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STUDY ON PIXEL INTENSITY CORRECTION IN PROJECTION TRANSFORM OF AN ALL-SKY AURORAL IMAGE

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Abstract: The algorithm now in use for the projection of an all-sky auroral image onto the geomagnetic/geographic coordinates is summarized in a matrix form and inspected in this note. It is found that in existing projection transforms, pixel intensity has not been corrected at all, and an assumption has been adopted that aurora luminosity is confined in a thin layer of the ionosphere. Under this assumption, namely, that aurora appears in a thin layer of the ionosphere, an expression for pixel intensity correction in a projection of an all-sky auroral image onto the geomagnetic/geographic coordinates is explicitly derived, in which the visual enhancement due to the all-sky imaging geometry and the extinction due to the atmospheric absorption are concerned. The correction may be represented by a diagonal matrix, and should be added to the existing algorithm.

1. Introduction

By using an all-sky/fish-eye lens and a highly sensitive TV camera, the effective monitoring radius and time resolution of a ground aurora observation have now been raised to 600 km and 1/30 s, respectively (for instance, when aurora is taken by a SIT-TV camera and recorded in the NTSC format). By incorporating a high-sensitivity imaging device (integrated from an image intensifier and a CCD (Charge Coupled Device) sensor) with an interference filter and a digital processing method, it also became possible to take a monochromatic image on an auroral emission line within several seconds (ONO *et al.*, 1987). Therefore, the all-sky aurora observation is now one of the most powerful means in the present ground-based observations for studying magnetospheric structures and dynamics.

Auroral particles precipitate along a geomagnetic field line, so that all-sky auroral images usually need to be projected onto the geomagnetic coordinates (MCILWAIN, 1966), for comparing them with satellite observations, or for studying the auroral conjugacy between the northern and southern hemispheres (MINATOYA *et al.*, 1995). Therefore, the projection transform of an all-sky auroral image is one of key steps in auroral data analysis.

2. The Algorithm of Projection Transform of an All-sky Auroral Image

For an ideal fish-eye lens, an all-sky image is formed with azimuth and zenith angles, and any pixel of it monitors the sky with an equal solid angle. In an all-sky polar coordinate system of Fig. 1a, its origin points to the zenith, and its polar axis is assumed towards the geographic north of the observation site. Figure 1b shows the observation geometry of an all-sky image. For any pixel P, its coordinates τ and r correspond to the azimuth relative to the geographic north and the zenith angle θ of the object P, respectively. The relationship between the polar distance r and the zenith angle θ satisfies

$$r=A(\theta / 90^\circ), \tag{1}$$

where A is the radius of an all-sky image.

Assuming that auroral intensity distributions in the all-sky polar coordinates (r, τ) and in the geomagnetic coordinates (ψ^m, λ^m) are given by $\mathbf{X}(r, \tau)$ and $\mathbf{Y}(\psi^m, \lambda^m)$, respectively, the projection transform of an all-sky image onto the geomagnetic coordinates can then be written as

$$\mathbf{Y}\ (\psi^m, \lambda^m)=\mathbf{A}\ (\psi^m, \lambda^m\ ;\ r, \tau)\ \mathbf{X}\ (r, \tau), \tag{2}$$

where $\mathbf{A}(\psi^m, \lambda^m ; r, \tau)$ is a projection operator. In the algorithm now in use for the projection of an all-sky image onto the geomagnetic coordinates (ONO *et al.*, 1987; MINATOYA *et al.*, 1994), the projection operator can be summarized as

$$\mathbf{A}(\psi^m, \lambda^m ; r, \tau)=\mathbf{A}_3(\psi^m, \lambda^m ; \psi^c, \lambda^c)\ \mathbf{A}_2(\psi^c, \lambda^c ; \psi^g, \lambda^g)\ \mathbf{A}_1(\psi^g, \lambda^g ; r, \tau), \tag{3}$$

where $\mathbf{A}_1(\psi^g, \lambda^g ; r, \tau)$, $\mathbf{A}_2(\psi^c, \lambda^c ; \psi^g, \lambda^g)$, and $\mathbf{A}_3(\psi^m, \lambda^m ; \psi^c, \lambda^c)$ are transform operators of the all-sky polar coordinates onto the geographic coordinates (ψ^g, λ^g), of the geographic coordinates (ψ^g, λ^g) onto the geocentric coordinates (ψ^c, λ^c), and of the geocentric coordinates (ψ^c, λ^c) onto the geomagnetic coordinates (ψ^m, λ^m) based on the IGRF model, respectively, when the auroral height is given. This projection operator sets up the geometric relation between the position of a pixel in an all-sky image and the corresponding area in the geomagnetic coordinates. Change of pixel intensi-

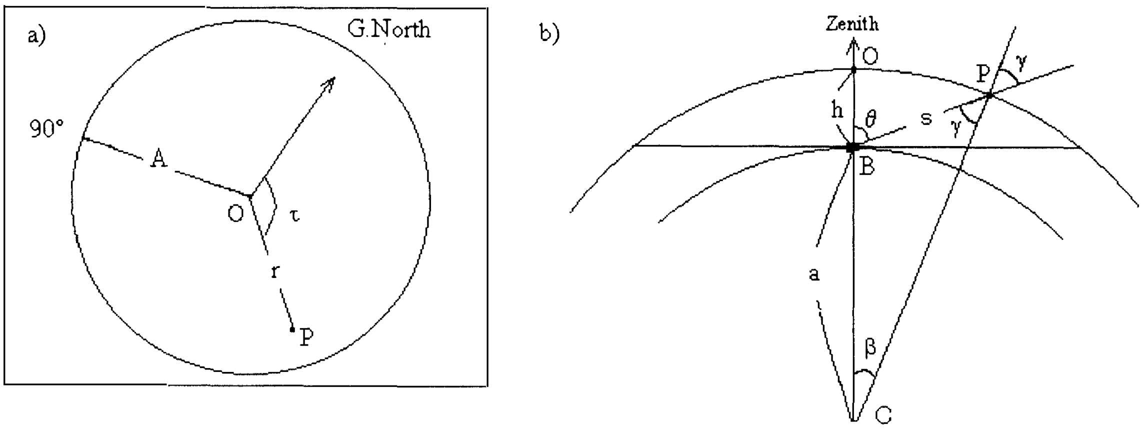

Fig. 1. The geometry of an all-sky auroral image.
(a) Polar coordinates of an all-sky auroral image, where the origin corresponds to the zenith, the polar axis is assumed to be in the geographic north of the observation site. The polar distance r is proportional to the zenith angle θ, and the polar angle τ represents the azimuth relative to the geographic north.
(b) The observation geometry of an all-sky image, where h is the auroral height, a is the earth radius, and θ is the zenith angle of an object P observed at the site B, respectively.

ty in the projection transform has, however, actually not been taken into account.

3. Pixel Intensity Correction

Consider the all-sky imaging geometry in Fig. 2. Suppose that the auroral luminosity of a given object P is $L(P)$ in Rayleighs, and its corresponding pixel intensity in an all-sky image is $I(P)$ given in a gray scale level of 8 bit digits without unit, namely, between 0 and 255. According to Fig. 2, $I(P)$ is related to $L(P)$ as

$$I(P)=k\ L(P)\ \mathrm{d}\sigma/s^2, \tag{4}$$

where $\mathrm{d}\sigma$ is the ionospheric area within the solid angle of the pixel P at the auroral altitude, s is the distance between the object P and the observation site B, and k is the response coefficient of an all-sky TV camera system. Especially on the central pixel of an all-sky image, eq. (4) is written as

$$I(O)=k\ L(O)\ \mathrm{d}\sigma_0/h^2, \tag{5}$$

where $\mathrm{d}\sigma_0$ is the ionospheric area observed by the central pixel O, h is the auroral height. Through the transform of an all-sky auroral image onto the geomagnetic coordinates at the auroral altitude, the pixel intensity represents the actual auroral luminosity. Equation (5) shows that the difference between the intensity of the central pixel $I(O)$ and the auroral luminosity in the local zenith $L(O)$ is only given by a constant factor of $k\ \mathrm{d}\sigma_0/h^2$. For the sake of convenience, we denote the intensity of a pixel P after transform as

$$I_c(P)=k\ L(P)\mathrm{d}\sigma_0/h^2. \tag{6}$$

Thus from eq. (4), for the central pixel O, $I_c(O)=I(O)$, and for any given pixel P,

$$I_c(P)=I(P)(s^2\ \mathrm{d}\sigma_0/h^2\mathrm{d}\sigma)\equiv \mathrm{c}\ I(P), \tag{7}$$

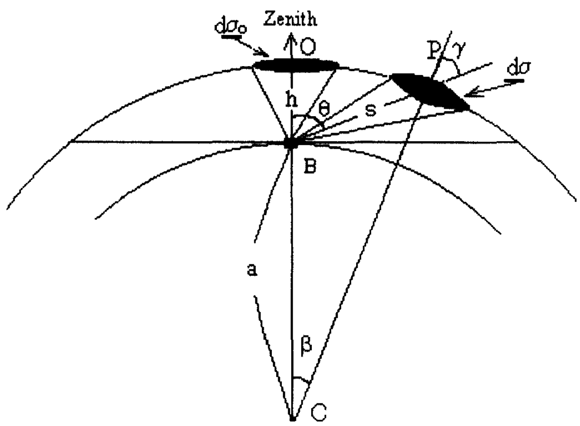

Fig. 2. All-sky imaging geometry.

Fig. 3. An all-sky auroral image taken at Syowa Station (69.00°S, 39.58°E), Antarctica, at 0018:12 UT, September 8, 1993.

where c is defined as a correction factor. $I_c(P)$ stands for the pixel intensity of an object P when it is observed on the ground at its local zenith. According to the all-sky imaging geometry shown in Fig. 2,

$$\mathrm{d}\sigma/\mathrm{d}\sigma_0=(s^2/h^2)/\cos\gamma,$$

where γ is the angle between the line of sight and the local zenith of an object P. So eq. (7) can be re-written as

$$I_c(P)=I(P)\cos\gamma. \tag{8}$$

Using the sine theorem on Fig. 2,

$$\sin\gamma/a=\sin\theta/(a+h), \tag{9}$$

and

$$\cos\gamma=\{1-[a\sin\theta/(a+h)]^2\}^{1/2}\equiv f(\theta, h). \tag{10}$$

The correction factor c is thus represented by

$$c=f(\theta, h). \tag{11}$$

Obviously the correction factor c depends only on the zenith angle θ when the auroral height h is given. In fact, it equals to the inverse of the van Rhijn function.

This correction factor c is to be applied to every pixel of an all-sky auroral image, so that, it can be written as a matrix $C(r)$ in the projection operator of an all-sky image onto the geomagnetic coordinates,

$$\begin{aligned}\mathbf{A}(\psi^m, \lambda^m; r, \tau)=&\mathbf{A}_3(\psi^m, \lambda^m; \psi^c, \lambda^c)\\ &\mathbf{A}_2(\psi^c, \lambda^c; \psi^g, \lambda^g)\,\mathbf{A}_1(\psi^g, \lambda^g; r, \tau)\,\mathbf{C}(r).\end{aligned} \tag{12}$$

Here $C(r)$ is in fact a diagonal matrix, of which each diagonal element satisfies eq. (11).

Figure 3 is an all-sky auroral image taken at Syowa Station (69.00°S, 39.58°E), Antarctica, at 0018:12 UT, September 8, 1993. It is an 8-bit gray-scale level image. Figure 4a, 4b shows projected images of Fig. 3 in the geomagnetic coordinates without/with above pixel intensity correction. The intensity and luminous area of aurora in Fig. 4b are obviously weaker and smaller than those in Fig. 4a. It clearly demon-

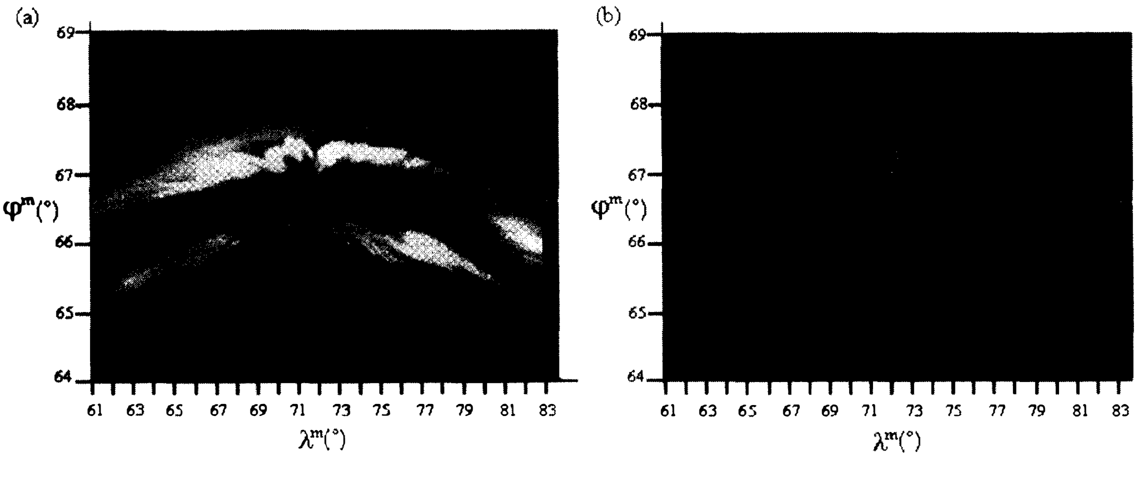

Fig. 4. Projected auroral images of Fig. 3 in geomagnetic coordinates (a) without, and (b) with pixel intensity correction on the van Rhijn effect.

strated the difference between projection transforms of an all-sky auroral image with and without the pixel intensity correction on the van Rhijn effect.

While the van Rhijn effect causes the visual enhancement of auroral luminosity, the atmospheric absorption, however, causes the extinction of auroral luminosity on the way (EATHER *et al.*, 1966). When atmospheric absorption is also taken into account, the luminosity arrived at the observation site and corresponding pixel intensity of any object P are

$$L'(P) = L(P)\ \exp(-\kappa s), \text{ and} \tag{13}$$
$$I(P) = k\ L'(P)\ \mathrm{d}\sigma/s^2, \tag{14}$$

where κ is the atmospheric absorption rate on per kilometer. In this case $I_c(P)$ should also be re-defined as

$$I_c(P) = k\ L(P)\ \exp(-\kappa h)\mathrm{d}\sigma_0/h^2. \tag{15}$$

Substitute eqs. (13) and (14) for eq. (15) and use the above result of $s^2\ \mathrm{d}\sigma_0/h^2\mathrm{d}\sigma$

$$I_c(P) = f(\theta,\ h)\ I(P)\ \exp\{\ \kappa\ (s - h)\}. \tag{16}$$

Using the sine theorem on the triangle CBP of Fig. 1 and eqs. (9) and (10), s satisfies

$$s = \{(a+h)^2 - (a\ \sin\theta)^2\}^{1/2} - a\ \cos\ \theta \equiv g(\theta,\ h). \tag{17}$$

So that when atmospheric absorption is taken into account, the correction factor c is

$$c = f(\theta,\ h)\ \exp\{\ \kappa\ [g(\theta,\ h) - h]\}. \tag{18}$$

After all, in a projection of an all-sky auroral image onto the geomagnetic coordinates, the pixel intensity change should be taken into account. It may be represented by a diagonal matrix $\mathbf{C}(r)$ and the projection operator is corrected to eq. (12). The diagonal elements of $\mathbf{C}(r)$ satisfy eq. (18), where the atmospheric absorption rate κ should be determined on an appropriate atmosphere model when the height and spectrometric characteristic of aurora are given.

When $\mathbf{A}_3(\psi^m,\ \lambda^m\ ;\ \psi^c,\ \lambda^c) = \mathbf{A}_2(\psi^c,\ \lambda^c\ ;\ \psi^g,\ \lambda^g) = \mathrm{I}$, where I is a unit matrix, eq. (12) reduces to

$$\mathbf{A}(\psi^g,\ \lambda^g\ ;\ r,\ \tau) = \mathbf{A}_1(\psi^g,\ \lambda^g\ ;\ r,\ \tau)\mathbf{C}(r). \tag{19}$$

This equation gives the algorithm for a projection of an all-sky image onto the geographic coordinates.

4. Conclusion and Discussion

The algorithm now in use for the projection of an all-sky auroral image onto the geomagnetic/geographic coordinates is summarized in a matrix form and inspected in this paper. It is found that in these algorithms, the aurora luminosity has been assumed to be confined in a thin ionospheric layer, and the pixel intensity, however, has not been corrected self-consistently. Corrections on the visual enhancement due to the van Rhijn effect and the extinction due to the atmospheric absorption are discussed, and an explicit expression is derived for them. It can be written in a diagonal matrix $\mathbf{C}(r)$, with its diagonal elements satisfying eq. (18), and the projection operator is corrected to eq. (12). The significance of the correction on the van Rhijn effect is demonstrated in the example given by Fig. 4. The atmospheric absorption, however, should be calculated on an appropriate atmosphere model when the height and spectrometric characteristic of aurora are given.

It is worth pointing out that the introduction of above pixel intensity correction is under an assumption that aurora luminosity is confined in a thin layer, or in other words, that the luminosity integral along a line of sight is approximate to that along the line of magnetic force. Unfortunately, this is a rare case, instead auroral luminosity usually distributes within a finite profile of the ionosphere. To this problem, multipoint observations may be one solution.

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