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GEOIDAL UNDULATION AND GRAVITY ANOMALY AROUND THE JAPANESE ANTARCTIC STATIONS ESTIMATED FROM BOTH SATELLITE ALTIMETER DATA AND SURFACE GRAVITY DATA

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Abstract: Gravity anomalies and geoidal undulations around the Japanese Antarctic stations (60°-80°S, 20°-50°E) have been newly estimated using both satellite altimeter data and surface gravity data. The method employed for the estimation is the Least Squares Collocation by which both data can be dealt with simultaneously with exact estimates of formal errors.

The data employed are surface gravity data mainly from JARE and GEOSAT altimeter data. These data are the most updated ones in the region, and the results obtained should be the most reliable. We believe that the results should contribute to both geophysical and geodetic applications in the region because there have been few gravity anomaly maps and no geoidal undulation map so far.

1. Introduction

Among the geophysical data generally limited in the Antarctic region, gravity and/or altimeter data cover relatively wide areas. In regard to the area around the Japanese Antarctic stations, a couple of gravity anomaly maps have already been published so far (SEGAWA *et al.*, 1984; FUKUDA *et al.*, 1988). In those maps, land gravity data, sea surface gravity data and SEASAT altimeter data were employed. After their compilations, GEOSAT data, which is a new data set of altimeter data (FUKUDA and SEGAWA, 1989), and additional surface gravity data (NAGAO and KAMINUMA, 1989) have become available. Expecting that these new data will contribute to gain our knowledge of the area, we have decided to compile a new gravity anomaly map.

On the other hand, the compilation of a geoidal undulation map, especially in the continental area, becomes much important recently, because it combines the satellite observation height and the conventional height. In the Antarctic region, there exist two kinds of positioning data, namely, the data obtained by the geodetic traverse method and those obtained by satellite Doppler observation. In the study of large scale topography on land, for instance, a geoidal undulation map plays an important role.

From the theoretical points of view, both gravity anomaly and geoidal undulation are the derivatives of the gravity field, and using the Least Squares Collocation (LSC), they are systematically estimated from both altimeter data and surface gravity data. In this study, therefore, we employ the LSC method to estimate the most reliable gravity anomaly and geoidal undulation around the Japaense Antarctic stations.

2. Least Squares Collocation

Supposing T is the anomalous potential, we have given n observations of the gravity field, l_i. Then n linear or linearized functionals, L_i, relate T to l_i as

$$l_i = L_i(T) + e_i, \quad (i=1, 2, \cdots, n) \tag{1}$$

where e_i is the observation error. In the case of gravity anomaly Δg, for instance, eq. (1) will be

$$\Delta g = -\partial T/\partial r - 2/r \cdot T, \tag{2}$$

and for geoidal undulation N,

$$N = T/\gamma, \tag{3}$$

where r is the mean radius of the Earth and γ is the normal gravity (MORITZ, 1980). These formulas show that once T would be estimated, any quantities of the gravity field could be systematically derived from T. Thus, our problem is to estimate the anomalous potential using the given observations as gravity data or altimeter data.

In general, T is a harmonic function outside the Earth, and only from a finite number of observations, we could not determine T uniquely. We need some assumptions to estimate T accordingly. LSC is a method to estimate T under the conditions of the minimum norm of T and the minimum error variance (MORITZ, 1980). A typical solution of LSC may be written as

$$s = C_{sl}(C_{ll} + D_{ll})^{-1} l, \tag{4}$$

where s is a row vector of the estimated quantities, C_{sl} is a covariance matrix of s and l, C_{ll} is a covariance matrix of observations and D_{ll} is a covariance matrix of observation error. Furthermore, LSC gives formal error estimates of s as

$$E_{ss} = C_{ss} - C_{sl}(C_{ll} + D_{ll})^{-1} C_{sl}^t. \tag{5}$$

This is one of the important benefits of LSC because we could hardly evaluate the estimation error by other methods.

The main drawback of LSC, on the other hand, is that we have to solve a large set of linear equations. The size of the matrix which we have to invert (*e.g.* $C+D$ in eq. (4)) is proportional to square of the number of observations. Because of computer limitations, it is desirable to limit the size of the matrix within several thousands at most. To decrease the size of the matrix, it is effective to remove long wavelength components of the gravity field by using a spherical harmonic gravity model (*e.g.* RAPP, 1983; ARABELOS and TSCHERNING, 1988). This procedure shortens the correlation length of the residual gravity field and makes it possible to estimate detailed gravity field.

Another point in the practical calculation is, as discussed in TSCHERNING (1979), the consideration of the topographic or the known structure effects. LSC requires a covariance function of the anomalous potential in the calculation, and we have to estimate it empirically from the observed data. In these processes, removing the effects of known mass structures makes the gravity field more random, and leads to better estimation. For this purpose, we employed the residual terrain modeling (RTM) method (FORSBERG and TSCHERNING, 1981). By the RMT method, topographic elevations are divided into two parts, namely, mean elevations which are obtained by means of moving average over some blocks, and residual elevations. And then, the effect of the masses associated with the residual topography on the gravity fields is calculated. This method, being simple and having small indirect effects, is suited for gravity field estimations.

3. The Data Set Employed

The area with which we are concerned is bounded by parallels 60°S–80°S and meridians 20°E–50°E. The land gravity data we have employed are from the Japanese Antarctic Research Expedition (JARE) (YANAI and KAKINUMA, 1971; YOSHIDA and YOSHIMURA, 1972; ABE, 1975; KAMINUMA and NAGAO, 1984; NAGAO and KAMINUMA, 1988) and from GRUSHINSKY *et al.* (1972). All of the surface ship gravity data, on the other hand, are from JARE (KASUGA *et al.*, 1983; FUKUDA *et al.*, 1988). The data from JARE-27 and 28 have been re-processed for this study.

The altimeter data are from 38 Exact Repeat Missions of GEOSAT. The raw data have been geophysically corrected and the orbit adjustments have been carried out locally. The detailed description of the processing is found in FUKUDA (1990).

There is another data set of altimeter data, SEASAT. However, we have not employed it because we were afraid of the effects of sea ice on the data (FUKUDA and SEGAWA, 1988) and besides it was not expected to improve the data coverage.

The gravity data and the altimeter data have been edited so that at most one point is included in a $5' \times 5'$ block because the number of data points is partly still too large for LSC calculation. The data distribution after editing is shown in Fig. 1. Total number of the gravity data and the altimeter data after editing amounts to 10672 and 21783, respectively.

In addition to these data, we have employed OSU-86F spherical harmonic model (RAPP and CRUZ, 1986) for the purpose of removing long wavelength components of gravity field and $5' \times 5'$ version of TUG-87 digital terrain model (WIESER, 1987) for the RTM. OSU-86F model provides the most updated coefficients completed up to degrees and orders 360. This model can represent rather short wavelength components of gravity field and is suited for the present purpose.

4. Estimation of the Gravity Field

As the first step of the gravity field estimation, we have to determine the covariance function of the anomalous potential. Outline of the procedure is as follows:

(1) calculate the reference field from OSU-86F and the RTM effect from TUG-87,

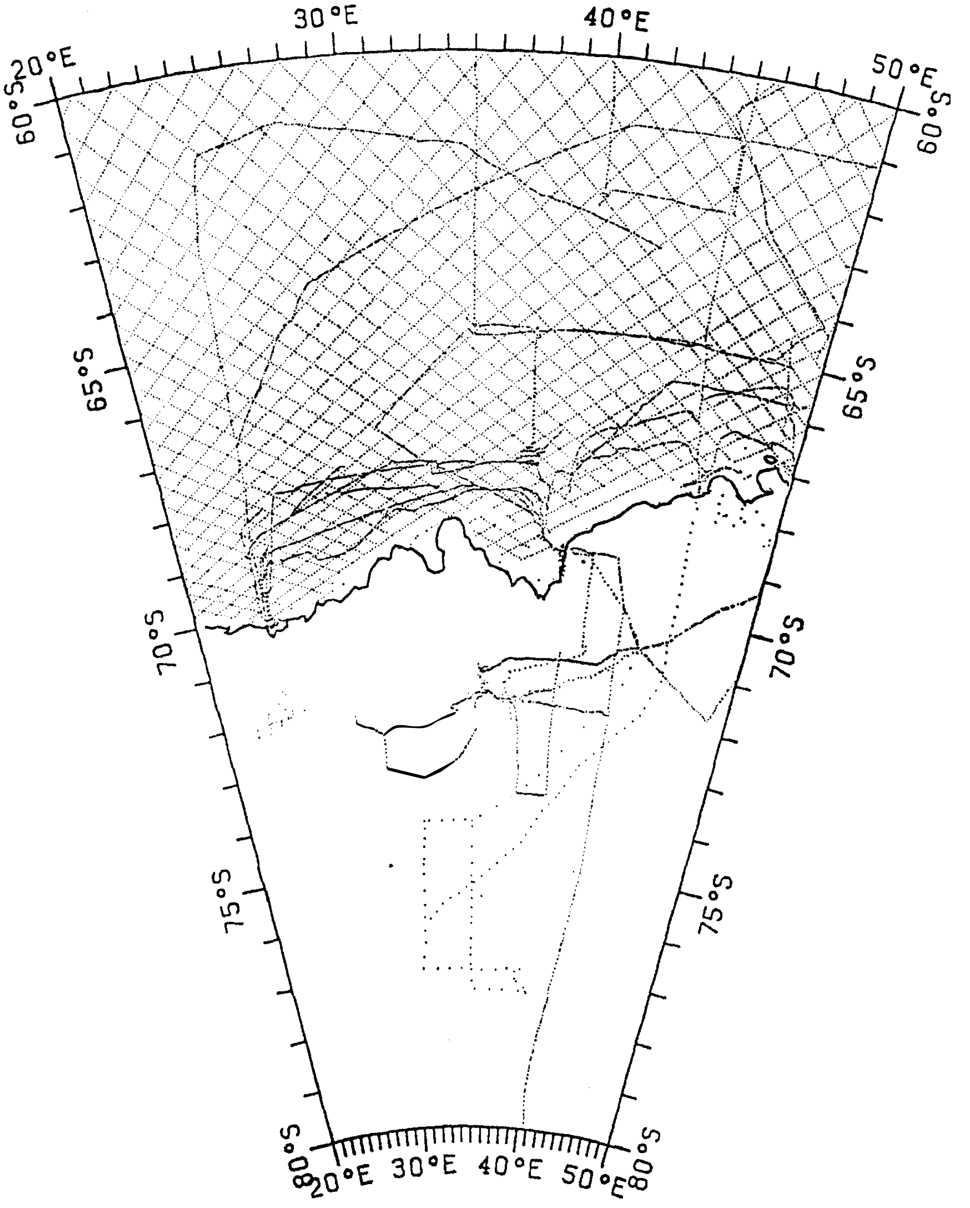

Fig. 1. Data distribution around the Japanese Antarctic stations.

(2) remove the reference field and the RTM effect from the original data,
(3) calculate the empirical covariance function, and
(4) determine an analytic model for the covariance function by fitting the model to the empirical one.

We employed the model for the covariance function of the form

$$K(\psi)=\alpha \sum_{i=2}^{N} \varepsilon_i (R_E^2/rr')^{i+1} P_i(\cos\psi) + \sum_{i=N+1}^{\infty} A/(i-1)(i-2)(i+24)(R_B^2/rr')^{i+1} P_i(\cos\psi), \qquad (6)$$

where

ε_i is the error degree variance related to the reference potential coefficient (OSU-86F),
N is the maximum degree of the reference coefficient (360),
r, r' are the radial distances of the points,
R_E is the mean earth radius,
R_B is the radius of a so-called Bjerhammar sphere,
and α and A are numerical parameters which should be determined to fit the empirical covariance function (TSCHERNING and RAPP, 1974).

Table 1 shows a basic statistics of the data. It should be noted that the average values after removing the OSU-86F field and the RTM effects are not zero. In the case of simultaneous use of both gravity and altimeter data, these differences of the bias should be adjusted to avoid the inconsistency of the data.

Table 1. Basic statistics of the gravity and altimeter data around the Japanese Antarctic stations.

Data	OSU-86F		RTM	
	AV	RMS	AV	RMS
Gravity (mgal)	−6.35	29.94	−5.50	28.59
Altimeter (m)	−0.60	1.62	−0.60	1.60

Generally speaking, terrain information in the Antarctic region is not so reliable. As seen in Table 1, remove of the RTM effects by using TUG-87 slightly decreases the rms value of the residual signals accordingly.

The final numerical values of α and A we have adopted are 1.2 and 100 mgal^2 respectively. Corresponding synthetic covariance functions against empirical ones are plotted in Fig. 2. As seen in the figure, the empirical covariance of geoidal undulation does not seem real because the data points are restricted on the GEOSAT ground tracks and not random. Thus we have put a larger weight on gravity data than on altimeter data when we determine the numerical parameters.

Each calculation by LSC has been carried out using both the gravity data and the altimeter data within a data block which has a width of 3° (latitude)$\times$5° (longitude). And point values of gravity anomaly and geoidal undulation at the center of $5'\times5'$ meshes within a $1°\times1°$ block located in the middle of the data block are estimated simultaneously. These calculations have primarily been carried out by shifting the blocks every one degree. In the case of computing geoidal undulations on land, however, this has produced rather large discontinuities along the block boundaries. We have thus shifted the blocks with 50% overlaps in this case, and taken the average values for final results. The computation was skipped if a data block contained fewer data than 10.

Figure 3 shows the estimated gravity anomalies. The amplitude of the gravity anomaly is not so large in this region. In the oceanic region, in particular, the gravity anomalies are almost zero. It should be noted that a thin, linear pattern observed at the northwestern corner of the figure (60°S, 24°E to 63°S, 20°E) may not be real but relics of GEOSAT orbit which remains unprocessed.

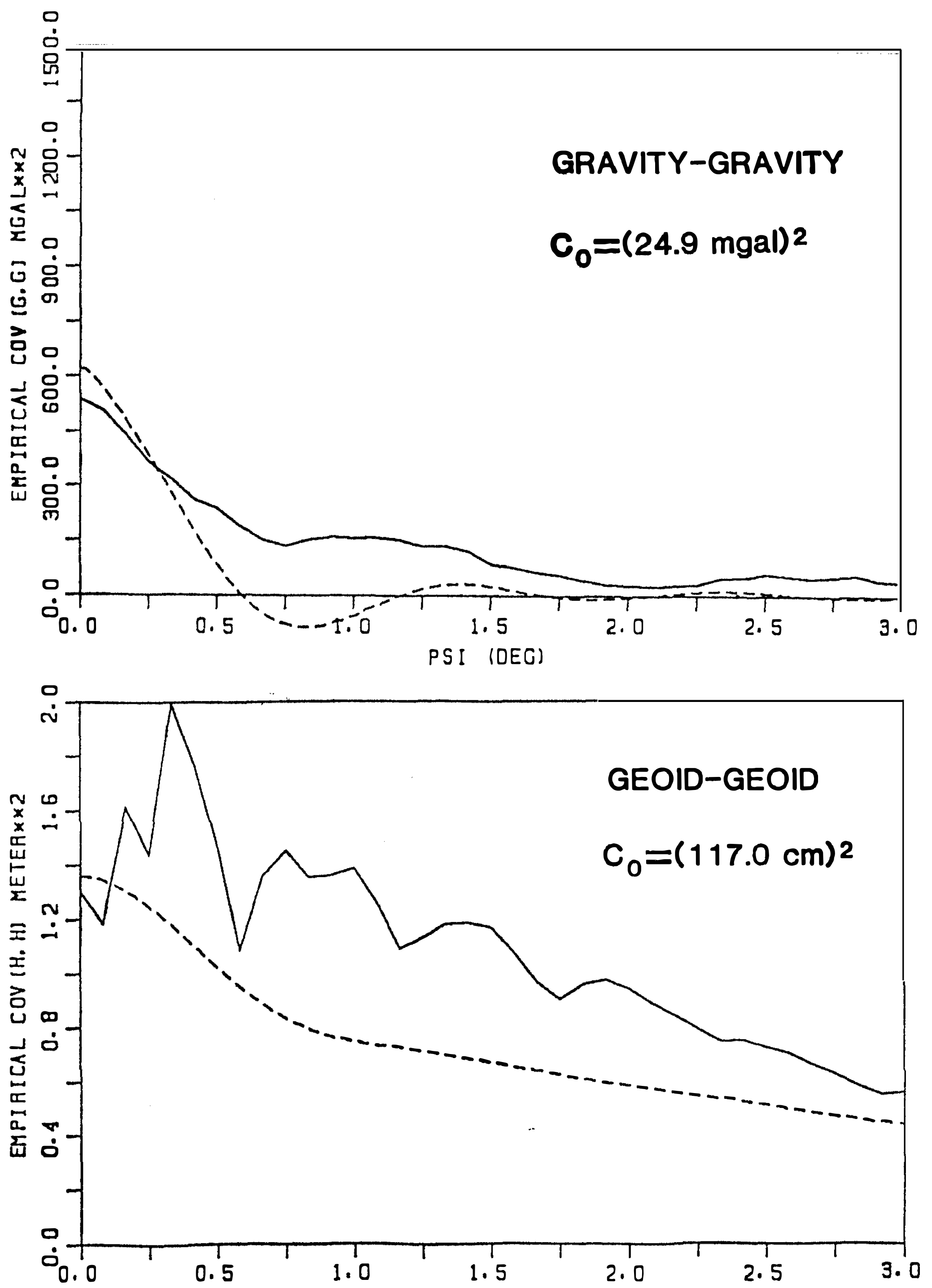

Fig. 2. Empirical (solid line) and synthesized (broken line) covariance functions.

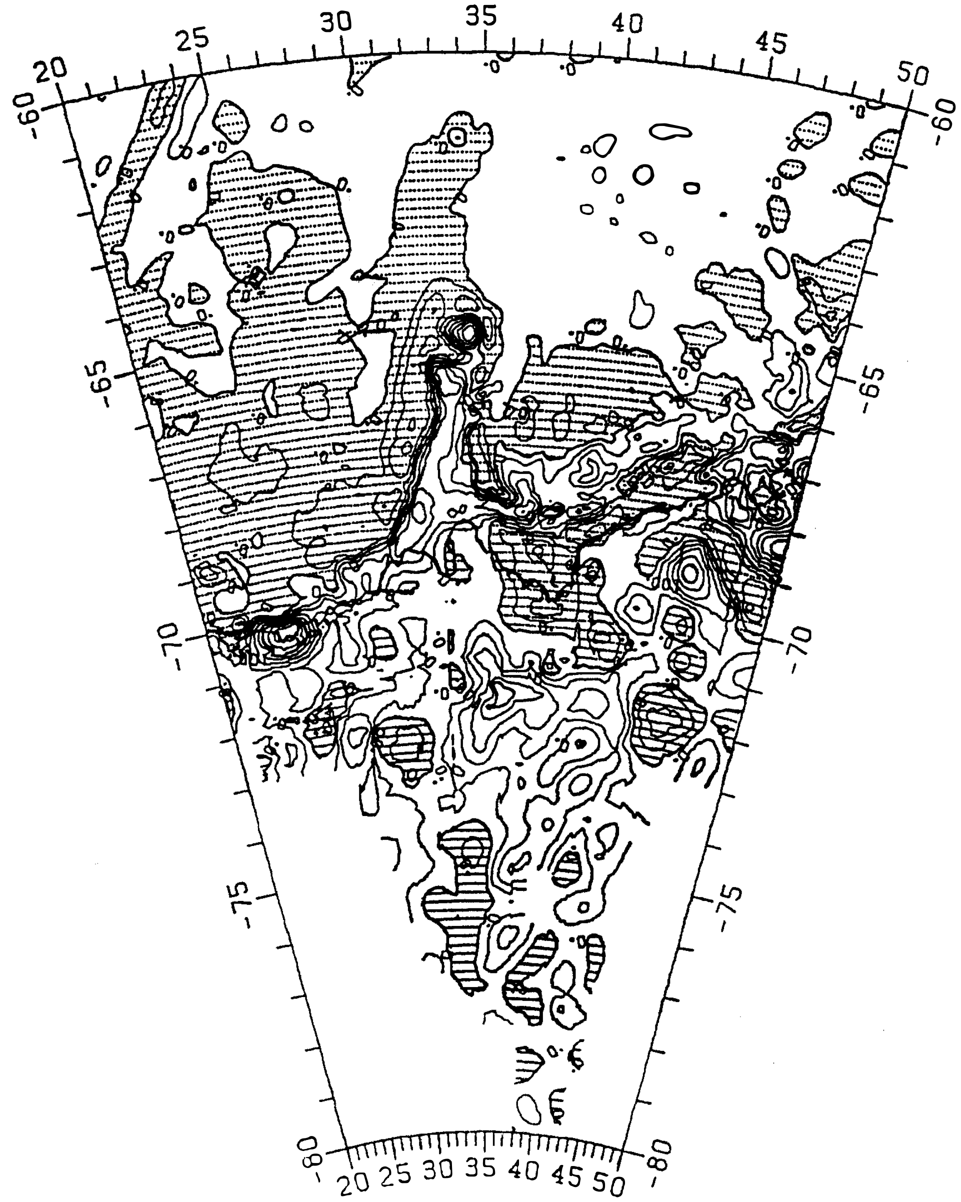

Fig. 3. Estimated gravity anomaly. Contour interval is 20 mgal. Dots represent the area of negative anomalies.

The formal error estimated of the gravity anomalies have also been calculated by using eq. (5). It is noted that the maximum value of the error is bounded by the assumed covariance function, *i.e.* $\sqrt{C_{ss}(0)}=24.93$ mgal. Since the coverage of GEOSAT data is quite uniform, the formal error estimates in the oceanic area have a uniform value around 10 mgal. In the land area, however, the errors are less than 10 mgal only near the data points, and they increase up to maximum value towards the areas with no data.

Figure 4 shows the estimated geoidal undulations. The trend from northeast to southwest represents one of the general features of geoidal undulations around Antarctica. Notches observed in the contour lines on land show inconsistency along

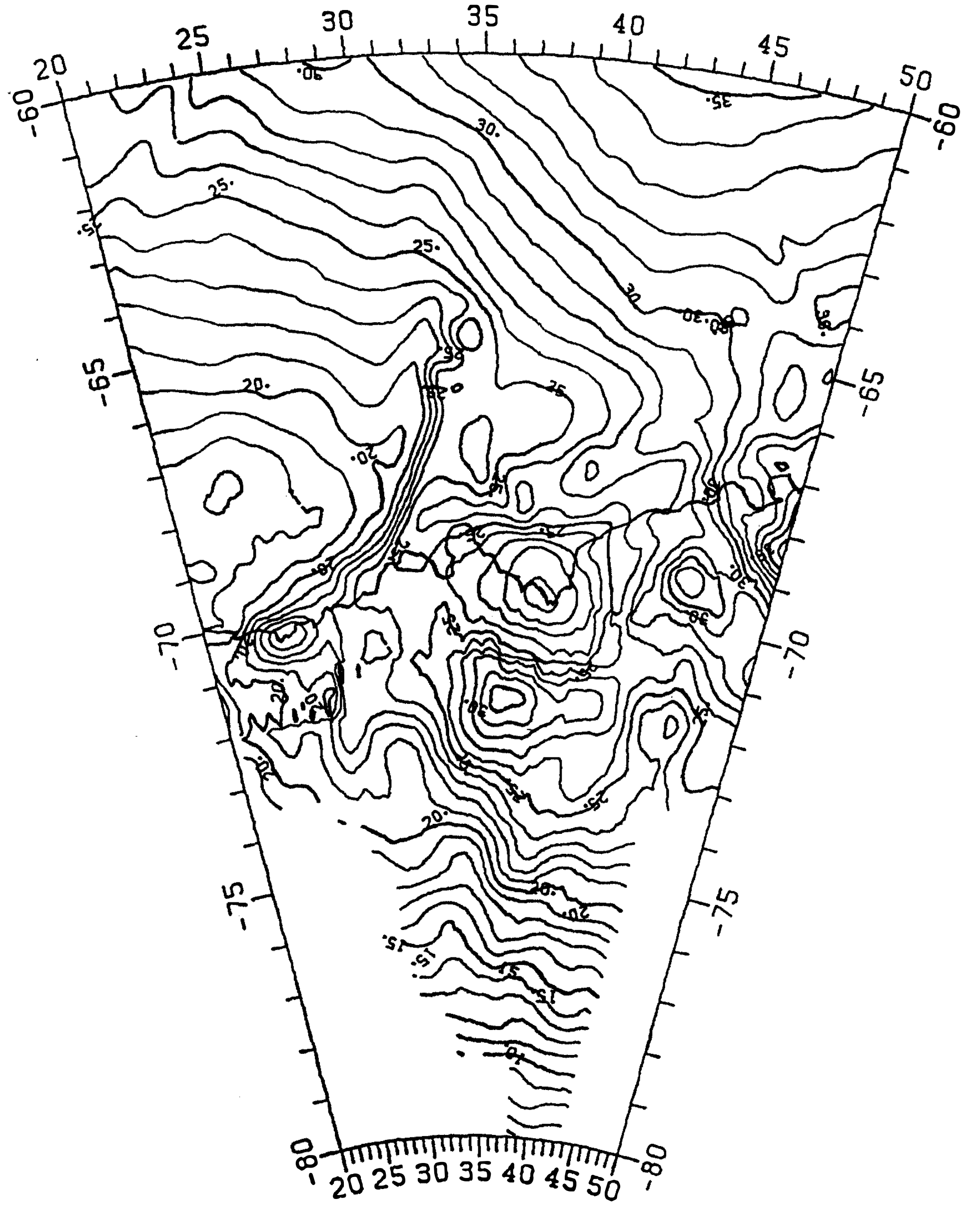

Fig. 4. Estimated geoidal undulation. Contour interval is 1 m.

block boundaries owing to sparse gravity data distribution. In the sea area, on the other hand, the employment of the GEOSAT data has yielded very smooth contour lines.

The maximum formal error of the geoidal undulations, which is determined from the assumed covariance function, is 1.17 m. This value is smaller than the rms of the residual field (see Table 1), thus multiplication by a factor of about 1.4 may give more reliable estimates. The formal errors in the sea area are less than 20 cm because of the employment of the GEOSAT data, but they exceed 80 cm in the land area. This is unavoidable if the sparse data distribution is considered. If the factor 1.4 is to be multiplied, the final accuracy of geoidal undulation on land may be 1.0 to 1.5 m.

5. Discussions

As discussed in the first section, a couple of gravity anomaly maps of this region have been published. Since the data available in the region are restricted, these maps appear different from each other. We believe that the maps presented in this study (Fig. 3) are the most reliable ones among the similar maps because of the number of data employed, the method of data processing and so on. Some of the features of these maps resemble, but some do not. The items of the features about the maps that can be stated definitely are as follows;

(1) high gravity anomaly associated with the Gunneras Ridge and low gravity

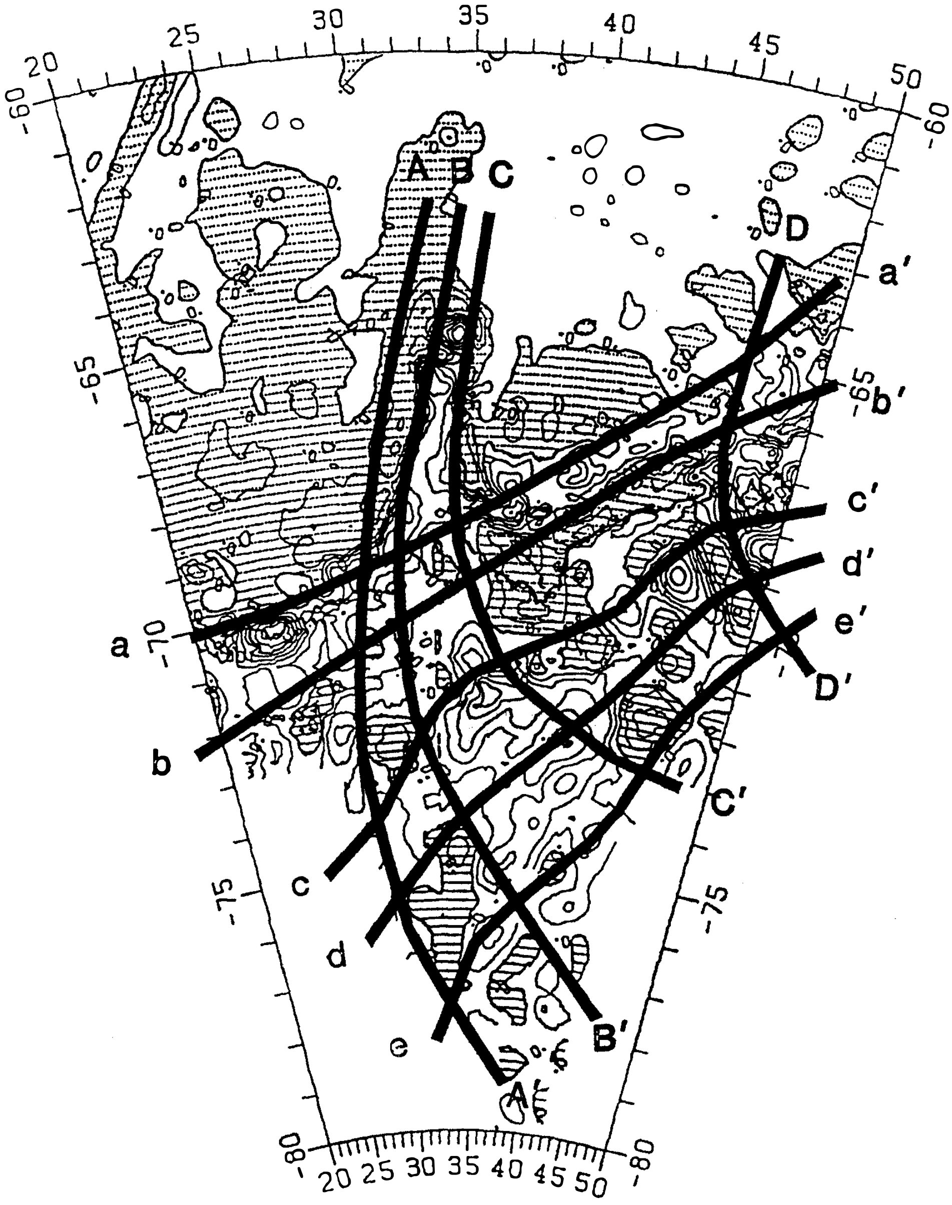

Fig. 5. Zonings of the gravity anomaly (Fig. 3). Solid lines in the figure show the boundaries of the gravity anomaly.

anomalies beside the ridge,
(2) high gravity anomaly in the Yamato Mountains,
(3) high gravity anomaly in the Napier Complex,
(4) high gravity anomaly in Breid Bay.
(5) low gravity anomaly in the area from the Shirase Glacier to Lützow-Holm Bay.

These features are commonly recognized in the maps. According to these anomalies, zonings of the gravity anomaly may be possible. This is shown by the lines AA′ to DD′ in Fig. 5. Possibility of the same zonings has already been pointed out by FUKUDA *et al.* (1988). Since the geological surveys in the area have been restricted to the surface outcorps of rocks, we can hardly refer to the relation between the zonings and the deep-seated geological structures. In spite of these hostile situations, it is inferred at least that line CC′ may correspond to the boundary between the Lützow-Holm Complex and the Yamato-Belgica Complex, and that line DD′ may correspond to the boundary between the Lützow-Holm Complex and the Rayner Complex.

In Fig. 3, another zoning may be possible in the different direction, as shown by the lines aa′ to ee′ in Fig. 5. These features have hardly been recognized in the gravity anomaly maps so far. Since we have little information concerning geological or topographical structure, it is not clear whether the zoning is of any significance or not. However, the zoning seems to suggest a large scale topography in the region.

The geoidal undulation map (Fig. 4) obtained in this study is the first one of this region and also it has a rather practical implication as discussed in the first section. We expect that the map would play an important role, for the continental area in particular, in relating the satellite observation height to the conventional height. However, there remains a problem in the absolute level of the geoid height because the geoid map obtained in the present study does not refer to a local geoid but to the OSU-86F field. We have to know at least one absolute geoid height to fix the map to the local geodetic system. Recently, an approach, which is based on the method combining the GPS interferometric surveying, the NNSS positioning and the tide gauge observation, has been taken to establish a local geoid height (SHIBUYA *et al.*, in preparation, 1990). Unfortunately, the attempt has not been satisfactory yet in its accuracy. But, in the near future, this should be accomplished because it has an important meaning to determine the absolute value of geoid height.

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