ESTIMATION OF LIQUID WATER AMOUNT BY A MICROWAVE RADIOMETER

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Abstract: The microwave radiative properties of water cloud was investigated by solving a radiative transfer equation in which the effect of scattering by cloud particles is taken into account. In the microwave band often used in cloud remote-sensing (0.8–3 cm), clouds can be grouped into two categories. A cloud composed only of waterdrops smaller than about 300 μm in radius, which generally corresponds to non-precipitating cloud, shows a unique relationship between the integrated liquid water and the effective emissivity, almost independently of the size distribution of drops. For a cloud including drops larger than 300 μm the above relationship strongly depends upon the size distribution.

The relationship between the integrated liquid water and the effective emissivity was further studied for several microwave wavelengths. For an optically thinner cloud a short wavelength of 0.81 cm can give a better and more sensitive estimation of integrated liquid water. For the cloud of large optical depth for which the microwave radiation at short wavelength becomes saturated, a combination of radiation at 0.81 and 2.0 cm wavelengths can reduce the effect of size distribution and give the better estimation of liquid water amount.

1. Introduction

The cloud liquid water amount and its horizontal distribution are of fundamental importance in meteorological and climatological studies, because it is one of the main components in the atmosphere and it plays an important part in radiative processes as well as cloud physical processes. Many remote sensing methods have been attempted to estimate the cloud liquid water amount. The passive observation of microwave radiation would be a very useful approach in quantitatively deriving the liquid water amount.

Estimations of the liquid water amount from satellite microwave data have been made by several authors. STAELIN *et al.* (1976) used linear empirical equation to infer the liquid water amount and the water vapor amount over the ocean from Nimbus 5 NEMS data, and they gave roughly a global distribution of precipitable water amount. WILHEIT *et al.* (1977) developed an estimation technique of the rainfall intensity by studying microwave radiative transfer in the rainy atmosphere over the ocean. It is stated by them that the rainfall intensity can be estimated from Nimbus 5 ESMR data with their retrieval technique within an accuracy of a factor of two over the range of 1 to 25 mm/h. RODGERS and SIDDALINGAIAH (1983) made some efforts of establishing an algorithm to differentiate the rains over the land by using the polarization data of Nimbus 7 SMMR. In China LÜ and LIN (1983) and LIN *et al.* (1984)

investigated the combined use of radars and ground-based microwave radiometers to improve the accuracy in measuring the rainfall intensity and the cloud liquid water amount. Recently the study of monthly averaged distributions of precipitable water amount by using Nimbus 7 SMMR data was reported by CHANG *et al.* (1984). In their study, a multiple regression technique was used similarly to that used by STAELIN *et al.* (1976), and a very good agreement with values estimated by other methods was indicated.

One of the problems to be first considered in estimating the liquid water amount is the sensitivity of microwave radiation to the change in liquid water amount. The radiation of shorter wavelength microwave will be sensitive in the range of small liquid water amount. But it becomes insensitive easily with increase in liquid water amount. Meanwhile, the radiation of longer wavelength has the advantage of high saturation level, but its sensitivity to liquid water changes is very low. A second problem is an unambiguous relationship between the microwave radiative intensity and the cloud liquid water amount. TAKEDA and NATSUKI (1982) examined the estimation of the liquid water amount by using Nimbus 5 ESMR data (19.35 GHz) and pointed out that the difference in waterdrop size distribution will considerably disturb the relationship between the brightness temperature and the integrated liquid water amount.

In the present study a thermal radiative transfer equation in the scattering atmosphere is adopted to investigate the radiative properties of clouds which have several different size distributions of waterdrops. We will show that only large drops contribute to the influence of the difference in drop size distribution and that the combined use of microwaves of 0.81 and 2.0 cm wavelengths can reduce the ambiguity caused by the difference in drop size distributions. We have not taken ice particles into account in this study. Microwave radiation caused by ice particles would be negligible for cloud not including such large ice particles as hailstones. The radiation resulting from a melting layer is also a problem to be solved, though it is not involved in this paper.

2. Microwave Radiative Transfer in the Cloudy Atmosphere

A preliminary calculation of microwave radiative transfer in the cloudy atmosphere has been carried out by TAKEDA and NATSUKI (1982). It is shown in their research that the effect of scattering by large drops cannot be neglected. In the present study, a thermal radiative transfer equation in which the effect of scattering is taken into account is adopted to discuss the microwave radiative properties of a cloud.

The thermal radiative transfer equation for a plane-parallel layer in local thermodynamic equilibrium can be written as follows, on the assumption that the scattering and emitting layer is isothermal (LIOU, 1973);

$$\mu\frac{\mathrm{d}I(\tau,\mu)}{\mathrm{d}\tau}=I(\tau,\mu)-\frac{1}{2}\int_{-1}^{+1}p(\mu,\mu')I(\tau,\mu')\mathrm{d}\mu'-(1-\omega_0)B(T)\,. \tag{1}$$

Here $I(\tau,\mu)$ represents radiative intensity in direction μ and at optical thickness τ

$$\tau=\int_{z_0}^{z}R_{\mathrm{ext}}\,\mathrm{d}z\,, \tag{2}$$

where R_{ext} is the extinction coefficient. $p(\mu,\mu')$ is the axially-symmetrical phase function. μ and μ' are the cosine of zenith angles of emergent and incident radiations. $B(T)$ is the Planck function for temperature T. ω_0 is the well-known single-scattering albedo which is a measure of the effect of scattering compared with extinction and it is given by

$$\omega_0 = \frac{R_{scat}}{R_{ext}} , \tag{3}$$

where R_{scat} is the scattering coefficient.

The discrete-ordinate method for radiative transfer was discussed theoretically and applied to a cloud by LIOU (1973). An analytic solution can be attained by four-stream solution of the method with a satisfactory accuracy (LIOU, 1974). It can be written as

$$I(\tau,\mu_i) = \sum_{j=1}^{2} [L_j W_j(\mu_i)\exp(-k_j\tau) + L_{-j} W_j(-\mu_i)\exp(k_j\tau)] + B(T), \quad (i=-2,\ -1,\ 1,\ 2)\,. \tag{4}$$

W_j and k_j are constants related to the cloud layer, and L_j and L_{-j} can be determined from boundary conditions.

For microwave band the RAYLEIGH-JEANS approximation holds and the radiative intensity can be expressed in terms of brightness temperature. Therefore, the solution is rewritten as

$$T_B(\tau,\mu_i) = \sum_{j=1}^{2} [L'_j W_j(\mu_i)\exp(-k_j\tau) + L'_{-j} W_j(-\mu_i)\exp(k_j\tau)] + T, \quad (i=-2,\ -1,\ 1,\ 2)\,. \tag{5}$$

Here an effective emissivity of the cloud layer is defined as

$$\varepsilon = \frac{T_B(\tau,\mu_i) - T_{B0}(\tau_0,\mu_i)}{T - T_{B0}(\tau_0,\mu_i)} , \tag{6}$$

where $T_{B0}(\tau_0,\mu_i)$ is the brightness temperature resulting from the rear-side boundary of the cloud layer (τ_0). The effective emissivity defined here is a measure of radiative ability of the whole cloud layer. It implies that the radiation received at the cloud top in a direction μ_i can be given by two portions—one is the emission resulting from the cloud layer with an average temperature T and an effective emissivity ε, and another is the contribution of boundary surface (optical thickness τ_0) in the same μ_i direction. Microwave radiation from the boundary is absorbed in the non-reflected cloud layer and added to the portion of cloud emission at the cloud top similarly to that given by Cox (1976).

Here, the value of 2 is taken as i, in accordance with the calculation of microwave intensity at about 30° in zenith angle.

Besides water particles, the absorption (and emission) of atmospheric gases also exists in the microwave region. The intensities of absorption lines or bands are generally large only near the centers and are sensitive to the concentrations of relevant atmospheric constituents. In the microwave band often used in cloud remote sensing, the following three resonance absorptions should be taken into account—the wing of 118.75 GHz oxygen line, 60 GHz oxygen band and 22.235 GHz water vapor line. Several detailed studies for the above spectra have been made. MEEKS and LILLEY (1963) provided a parameterized expression of microwave absorption coefficient of oxygen for atmospheric temperature and pressure on the basis of laboratory experiments and theoretical analysis. Water vapor resonance line at 22.235 GHz was discussed by BARRETT and CHUNG (1962) and its absorption coefficient was expressed as a function of concentration of water vapor.

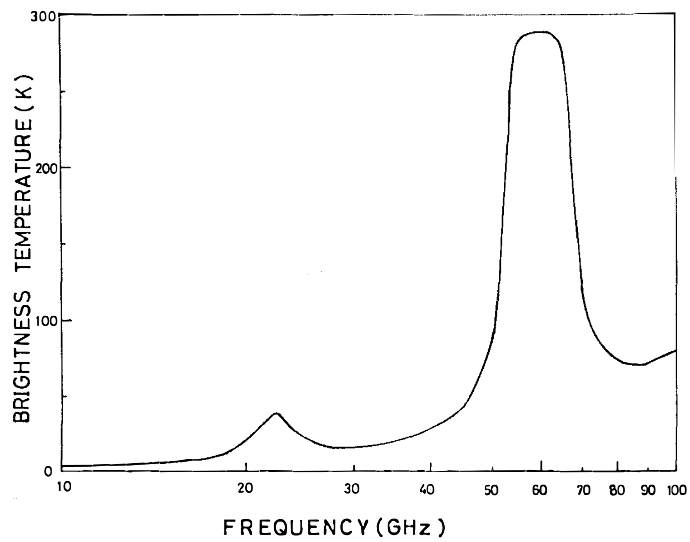

Fig. 1. Brightness temperature of U.S. Standard Atmosphere at microwave band. Radiation resulting from rear-side boundary is not taken into account.

Figure 1 shows the brightness temperature corresponding to the whole layer of U. S. Standard Atmosphere (ULABY *et al.*, 1981) at different frequencies of microwave band. The contribution of about 20 K in the window near 30 GHz is nearly equivalent to the contribution of a cloud layer with integrated liquid water of about 0.02 g/cm^2 at the frequency, and the contribution of 10 K in the region lower than 20 GHz is nearly the same as that of a cloud layers of 0.04 g/cm^2. In resonance regions the effects of atmospheric gases are larger and sensitive to the change of the concentrations of relevant atmospheric constituents.

The contribution of atmospheric absorption is often estimated by using a channel near the resonance regions or is obtained directly from radiosonde data in practical experiments.

In this paper, the effects of size of liquid waterdrops are mainly discussed. All of wavelengths adopted in this study are in window regions and the absorption of atmospheric gases is not taken into consideration. The absorption of these atmospheric gases should be considered in using the results of our calculation in an algorithm for estimating the liquid water amount in practical experiments.

3. Microwave Radiative Properties of a Model Cloud

When the absorption of atmospheric gases is neglected, the microwave radiation from a cloud layer will be the only contribution of waterdrops which have various sizes. The scattering and absorption of a spherical particle are governed by three parameters of the electromagnetic wavelength, the complex index of refraction and the size of the drop. If the water substance is considered at constant temperature and wavelength, the governing parameter is drop size only. Emission, absorption and scattering of waterdrops with various sizes make radiative transfer processes complicated. It would be helpful to investigate first the feature of microwave radiation from a model cloud which is composed of drops of uniform size.

The model cloud is 4 km in thickness and 273.15 K in temperature. We assume that the liquid water content of it is always 1.0 g/m^3 regard less of the drop size, and the integrated liquid water is 0.4 g/cm^2.

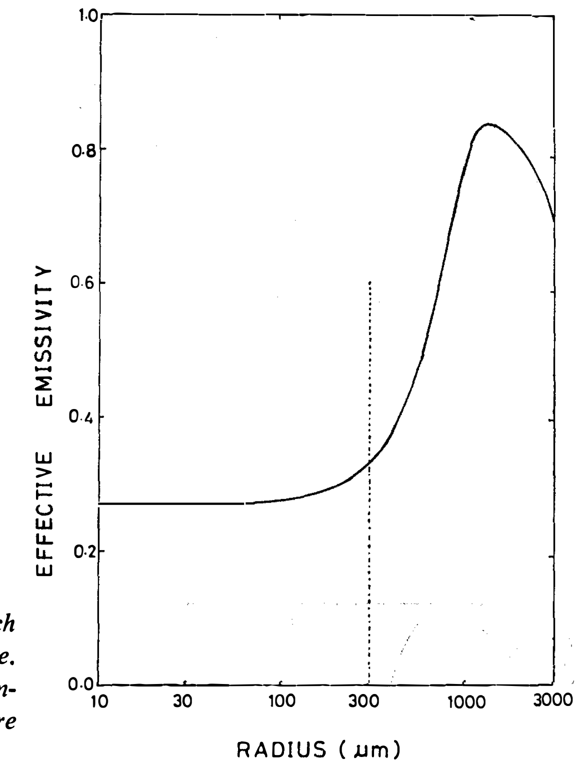

Fig. 2. Effective emissivity of a model cloud which is composed of drops of uniform size. Wavelength is 1.55 cm (19.35 GHz). Temperature and integrated liquid water are 273.15 K and 0.4 g/cm², respectively.

The effective emissivity of the model cloud at wavelength 1.55 cm (19.35 GHz) is shown in Fig. 2. Because the integrated liquid water here is the same, a horizontal straight line would be achieved for the relationship between the effective emissivity and the drop size if the radiation from the cloud is independent of drop size. The line is nearly straight only for drop size smaller than about 300 μm. The radiative intensities of clouds which are composed only of drops smaller than this critical radius are independent of the drop size. But if there are some larger drops in a cloud, the radiative intensity of the cloud would depend on the size distribution of drops. The effective emissivity of the model cloud for several other wavelengths was also investigated in the same method. As expected, the longer the wavelength, the larger is the critical size of drops. The critical radius for wavelength of 0.81 cm is about 100 μm and it is about 500 μm for wavelength of 3.2 cm.

In the following discussion, we will define two kinds of clouds for convenience. One is a "non-precipitating cloud" which is composed only of drops smaller than 300 μm, and another is a "precipitating cloud" which contains drops larger than 300 μm.

Many formulae have been proposed to express the size distributions of cloud drops and raindrops. Adopted here is the formula presented by DEIRMENDJIAN (1964)

$$n(r)=ar^{\alpha}\exp(-br^{\gamma}), \tag{7}$$

where $n(r)$ is number concentration in radius interval of r to $r+\mathrm{d}r$, and a, b, α and γ are constants. We will assume $\gamma=1$ and $\alpha=2$ for drops smaller than 300 μm as given by PRUPPACHER and KLETT (1978) and $\gamma=1$ and $\alpha=0$ for drops larger than 300 μm similarly to the well-known distribution in MARSHALL and PALMER (1948). Then the size distribution is given by

$$n(r)=a_1r^2\exp(-b_1r) \qquad (r\leqq 300\ \mu\text{m})\,, \tag{8}$$

$$n(r)=a_2\exp(-b_2r) \qquad (300<r\leqq 3000\ \mu\text{m})\,. \tag{9}$$

If the liquid water content is the same, larger values of b_1 and b_2 mean a smaller ratio of large particles to small ones. We will take values of b_1 to be 1, 0.3, 0.15, 0.06 or 0.03 μm^{-1} (mode radius is 3, 10, 20, 50 or 100 μm), and the value of b_2 to be 0.004, 0.005, 0.006, 0.008 or 0.01 μm^{-1}. The values of a_1 and a_2 determine the total number concentration or liquid water content when b_1 and b_2 are fixed. The size distributions of drops in non-precipitating cloud are expressed by eq. (8). For a precipitating cloud we assume that the size distribution of drops is expressed by eq. (8) with b_1 of 0.3 if the integrated liquid water is less than 0.15 g/cm². When the integrated liquid water exceeds 0.15 g/cm², the large drops whose size distribution is expressed by eq.

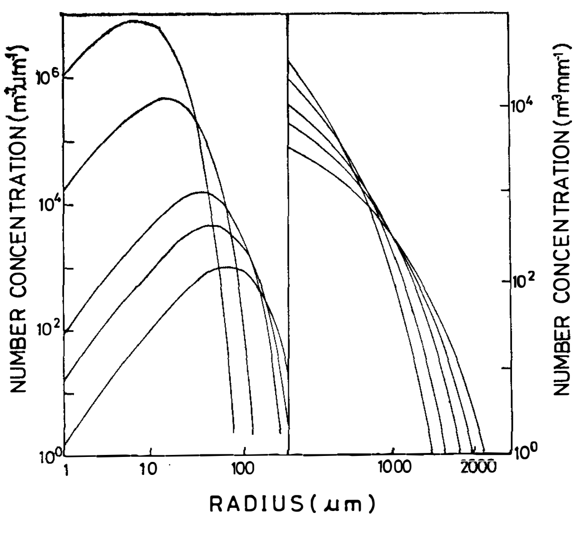

Fig. 3. Size distributions of drops smaller than 300 μm (left side) and larger than 300 μm (right side) in the case of liquid water content 1.0 g/m³. The scales of left and right ordinates are different.

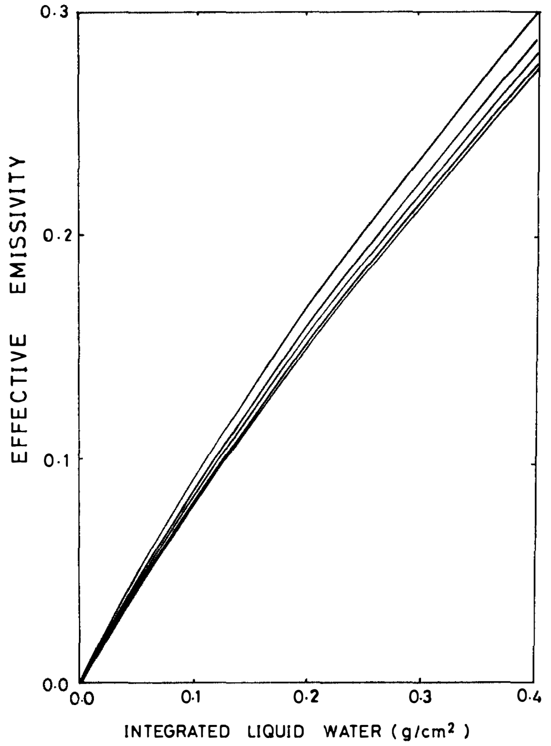

Fig. 4a. Effective emissivity of non-precipitating cloud for wavelength of 1.55 cm. Five kinds of size distribution are adopted.

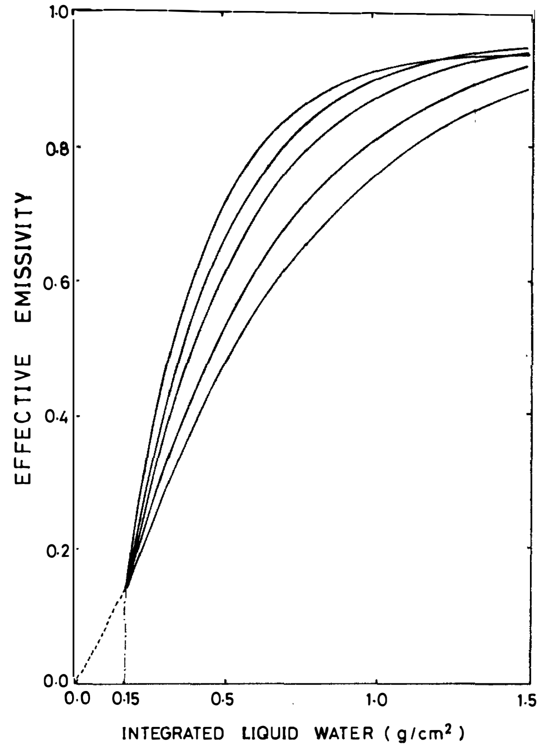

Fig. 4b. Effective emissivity of precipitating cloud for wavelength of 1.55 cm. Five kinds of size distribution are adopted.

(9) are added to drops of 0.15 g/cm^2. The choice of amount 0.15 g/cm^2 is rather arbitrary. In some natural clouds drops larger than 300 μm would also exist even when the integrated liquid water is less than this value. The size distributions of drops for these two kinds of clouds are illustrated in Fig. 3.

Figures 4a and 4b show the effective emissivities of non-precipitating and precipitating clouds for wavelength 1.55 cm (19.35 GHz). The integrated liquid water of a non-precipitating cloud is taken to be less than 0.4 g/cm^2 because larger values are not reasonable for non-precipitating clouds in the real atmosphere. In both non-precipitating cloud and precipitating clouds, five kinds of size distributions were adopted as mentioned above. As long as the radii of cloud drops are smaller than 300 μm (non-precipitating cloud), the microwave radiation is almost independent of the size distribution and the effective emissivity can be determined unambiguously only by the integrated liquid water. When larger drops are included in the cloud (precipitating cloud), however, the shape of size distribution has a considerable influence on the relationship between the effective emissivity and the integrated liquid water. This influence would cause us some difficulty when we attempt to estimate the liquid water amount inversely from the microwave radiative intensity.

The effective emissivity of the non-precipitating cloud for microwave of wavelength 0.81 cm is shown in Fig. 5a. It can be noticed that the radiation of wavelength 0.81 cm is much more sensitive to the change in the integrated liquid water than

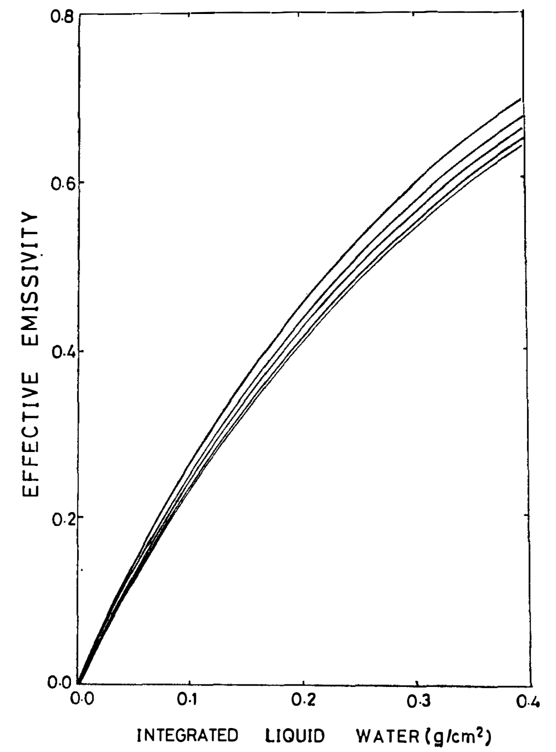

Fig. 5a. *Same as Fig. 4a except for wavelength of 0.81 cm.*

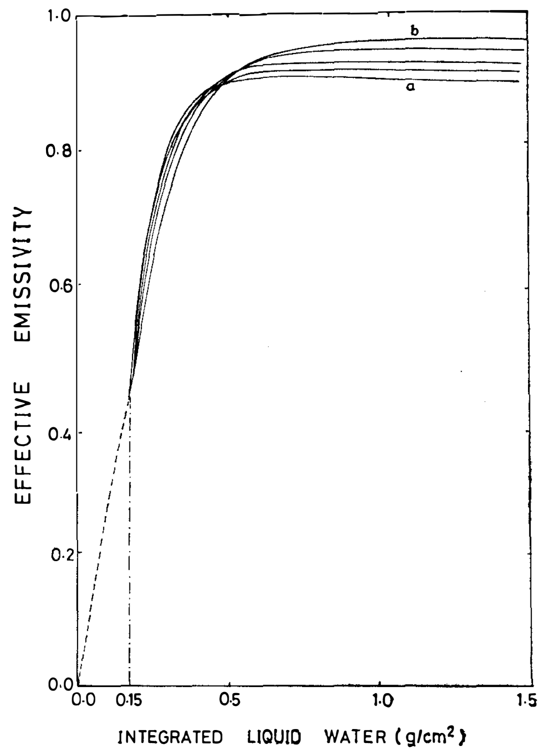

Fig. 5b. *Same as Fig. 4b except for wavelength of 0.81 cm.*

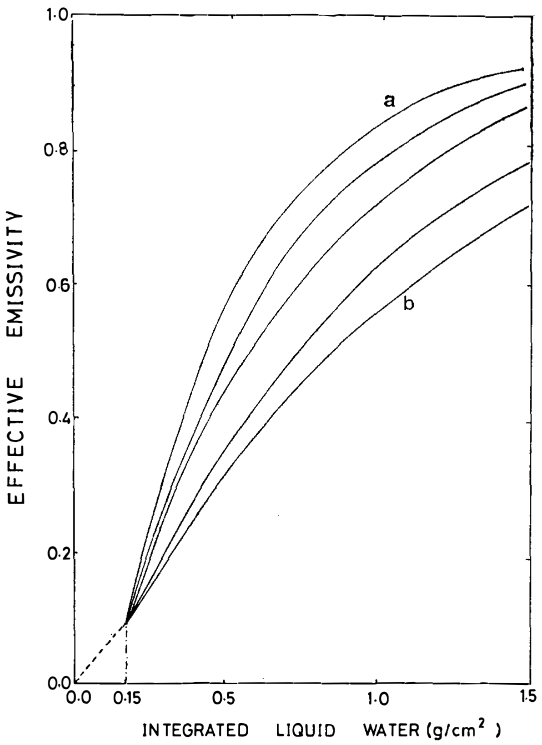

Fig. 6. *Same as Fig. 4b except for wavelength of 2.0 cm.*

wavelength 1.55 cm. It would be more suitable to choose the wavelength of 0.81 cm in the observation of non-precipitating clouds.

The effective emissivity of the precipitating cloud for wavelength of 0.81 and 2.0 cm is shown in Fig. 5b and Fig. 6. Line *a* corresponds to the size distribution containing many large drops (rainfall intensity 30 mm/h) and the line *b* expresses the case of few large drops (rainfall intensity 0.5 mm/h). Until the effective emissivity reaches a saturation value of about 0.9, the microwave of 0.81 cm gives a satisfactory relationship between the integrated liquid water and the effective emissivity for various size distributions. When the cloud is optically thicker, however, the effective emissivity at wavelength 0.81 cm hardly increases with integrated liquid water and it is in a saturation state. Meanwhile, the relationship between the integrated liquid water and the effective emissivity at wavelength 2.0 cm is much more dependent of the size distribution of drops. When the cloud is so thick that radiation at 0.81 cm is in a saturation state, the effect of drop size distribution on the relationship is very different between these two wavelengths. For a given integrated liquid water, the increase in the ratio of large drop concentration to small drop concentration enhances the radiative capability at wavelength of 2.0 cm (brightening), but it makes the radiation at wavelength of 0.81 cm darker.

The different effect of size distribution on the effective emissivity at two different wavelengths suggests a possibility that the effect of size distribution can be weakened by combining these two wavelengths. Figure 7 shows the combined effective emissivity of 0.81 and 2.0 cm defined by

$$\tilde{\varepsilon}=0.6\times\varepsilon_1+0.4\times\varepsilon_2\,, \tag{10}$$

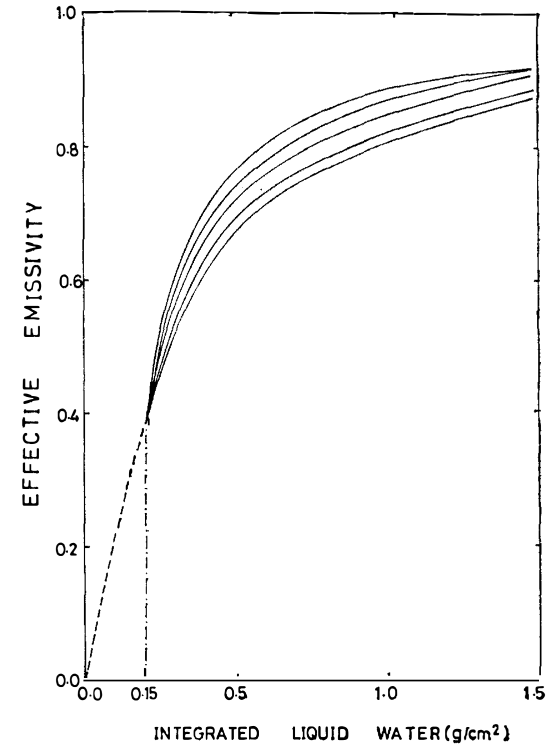

Fig. 7. Effective emissivity of precipitating cloud in the combination of microwaves of wavelengths 0.81 and 2.0 cm. Five kinds of size distribution are adopted.

where ε_1 is effective emissivity at 0.81 cm and ε_2 is that at 2.0 cm. The coefficients of 0.6 and 0.4 were determined as the best values by testing several combinations.

It can be seen by comparing the combined effective emissivity with that at single wavelength that there are at least two merits in the combined emissivity. Firstly the effect of size distribution is reduced because of the different effect of size distribution at two wavelengths. Secondly the combined effective emissivity does not show a saturation state for integrated liquid water of 1.5 g/cm^2 yet. It can be understood from Fig. 5b and Fig. 6 that the radiation at wavelength 0.81 cm is weakly dependent on size distribution and the radiation at wavelength 2.0 cm remains widely unsaturated. The combination of these two wavelength takes advantages of features of radiation at both wavelengths, and reduces error in the estimation of integrated liquid water for large microwave optical-depth. The combination of radiations at 0.81 and 1.55 cm wavelengths was also investigated, but it did not work so good as that of radiations at 0.81 and 2.0 cm.

4. Summary

In order to examine the estimation of liquid water amount in the atmosphere by microwave radiometers, the radiative transfer equation was solved using a 4-stream method. The result of calculation shows that microwave radiation is largely governed by the size distribution of large drops.

As a result of calculation, the following conclusions were attained. In the cloud (non-precipitating cloud) which is composed only of waterdrops smaller than a critical size, the microwave radiation is not dependent on drop size distribution. However, in the cloud (precipitating cloud) in which larger drops are included, it is a function of drop size distribution. Even for the precipitating cloud a satisfactory relationship exists between the effective emissivity and the integrated liquid water at wavelength 0.81 cm if the cloud is optically thin. When the cloud contains large drops and it is optically thick, the effect of drop size distribution on microwave radiation can be reduced by a combination of wavelength 0.81 and 2.0 cm. The sensitivity of microwave radiation to the change in integrated liquid water can remain unsaturated over the wide range of integrated liquid water as a result of this combination.

The measurement of integrated liquid water, especially integrated supercooled cloud water, in the polar region is very important for studying the formation mechanism of clouds and precipitation and the radiation properties of clouds in that region. The result of the calculation in the present study suggests that the measurement of radiation at wavelength 0.81 cm is suitable for the estimation of integrated liquid water in non-precipitating clouds, and the reasonable estimation for precipitating clouds can be attained by combining the radiation at two wavelengths. It is to be studied in more detail in future what combination of radiation at two wavelengths is the most useful.

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(Received April 1, 1985; Revised manuscript received September 13, 1985)