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Research note

The effect of magnetic mirror force on the field-aligned acceleration of plasmas

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Abstract: A magnetic mirror effect on the field-aligned acceleration of plasma flow is discussed for anisotropic plasma conditions by incorporating double adiabatic equations of state. In a non-uniform distribution of the field magnitude along the field lines, it is found that the field-aligned acceleration is toward higher field intensity region for the fluid of low thermal energy, while the acceleration is toward lower field intensity region for the fluid of high thermal energy. We infer that perpendicular pressure would cause such an energy-dependent behavior of the field-aligned acceleration through the magnetic mirror force.

key words: anisotropic plasma pressure, diamagnetic condition, field-aligned acceleration

1. Introduction

MHD waves in space plasmas are categorized as Alfvén, fast mode and slow mode waves. The Alfvén wave has an incompressible nature. It carries a curl of the cross-field plasma displacement, $\xi_\perp$ (Southwood and Kivelson, 1991), which is the reason why the Alfvén wave has been suggested to carry field-aligned currents. On the other hand, the fast and slow mode waves can carry the divergence of the cross-field plasma displacement $\xi_\perp$ (*e.g.*, Kadomtsev, 1976). For this reason, the fast and slow mode waves are referred to as a compressible mode. For those compressible modes, a spatial gradient of div$\xi_\perp$ along the field line would result in a field-aligned inertial force. Such a force would drive the plasma flows along the field lines. In anisotropic plasma where the perpendicular and parallel pressures differ, the magnetic mirror force averaged over the particles and the net field-aligned force associated with the changing area along the flux tube are added (Comfort, 1988).

In this report, we examine the field-aligned acceleration of plasmas in anisotropic fluid pressure.

2. An equation of motion for field-aligned plasma acceleration

We examine field-aligned flow acceleration by assuming anisotropic plasma pressure condition ($P_\perp \neq P_z$). Here, $\perp$ and z denote "perpendicular" and "parallel" to the field lines.

We start with the field-aligned acceleration of the plasma flow by the following equation (*e.g.*, Parks, 1991).

$$\rho \frac{d\mathbf{V}_z}{dt} = -\nabla_z P_z - (P_z - P_\perp)\mathbf{b}(\nabla \cdot \mathbf{b}). \tag{1}$$

Here, $\mathbf{b}$ is unit vector of the magnetic field $\mathbf{B}$, ρ is mass density, $\mathbf{V}_z$ is field-aligned component of the fluid velocity $\mathbf{V}$. The second term of the right-hand side of the equation can also be written as $-[(P_z - P_\perp)/B]\nabla_z B$, because $\mathbf{b}(\nabla \cdot \mathbf{b}) = -(\mathbf{B}/B^2)\nabla_z B$. Equations of motion perpendicular to the field lines in anisotropic plasma can be given by the equation (*e.g*, Parks, 1991)

$$\rho \frac{d\mathbf{V}_\perp}{dt} = \mathbf{j} \times \mathbf{B} - \nabla_\perp P_\perp - (P_z - P_\perp)(\mathbf{b} \cdot \nabla)\mathbf{b}. \tag{2}$$

Here, $\mathbf{j}$ is current vector.

From the field equation, $\partial \mathbf{B}/\partial t = \nabla \times (\mathbf{V} \times \mathbf{B})$, we can relate field and plasma displacement $\boldsymbol{\xi}$ as,

$$\mathbf{B} = \nabla \times (\boldsymbol{\xi} \times \mathbf{B}). \tag{3}$$

In addition, double adiabatic equations of state are given by (*e.g.*, Parks, 1991)

$$\frac{d}{dt}\left(\frac{P_\perp}{\rho B}\right) = 0, \tag{4}$$

$$\frac{d}{dt}\left(\frac{P_z B^2}{\rho^3}\right) = 0. \tag{5}$$

We examine first order perturbations of eqs. (1) through (5) assuming uniform background field geometries and uniform distribution of background plasma density, pressure. The pressure is however anisotropic. Those are expressed as,

$$\rho_0 \frac{\partial \mathbf{V}_{1z}}{\partial t} = -\nabla_z P_{1z} + \frac{(P_{0z} - P_{0\perp})}{B_0} \nabla_z B_1, \tag{1$'$}$$

$$\mathbf{j}_1 \times \mathbf{B}_0 = \nabla_\perp P_{1\perp}, \tag{2$'$}$$

$$\mathbf{B}_1 = \nabla \times (\boldsymbol{\xi}_1 \times \mathbf{B}_0), \tag{3$'$}$$

$$P_{1\perp} = \frac{P_{0\perp}}{\rho_0}\rho_1 + \frac{P_{0\perp}}{B_0} B_1, \tag{4$'$}$$

$$P_{1z} = -2\frac{P_{0z}}{B_0} B_1 + 3\frac{P_{0z}}{\rho_0}\rho_1. \tag{5$'$}$$

Here, the suffix "0" and "1" indicate the background and perturbed quantities. We assume no background flow, $\mathbf{V}_0$ and no background currents, $\mathbf{J}_0$. To derive the eq. (2$'$), we assumed that the force balance across the field lines is maintained primarily by the diamagnetic condition where the inertial currents in the left-hand side of the equation and centrifugal term ($(\mathbf{b} \cdot \nabla)\mathbf{b} = \mathbf{n}/R$, $\mathbf{n}$ is unit vector normal to the field lines, R is radius of the field line curvature) in the last part of the right-hand side of the eq. (2) are neglected. Such conditions can be justified when field variations are slow to be able to neglect the inertial force as compared to the pressure gradient force and when scale length of the pressure gradient across the field lines is smaller than the radius of the field line curvature.

From the eq. (3′) we can calculate the first order perturbation of the field line vector $\mathbf{B}_1 = -\mathbf{B}_0 \mathrm{div}\xi_\perp + (\mathbf{B}_0 \cdot \nabla)\xi_\perp$ (Kadomtsev, 1976). Here, $\xi_\perp$ is plasma displacement across the field lines. The first order quantity of the field magnitude can be obtained by the following relation.

$$B_1 = \sqrt{(\mathbf{B}_0 + \mathbf{B}_1)^2} - \sqrt{(\mathbf{B}_0)^2} \approx \frac{\mathbf{B}_0 \cdot \mathbf{B}_1}{B_0}.$$

Substituting $\mathbf{B}_1 = -\mathbf{B}_0 \mathrm{div}\xi_\perp + (\mathbf{B}_0 \cdot \nabla)\xi_\perp$ into the above relation, we have the following expression.

$$B_1 = -B_0 \cdot \mathrm{div}\xi_\perp. \qquad (3'')$$

The left-hand side of the eq. (2′) may be expressed as (Kadomtsev, 1976),

$$-\frac{1}{\mu_0}\nabla_\perp(\mathbf{B}_0 \cdot \mathbf{B}_1) + \frac{B_0^2}{\mu_0}(\mathbf{b} \cdot \nabla)\mathbf{b}.$$

We again neglect the centrifugal term in the equation. Then, pressure perturbations in the right-hand side of eq. (2′) may be written as,

$$P_{1\perp} = \rho_0 C_A^2 \cdot \mathrm{div}\xi_\perp. \qquad (2'')$$

Here, C_A is Alfvén velocity defined by $B_0/\sqrt{\mu_0\rho_0}$, μ_0 is the magnetic permeability in vacuum. Because the term "$\mathrm{div}\xi_\perp$" in the eqs. (2″) and (3″) is common for both the perturbations B_1 and $P_{1\perp}$, the field-aligned gradient of the pressure and field magnitude along the flux tube can be related as,

$$\nabla_z B_1 = -\frac{\mu_0}{B_0} \cdot \nabla_z P_{1\perp}. \qquad (6)$$

Substituting eq. (6) into (1′), we obtain

$$\rho_0 \frac{\partial \mathbf{V}_{1z}}{\partial t} = -\nabla_z P_{1z} + \frac{(P_{0\perp} - P_{0z})}{B_0} \frac{\mu_0}{B_0} \nabla_z P_{1\perp}. \qquad (7)$$

Substituting (2″), (3″), (4′) and (5′) into eq. (7), we have

$$\rho_0 \frac{\partial \mathbf{V}_{1z}}{\partial t} = -\frac{P_{0z}}{B_0}\left[\frac{P_{0\perp}}{P_{0z}} - 3 \cdot \left(2 + \frac{B_0^2}{\mu_0 P_{0\perp}}\right)\right]\frac{\partial B_1}{\partial z}. \qquad (8)$$

We define the pressure anisotropy, α, and the ratio of field and fluid energy density, γ by the equations,

$$\alpha = \frac{P_{0\perp}}{P_{0z}},$$

$$\gamma = \frac{1}{N_0 E_0} \cdot \frac{B_0^2}{\mu_0}.$$

Here, N_0 is number density, E_0 is total fluid energy define by the following relations,

$$N_0 E_0 = P_0, \text{ and } P_0 = P_{0\perp} + P_{0z}.$$

Substituting above relations into (8), we have

$$\rho_0 \frac{\partial \mathbf{V}_{1z}}{\partial t} = -\frac{P_0}{B_0}\left[\frac{\alpha-6}{\alpha+1} - \frac{3}{\alpha}\cdot\gamma\right]\frac{\partial B_1}{\partial z}. \tag{9}$$

The field-aligned acceleration is in the parallel/anti-parallel direction of $\partial B_1/\partial z$ as the total fluid energy is above/below the critical value, E_{0c} defined by

$$E_{0c} = \frac{B_0^2}{N_0\mu_0}\cdot\frac{3(\alpha+1)}{\alpha(\alpha-6)}.$$

3. Discussion

The magnetic mirror force is force acting on a single particle. It is a multiple of a perpendicular kinetic energy and gradient of the field magnitude. When the magnetic mirror force of individual particle is averaged over the particles in a volume element, it can be written by $-(P_\perp/B)\nabla_z B$. In addition, a net parallel pressure associated with the changing area along the flux tube can be shown as $(P_z/B)\nabla_z B$ (*e.g.* Comfort, 1988). Therefore the second term of the right-hand side of the eq. (1′) consists of the magnetic mirror force averaged over the particles and net parallel pressure force, though the field gradient is in a first order quantity. The direction of magnetic mirror force is opposite to both the pressure gradient force of fluid perturbation in the first term and the net parallel pressure force. When the background perpendicular pressures exceed that of the parallel pressures, the magnetic mirror force might exceed the sum of the remaining forces. Then, it is apparent that field-aligned acceleration would show two different behaviors as the anisotropy of fluid pressure varies. Because the fluid pressure can be expressed by the thermal energy of fluid, the field-aligned acceleration is a function of the pressure anisotropy and thermal energy. In our calculation, the field-aligned flow acceleration and $\partial B_1/\partial z$ are oppositely directed, when the fluid energy was above E_{0c}. This means that the fluid energy above E_{0c} is accelerated toward weaker field intensity region, if the field intensity changes along the field lines. Substituting typical plasma sheet parameters, $B_0 = 10$ nT, $N_0 = 10^{-1}$cm^{-3}, E_{0c} is estimated to be 16.8 keV when the pressure anisotropy α is 7.0. The fluid energy E_{0c} decreases to 4.0 keV at $\alpha = 10.0$. There appears no upper limit in E_{0c} when α is below 6.0. It asymptotically approaches to zero as α increases.

In the present study, we examined the pressure anisotropy effect on the field-aligned flow acceleration by assuming the diamagnetic condition where inertia force and the centrifugal term are neglected. The results presented here are valid when $\nabla_\perp P_\perp \gg \rho(\partial V_\perp/\partial t)$ and $\nabla_\perp \gg 1/R$, R is radius of the field line curvature.

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