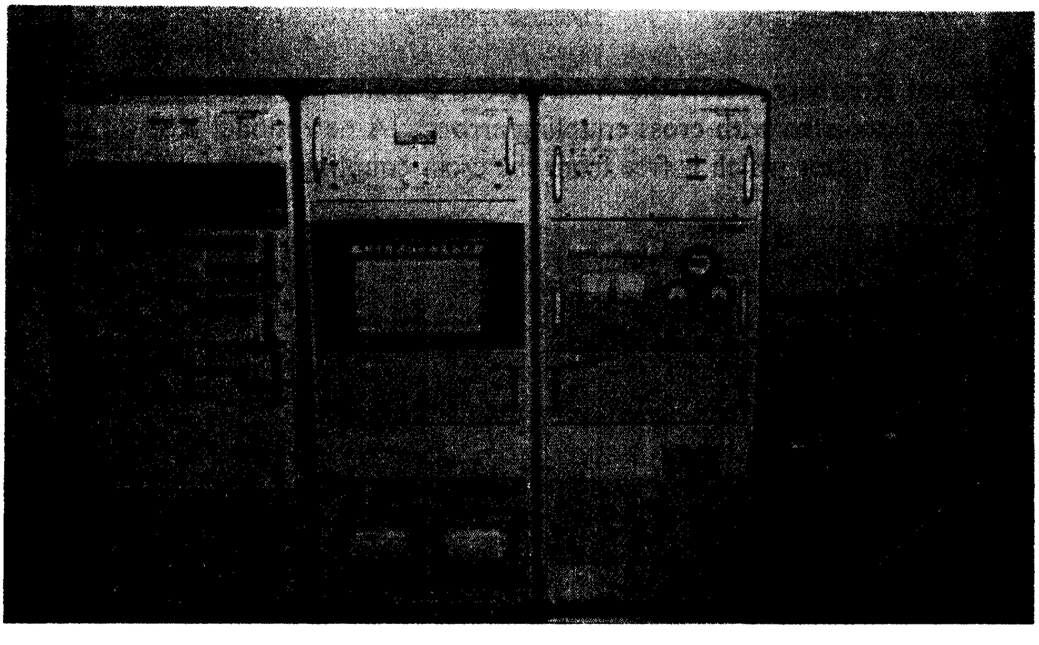

Fig. 3. LaCoste and Romberg gravimeter Model S.

2) Askania seagravimeter KSS 30 (Bodenseewerk Geosystem, 1978): The Askania seagravimeter was developed as early as the LaCoste and Romberg meter and is known to the world as well. The earlier model was named Gss 2, in which a short period vertical pendulum was used as a sensor. This meter consists of a sensor, a gyro-stabilized platform and a data processor. As is the case with the

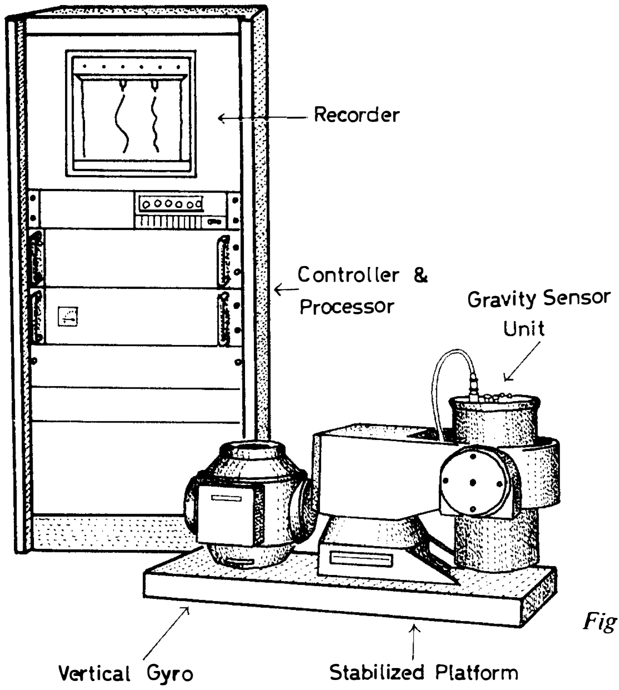

Fig. 4. Askania seagravimeter KSS 30.

LaCoste and Romberg meter, the Askania Gss 2 meter has a moving beam sensor so that it is associated with cross coupling errors. A new model KSS 30 is a radically improved meter which is free from the cross coupling error. This new model

Fig. 5. Tokyo Surface Ship gravimeter.

is very different from the old one in many respects: Its sensor is not a short period pendulum but a force-rebalance type vertical accelerometer which is not subject to cross coupling effect. The gyro-stabilized platform is a platform of repeater type which follows a vertical gyro placed outside the platform. Nominal accuracy of this meter is better than 1 mgal. Fig. 4 shows the assembly of the Askania seagravimeter KSS 30.

3) Tokyo Surface Ship gravimeter (TOMODA and KANAMORI, 1962; TOMODA *et al.*, 1968; SEGAWA, 1970): This meter was developed at the Ocean Research Institute, University of Tokyo in the 1960's. A significant difference from the previous two meters lies in the sensor which is of the vibrating string type. Since the frequency of vibration of a string suspended vertically is proportional to the square root of the string tension exerted by gravity, it is possible to measure gravity through the measurement of vibration frequency. However, some correction for the nonlinearity inherent in the sensor has to be applied when there are ship's heave accelerations. This meter also uses a gyro-stabilized platform and a digital processor. Fig. 5 shows the assembly of the Tokyo Surface Ship gravimeter.

3. Some Remarks on Gravity Measurement in the Antarctic Region

Observations required of gravity measurements at sea include

1. gravimeter operation
2. ship positioning
3. ship velocity
4. sea depth.

In the gravity observations on moving vessels item 2 through 4 are as important as item 1. The accuracy of ship positioning determines the final accuracy of the gravity values obtained. Positioning accuracy better than 1 nautical mile is desired in any time and place. Navigation satellite system NNSS together with a two channel satellite receiver will serve for this purpose. Loran C system is also a good positioning system, but it is not available in the southern hemisphere. Although the OMEGA system is the one globally available, its accuracy is unsatisfactory. The ship's velocity is important for the Eötvös correction. It is desired that a 0.1 knot velocity accuracy could be attained. Ship's velocity can be measured either from the time derivative of position, from ship's speed log, from the inertial navigation system or from the doppler sonar system. In case when a ship maneuvers in the frozen polar sea it is hard to obtain accurate velocities only by differentiating position with time, and so a method of continuous velocity measurement such as the inertial navigation system may be necessary.

If the objective of gravity measurement is to know geoidal undulations from gravity anomalies, for which only free air gravity anomalies are important, the measurement of sea depth may be unnecessary. However, if the objective includes the prospecting of subbottom structures, correction for water depth has to be applied to measured gravity in order to obtain Bouguer gravity anomalies. To do this, a precision depth sounder capable of measuring depth as deep as 10000 m is desired to be installed on board. Because the Bouguer correction at sea has a rate of about 70 mgal/1000 m, the accuracy better than 10 m in depth measurement is desired.

Gravity is usually measured continuously or at intervals from 1 to 10 min, depending on the meter characteristics or on the objective of measurement. Therefore, a smooth combination of gravity, position, Eötvös correction and depth becomes necessary. A proper computing and control system will be necessary for this purpose.

3.1. Other requirements for better operation

Marine gravimeters are very sensitive to the environments in which they are placed. Particularly, electronic noises from the ship's power lines, strong vibrations of the ship's floor and a large change in room temperature should be avoided. In addition, the best state of ship's cruising can be attained when the ship is autopiloted, and unnecessary manual steerings, which are apt to take place in orienting the ship on a scheduled route, may cause an abrupt change of ship's acceleration and have an undesirable effect on gravity measurements.

4. Some Gravity Data for the Antarctic Region

Gravity data in the Antarctic region are not sufficient. This is partly because the environment is so severe that it has been difficult for human beings to approach, but partly because the gravity data, if ever obtained in the region, have not been made public for some reason or other. There are several gravity maps for the sea regions that were published during the last ten years (*e. g.*, NOAA, U.S. DEPARTMENT OF COMMERCE, 1975), but all these maps lack the data in the Antarctic region south of 60°S, presumably because they were deleted. Gravity data in the Antarctic Continent, however, were summarized and made public by the Burean Gravimetrique International, Paris (1974).

The only surface ship gravity data available for the Antarctic Sea (or Southern Sea) are those measured in 1969 by the Tokyo Surface Ship gravity meter (TOMODA, 1974). The area of measurements are from New Zealand to the Ross Sea as far as 70°S. There are no surface ship data for the other regions except for those obtained from satellite observations. Recent studies of global gravity and geoid through orbital perturbations of the artificial satellite and satellite altimetry are remarkable. The compilation of global gravity by combining surface gravity, satellite perturbation and altimetry gravity (by GEOS 3) has resulted in a detailed mapping of gravity anomalies all over the earth with the resolution of 200 km in wavelength. Fig. 6 shows free air gravity anomalies computed from the Earth Model GEM 10 (Goddard Earth Model 10) by LERCH *et al.* (1977), and Fig. 7 shows a rough distribution of

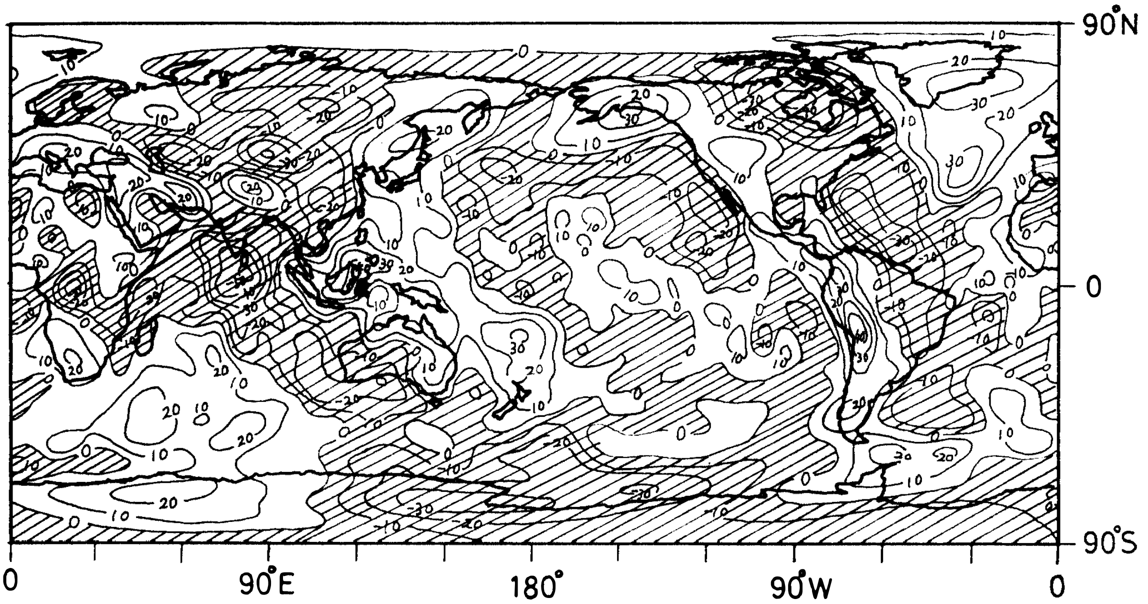

Fig. 6. Free air gravity anomalies computed from the Earth Model GEM 10 (unit in mgal).

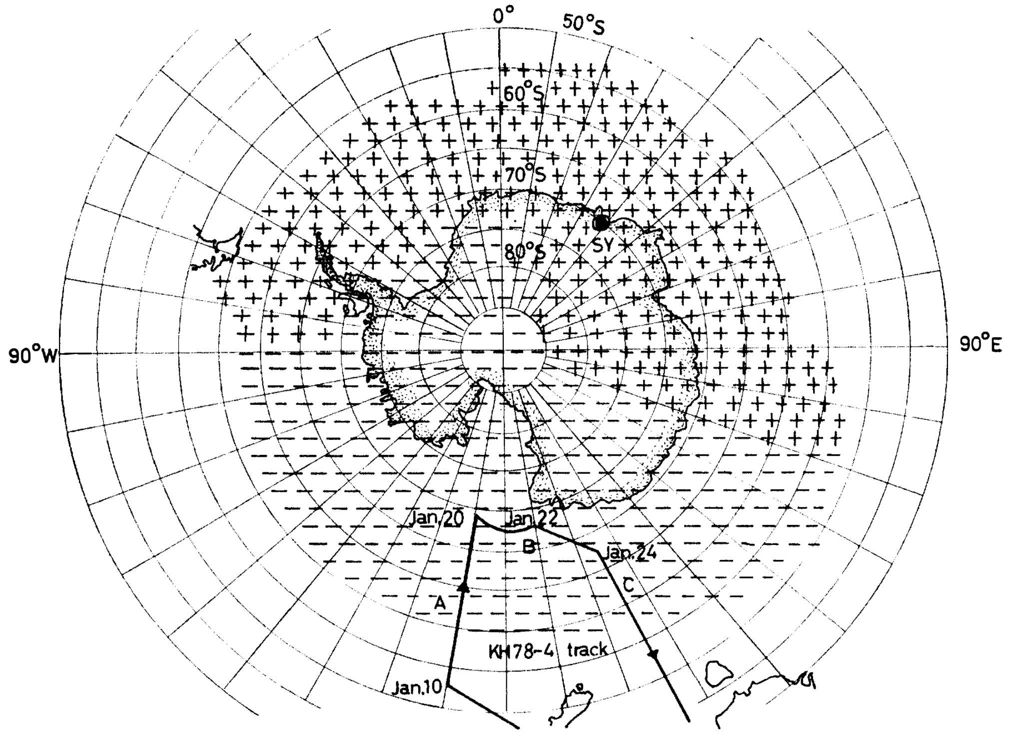

Fig. 7. A general trend of free air anomalies over the Antarctic and its vicinity. Thick solid lines show the tracks of the HAKUHO-MARU cruise KH-68-4 from 1968 to 1969.

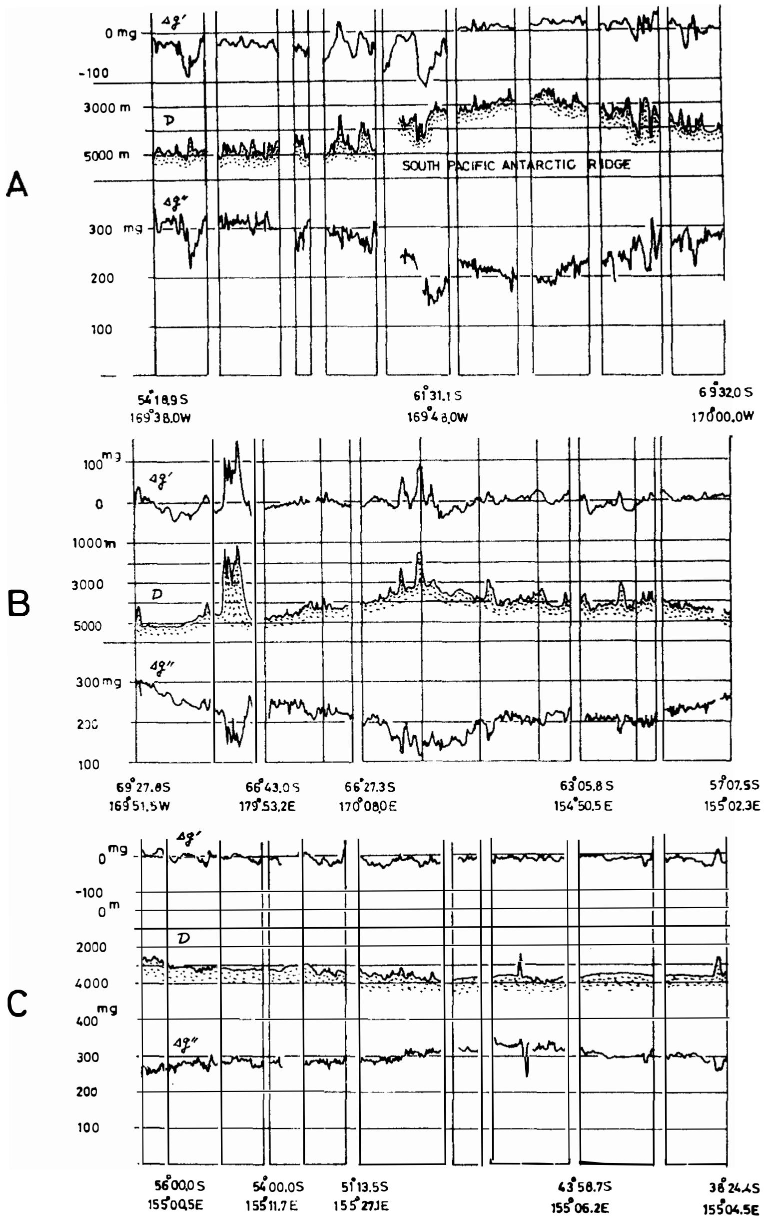

Fig. 8. Profiles of free air ($\Delta g'$) and Bouguer ($\Delta g''$) gravity anomalies along the HAKUHO-MARU *tracks of 1969 in the Antarctic Sea. D shows bathymetry.*

free air anomalies over the Antarctic and its vicinity together with the track of the HAKUHO-MARU cruise KH–68–4. In Fig. 8 the profiles of free air and Bouguer gravity anomalies along the HAKUHO-MARU tracks of 1969 near the Antarctic are shown. From these maps it is clear that the free air gravity anomalies are roughly positive in most part of East Antarctica and negative in West Antarctica. In Fig. 9 are shown shorter wavelength free air gravity anomalies that have resulted from the

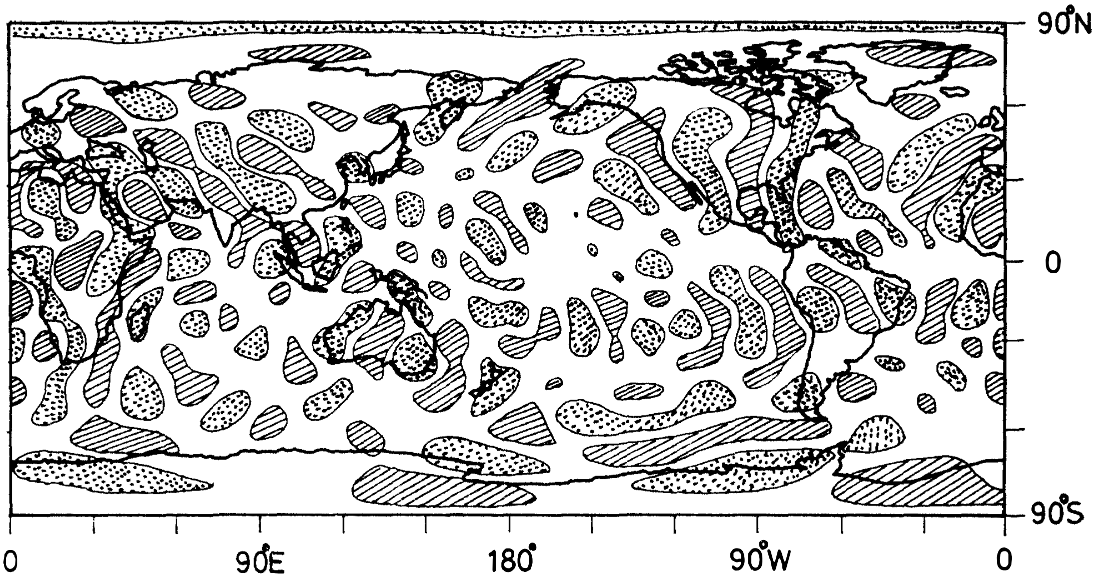

Fig. 9. Free air gravity anomalies computed from the Earth Model GEM 9 using co-efficients of shorter terms of degree 13 to 22. Dotted: higher than 4 mgal. Hatched: lower than —4 mgal. Blank: between 4 and —4 mgal.

synthesis of spherical harmonic terms of degree from 13 to 22 (LERCH *et al.*, 1977). From this map it is seen that remarkable negative anomalies are found from Wilkes Land to the Ross Sea and from Coats Land to the Weddell Sea. Positive anomalies, on the other hand, are found in Queen Maud Land and Ellsworth Land.

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