THE GROUND MAGNETIC EFFECT OF A THREE-DIMENSIONAL CURRENT SYSTEM IN THE IONOSPHERE-MAGNETOSPHERE

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Abstract: The theorem that no ground magnetic effect is produced by the combination of the vertical current and the associated ionospheric irrotational current holds in the ULF range which is prescribed by wave-period of $T \gtrsim 1$ s and horizontal wave-length of $\lambda \lesssim 10000$ km when the ionosphere is uniform. The magnetic field observed on the ground is that with no vertical current and it consists of the part produced by the magnetospheric current closing two-dimensionally in the horizontal plane and that produced by the ionospheric source-free current except for that due to the induction current within the earth. It has been found that for longer wave-periods ($T \gtrsim 100$ s) the contribution of the magnetic field produced by the magnetospheric current to the ground magnetic field is not so much important in the auroral zone except for the case of the wave-length in the east-west direction less than that in the north-south direction.

1. Introduction

The magnetic field of hydromagnetic waves incident from the magnetosphere experiences a modification due to the ionosphere during its passage to the ground. When the toroidal (slow, Alfvén)-mode field is incident from the magnetosphere, the Pedersen current in the ionosphere almost perfectly screens the incident magnetic field from the ground and the associated Hall current re-emits the poloidal-mode field above and below the ionosphere. The magnetic perturbation vector is then rotated through 90° in a left-handed direction with respect to the earth's magnetic field during its passage through the ionosphere. When the wave incident from the magnetosphere consists of the poloidal (fast, compressional) mode, the modification by the ionosphere is insignificant since the magnetic field due to the ionospheric current is smaller in magnitude than that observed on the ground. In the past many workers considered this effect from a viewpoint of the wave incidence theoretically (DUNGEY, 1963; NISHIDA, 1964; TAMAO, 1964) or numerically (INOUE, 1973; HUGHES, 1974; HUGHES and SOUTHWOOD, 1976).

FUKUSHIMA (1969, 1976), on the other hand, investigated the modification of the magnetic field by the ionosphere from a standpoint of the ground magnetic effect of a three-dimensional current system in the ionosphere-magnetosphere and presented the theorem that no ground magnetic effect was produced by the combination of the vertical current into or out of the ionosphere and the associated Pedersen current in the ionosphere. His theorem, however, dealt with only a static field. Since it is important to reconsider the modification of the incident wave field (non-static field) due to the ionosphere from a viewpoint of the magnetic effect of the three-dimensional current system, we intend to extend the theorem to the non-static field in the present paper. Further, we will examine basic relationships between the magnetic field and the ionospheric and magnetospheric currents.

2. Model and Formal Separation of Magnetic Fields and Currents

For simplification, the space is divided into four horizontally stratified regions: magnetosphere, ionosphere, atmosphere, and earth (Fig. 1). The magnetosphere ($z>d$) is regarded as a uniform hydromagnetic region, the ionosphere ($z=d$) as a uniform anisotropic conducting sheet, the atmosphere ($0<z<d$) as a vacuum region, and the earth ($z<d$) as a uniform conductor. In our model $d=100$ km. The ambient magnetic field, $\boldsymbol{B}_0$, is assumed to be uniform and laid in the x–z plane. The positive ψ (dip angle) corresponds to the northern hemisphere and the negative ψ to the southern hemisphere. In our model system the incident magnetic fields from the magnetosphere are considered to be input signals and the magnetic field on the ground, the ionospheric current, and so on to be output signals.

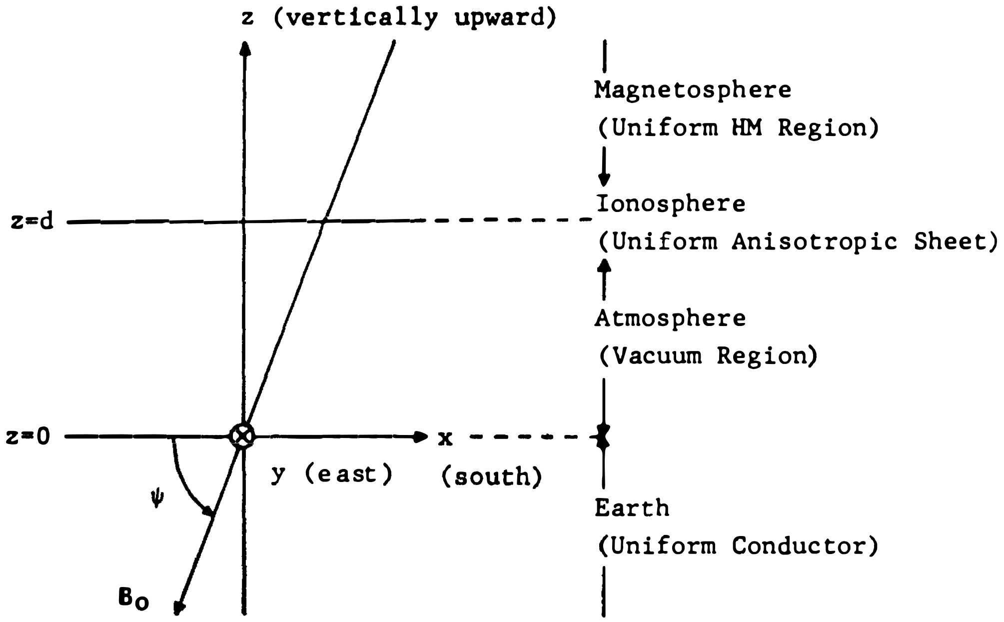

Fig. 1. Model of space.

In studying the magnetic effect of the three-dimensional current system in the ionosphere-magnetosphere, we first of all begin with the Fourier transform of the magnetic field $\boldsymbol{B}(x, y, z, t)$ with respect to x and y.

$$\boldsymbol{B}(x, y, z, t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \boldsymbol{B}(k_x, k_y, z, t) \exp[-i(k_x x+k_y y)]\mathrm{d}k_x \mathrm{d}k_y \, . \tag{1}$$

The horizontal component $\boldsymbol{B}_h(k_x, k_y, z, t)$ of the Fourier transform can be divided into

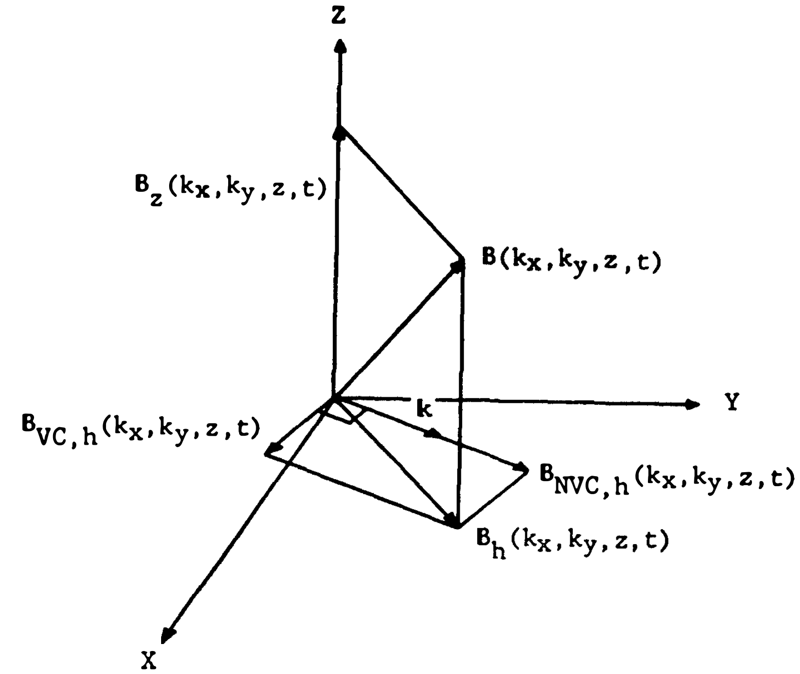

Fig. 2. Separation of $\boldsymbol{B}$ into $\boldsymbol{B}_{VC}$ and $\boldsymbol{B}_{NVC}$.

two parts: $\boldsymbol{B}_{VC,h}$ perpendicular and $\boldsymbol{B}_{NVC,h}$ parallel to the horizontal wave vector $\boldsymbol{k}$ (Fig. 2). From these horizontal Fourier components, $\boldsymbol{B}_{VC,h}$ and $\boldsymbol{B}_{NVC,h}$, and the vertical Fourier component, $\boldsymbol{B}_z$, two parts of the magnetic field $\boldsymbol{B}(x, y, z, t)$ are constructed by the inverse Fourier transform as follows:

$$\boldsymbol{B}_{VC}(x, y, z, t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\boldsymbol{B}_{VC,h}(k_x, k_y, z, t)\exp[-i(k_x x+k_y y)]\mathrm{d}k_x\mathrm{d}k_y\ , \tag{2}$$

$$\begin{aligned}\boldsymbol{B}_{NVC}(x, y, z, t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}&[\boldsymbol{B}_{NVC,h}(k_x, k_y, z, t)\\ &+\boldsymbol{B}_z(k_x, k_y, z, t)]\exp[-i(k_x x+k_y y)]\,\mathrm{d}k_x\mathrm{d}k_y\ .\end{aligned} \tag{3}$$

Thus, the magnetic field $\boldsymbol{B}$ can be uniquely separated into two parts, $\boldsymbol{B}_{VC}$ and $\boldsymbol{B}_{NVC}$, which is well known as Helmholtz's theorem in vector analysis. As seen from the definition, $\boldsymbol{B}_{VC}$ and $\mathbf{B}_{NVC}$ satisfy the following equations:

$$(\mathrm{rot}\ \boldsymbol{B}_{VC})_z=\frac{4\pi}{c}j_z+\frac{1}{c}\frac{\partial}{\partial t}E_z\ , \tag{4}$$

$$(\mathrm{rot}\ \boldsymbol{B}_{NVC})_z=0\ . \tag{5}$$

Hence, we shall call $\boldsymbol{B}_{VC}$ the magnetic field with a vertical current and $\boldsymbol{B}_{NVC}$ the magnetic field with no vertical current.

In relation to the separation of the magnetic field, the ionospheric sheet current $\boldsymbol{J}$ can be also uniquely divided into two parts, $\boldsymbol{J}_C$ and $\boldsymbol{J}_{SF}$, which are defined by

$$\left\{\begin{aligned} \frac{4\pi}{c} J_{\mathrm{C},x} &= -B_{\mathrm{VC},y}(d+0)+B_{\mathrm{VC},y}(d-0) \\ \frac{4\pi}{c} J_{\mathrm{C},y} &= B_{\mathrm{VC},x}(d+0)-B_{\mathrm{VC},x}(d-0) , \end{aligned}\right. \tag{6}$$

and

$$\left\{\begin{aligned} \frac{4\pi}{c} J_{\mathrm{SF},x} &= -B_{\mathrm{NVC},y}(d+0)+B_{\mathrm{NVC},y}(d-0) \\ \frac{4\pi}{c} J_{\mathrm{SF},y} &= B_{\mathrm{NVC},x}(d+0)-B_{\mathrm{NVC},x}(d-0) . \end{aligned}\right. \tag{7}$$

From eqs. (4)–(7) and Maxwell's equations we have

$$\mathrm{div}\, \boldsymbol{J}_{\mathrm{C}} = -[j_z(d+0)-j_z(d-0)] - \frac{\partial}{\partial t}\rho , \tag{8}$$

$$\mathrm{div}\, \boldsymbol{J}_{\mathrm{SF}} = 0 , \tag{9}$$

and

$$(\mathrm{rot}\, \boldsymbol{J}_{\mathrm{C}})_z = 0 , \tag{10}$$

where j_z is the vertical current density and ρ is the surface charge density at the ionosphere. Since the atmosphere $(0<z<d)$ is assumed to be a vacuum region, $j_z(d-0)$ in the right-hand side of eq. (8) vanishes. Further, the term $\partial\rho/\partial t$ is neglected compared with $j_z(d+0)$ in the ULF range of wave-period $T \gtrsim 1$ s and horizontal wavelength $\lambda \lesssim 10000$ km which we shall consider in the following. Since $\boldsymbol{J}_{\mathrm{C}}$ and $\boldsymbol{J}_{\mathrm{SF}}$ satisfy eqs. (8) and (9) respectively, we shall call $\boldsymbol{J}_{\mathrm{C}}$ the closing current via a vertical current and $\boldsymbol{J}_{\mathrm{SF}}$ the source-free current after KAMIDE and MATSUSHITA (1979). $\boldsymbol{J}_{\mathrm{C}}$ is also called the irrotational current because eq. (10).

Assuming static fields, $\boldsymbol{B}_{\mathrm{VC}}$ disappears in the atmosphere because of rot $\boldsymbol{B}=0$. Hence, as seen from eq. (6), no ground magnetic effect is produced by the combination of the vertical current in the magnetosphere and the associated ionospheric current $\boldsymbol{J}_{\mathrm{C}}$ which leads to the Pedersen current when the ionosphere is uniform and the ambient magnetic field, $\boldsymbol{B}_0$, is vertical. This is the generalized expression of the theorem given by FUKUSHIMA (1969, 1976).

3. Extension of Fukushima's Theorem to the ULF Range

In this section we confirm that the theorem by FUKUSHIMA (1969, 1976) also holds in the ULF range which is prescribed by wave-period of $T \gtrsim 1$ s and horizontal wavelength of $\lambda \lesssim 10000$ km when the ionosphere is uniform. If a horizontal spatial and time variation of the form $\exp[-i(kx-\omega t)]$ is assumed, B_x and B_y are the magnetic fields with no vertical current and with a vertical current, respectively. Regarding the earth as a perfect conductor, we have

$$\frac{B_y(z)}{B_x(z)} = \left(\frac{\omega}{kc}\right)^2 \frac{E_x(d)}{E_y(d)} , \tag{11}$$

in the atmosphere. Note that the vertical current in the atmosphere is the displacement current, $(1/c)(\partial E_z/\partial t)$. For $T\,(=2\pi/\omega)\gtrsim 1$ s and $\lambda\ (=2\pi/k)\lesssim 10000$ km,

$$\left(\frac{\omega}{kc}\right)^2 \lesssim 10^{-3}\,.$$

Hence, if $E_x(d)$ is not much greater in magnitude than $E_y(d)$, $|B_y(z)/B_x(z)|\ll 1$, that is, $\boldsymbol{B}_{\mathrm{VC}}$ is neglected compared with $\boldsymbol{B}_{\mathrm{NVC}}$ in the atmosphere. When the ambient magnetic field, $\boldsymbol{B}_0$, is vertical, the ratio of $E_x(d)$ to $E_y(d)$ is given as follows:

$$\frac{E_x(d)}{E_y(d)}=\frac{\dfrac{4\pi}{c}\Sigma_{\mathrm{P}}+\dfrac{\gamma^{\mathrm{RF}}c}{\omega}-\dfrac{ikc}{\omega}\coth(kd)}{\dfrac{4\pi}{c}\Sigma_{\mathrm{H}}}\,, \tag{12}$$

for the slow (Alfvén) wave incidence and

$$\frac{E_x(d)}{E_y(d)}=-\frac{\dfrac{4\pi}{c}\Sigma_{\mathrm{H}}}{\dfrac{4\pi}{c}\Sigma_{\mathrm{P}}+\dfrac{c}{V_{\mathrm{A}}}}\,, \tag{13}$$

for the fast (compressional) wave incidence, where Σ_{P} and Σ_{H} are the height-integrated Pedersen and Hall conductivities, respectively, V_{A} is the Alfvén velocity in the magnetosphere, and γ^{RF} is given in Appendix. As for the derivation of eqs. (12) and (13), refer to eq. (A-6). It is understood from eqs. (12) and (13) that the ratio $E_x(d)/E_y(d)$ is

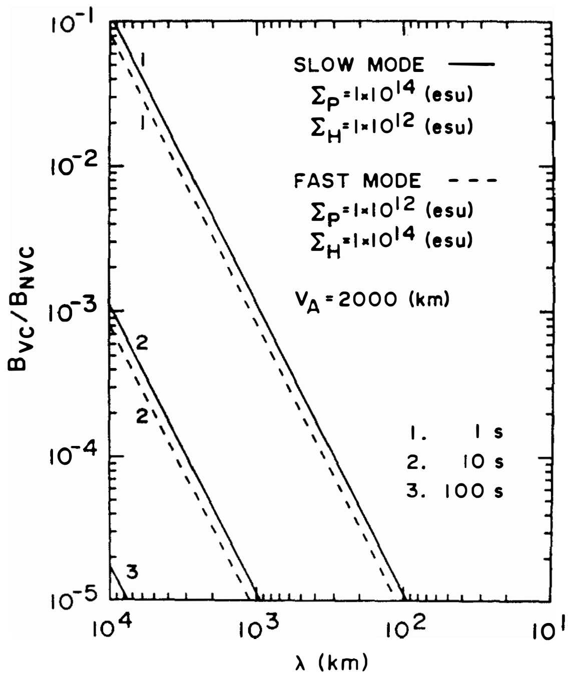

Fig. 3. Dependence of magnitude of the ratio $B_{\mathrm{VC}}/B_{\mathrm{NVC}}$ in the atmosphere upon horizontal wavelength λ in the case of $\Sigma_{\mathrm{P}}\neq\Sigma_{\mathrm{H}}$. It is assumed that the ambient magnetic field is vertical ($\psi=90°$) and the earth is a perfect conductor. The solid and broken lines represent the slow and fast wave incidences, respectively.

determined basically by the ratio Σ_H/Σ_P.

When $\Sigma_P=\Sigma_H=\Sigma$ and $\Sigma \gtrsim 10^{11}$ esu ($\sim$0.1 mho), $\boldsymbol{B}_{VC}$ is neglected compared with $\boldsymbol{B}_{NVC}$ in the atmosphere in the ULF range as expected from eqs. (12) and (13). Figure 3 shows the dependence of the ratio of $B_y(z)$ to $B_x(z)$ in the atmosphere upon the horizontal wave-length λ in the case of $\Sigma_P \neq \Sigma_H$. The solid and broken lines represent the slow and fast wave incidences, respectively. It is found in this figure that $\boldsymbol{B}_{VC}$ is also negligibly small compared with $\boldsymbol{B}_{NVC}$ in the atmosphere in the ULF range except for the extreme case of $T\sim 1$ s and $\lambda \sim 10000$ km even if the ratio of both conductivities reaches the order of 10^2. Further, it is seen that as anticipated from eq. (11), the ratio $B_y(z)/B_x(z)$ in the atmosphere becomes smaller as T increases or λ decreases. Although the assumption of the vertical ambient magnetic field and the perfectly conducting earth was made in the above example, it was confirmed through many calculations that the results obtained did not depend on such an assumption. Figure 4 is an example of such calculations, where a horizontal spatial and time variation of the form $\exp[-i(kx+ly-\omega t)]$ is assumed and the dependence of the ratio B_{VC}/B_{NVC} in the atmosphere upon λ_x $(=2\pi/k)$ is displayed for λ_y $(=2\pi/l)=10000$ km. In this case the ambient magnetic field, $\boldsymbol{B}_0$, is nearly horizontal ($\psi=10°$) and a typical value of the earth conductivity ($\sigma=1\times10^8$ esu $\sim$0.01 mho/m) is employed.

Thus, we may conclude that the theorem that no ground magnetic effect is produced by the combination of the vertical current and the associated ionospheric current $\boldsymbol{J}_C$ also holds in the ULF range under the condition of $\Sigma_P \gtrsim 10^{11}$ esu, $\Sigma_H \gtrsim 10^{11}$ esu,

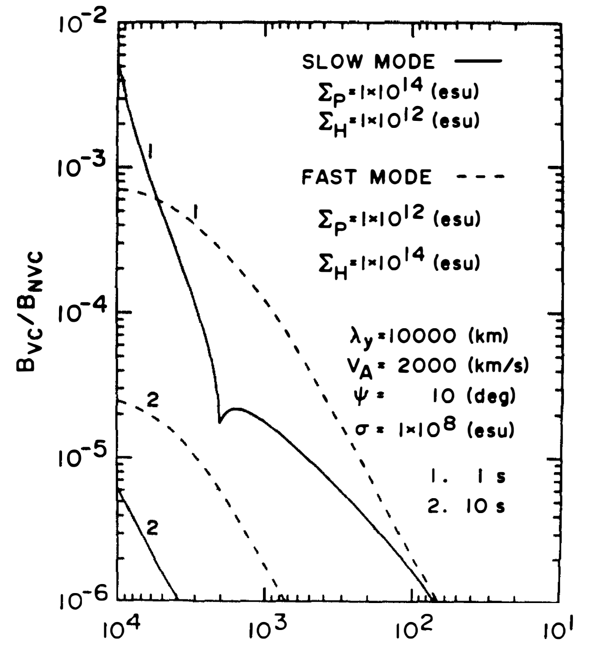

Fig. 4. Dependence of magnitude of the ratio B_{VC}/B_{NVC} in the atmosphere upon λ_x for $\lambda_y=10000$ km. In this case the ambient magnetic field is nearly horizontal ($\psi=10°$) and a typical value of the earth conductivity ($\sigma=10^8$ esu$\sim$0.01 mho/m) is employed. The solid and broken lines represent the slow and fast wave incidences, respectively. The singular variation seen around $\lambda_x=2000$ km for $T=1$ s may be due to the transition of the reflected fast mode from propagation to evanescence.

and $10^{-2} \lesssim \Sigma_H/\Sigma_P \lesssim 10^2$ when the ionosphere is uniform. It should be noted that the validity of the theorem does not depend on whether the ambient magnetic field, $\boldsymbol{B}_0$, is vertical or not.

When the ambient magnetic field, $\boldsymbol{B}_0$, is vertical, the magnetospheric current associated with the slow wave has a vertical current but that associated with the fast wave has no vertical current (Fig. 5). Figure 5 illustrates wave fields for the slow and fast modes under the condition that the ambient magnetic field is laid along the

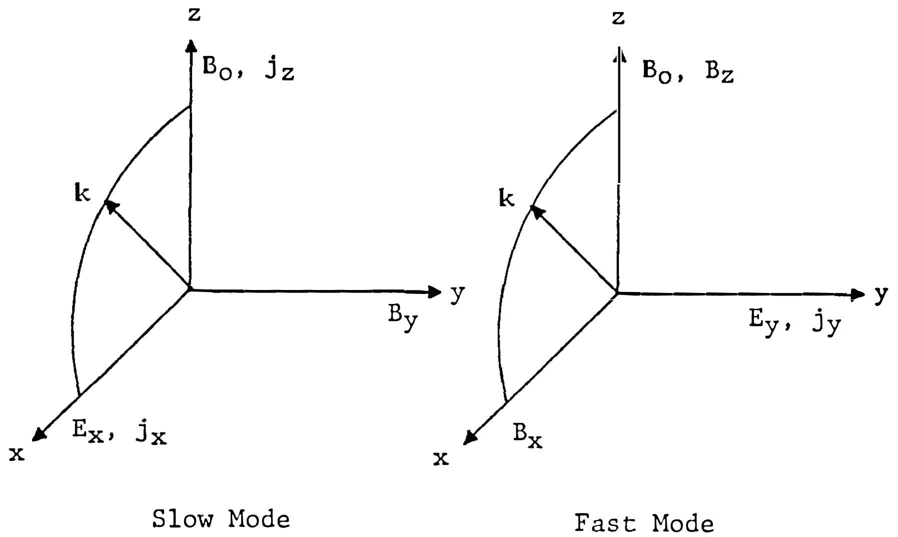

Fig. 5. Wave fields for the slow and fast modes under the condition that the ambient magnetic field is laid along the z axis and the wave vector $\boldsymbol{k}$ *in the x–z plane.*

z axis and the wave vector $\boldsymbol{k}$ in the x–z plane. Therefore, in case of the vertical ambient magnetic field no ground magnetic effect is yielded by the combination of the magnetospheric current associated with the slow wave and the ionospheric current $\boldsymbol{J}_C$. On the other hand, the magnetospheric current associated with the fast wave gives rise to the ground magnetic field in co-operation with the ionospheric current $\boldsymbol{J}_{SF}$. In this paragraph the case of the vertical ambient magnetic field has been considered. A detailed discussion on the oblique ambient magnetic field will be made in the next section.

4. Application of Theorem to the Oblique Magnetic Field Line Problem

The theorem states that no ground magnetic effect is produced by the combination of the vertical current and the associated ionospheric current $\boldsymbol{J}_C$, that is, $\boldsymbol{B}_{VC}$ is neglected compared with $\boldsymbol{B}_{NVC}$ in the atmosphere in the ULF range. If we neglect the presence of the earth, accordingly, the magnetic field due to the induction current within the earth, the magnetic field below the ionosphere, $\boldsymbol{B}^A$, will be composed of the part due to the magnetospheric current closing two-dimensionally in the horizontal plane, $\boldsymbol{B}^M_{NVC}$, and that due to the ionospheric source-free current, $\boldsymbol{B}^I_{NVC}$.

$$\boldsymbol{B}^A = \boldsymbol{B}^A_{NVC} = \boldsymbol{B}^M_{NVC} + \boldsymbol{B}^I_{NVC} \ . \tag{14}$$

When a horizontal spatial and time variation of the form $\exp[-i(kx+ly-\omega t)]$ is assumed, the equation,

$$(k-\gamma^{\mathrm{IS}}\cot\psi)B_x^{\mathrm{IS}}+l\,B_y^{\mathrm{IS}}=0\ , \tag{15}$$

is obtained for the incident slow (Alfvén) wave, where

$$\gamma^{\mathrm{IS}}=-k\cot\psi\mp\frac{\omega}{V_{\mathrm{A}}\sin\psi}\quad(\psi\gtrless 0^\circ)\ .$$

The equation similar to eq. (15) is also taken for the reflected slow wave. Since the horizontal components of $\boldsymbol{B}_{\mathrm{NVC}}$ are given by

$$\begin{cases} B_{\mathrm{NVC},x}=\dfrac{k}{k^2+l^2}(kB_x+lB_y) \\ B_{\mathrm{NVC},y}=\dfrac{l}{k^2+l^2}(kB_x+lB_y)\ , \end{cases} \tag{16}$$

it is seen from eq. (15) that in the case of $\psi\neq\pm 90^\circ$ and $l\neq 0$ the magnetic field of the slow wave has the part with no vertical current, that is, the magnetospheric current associated with the slow wave has the part closing two-dimensionally in the horizontal plane. Then, if the ambient magnetic field, $\boldsymbol{B}_0$, be oblique ($\psi\neq\pm 90^\circ$) and $l\neq 0$, a part of the magnetospheric current associated with the slow wave can contribute to the ground magnetic field.

In the oblique magnetic field line problem with the slow wave incidence, which is important in studying the events connected with the field-aligned current, we investigate the ratio which $\boldsymbol{B}_{\mathrm{NVC}}^{\mathrm{M}}$ bears to $\boldsymbol{B}_{\mathrm{NVC}}^{\mathrm{I}}$. For that purpose we calculate

$$\frac{B^{\mathrm{M}}}{B^{\mathrm{I}}}\equiv\frac{B_{\mathrm{NVC},x}^{\mathrm{M}}(d-0)}{B_{\mathrm{NVC},x}^{\mathrm{I}}(d-0)}=\frac{B_{\mathrm{NVC},y}^{\mathrm{M}}(d-0)}{B_{\mathrm{NVC},y}^{\mathrm{I}}(d-0)}\ . \tag{17}$$

The second equality in eq. (17) is valid since $B_{\mathrm{NVC},y}^{\mathrm{M}}/B_{\mathrm{NVC},x}^{\mathrm{M}}=B_{\mathrm{NVC},y}^{\mathrm{I}}/B_{\mathrm{NVC},x}^{\mathrm{I}}=l/k$. In eq. (17) $\boldsymbol{B}_{\mathrm{NVC}}^{\mathrm{I}}(d-0)$ is given by

$$\begin{cases} 2B_{\mathrm{NVC},x}^{\mathrm{I}}(d-0)=-\dfrac{4\pi}{c}J_{\mathrm{SF},y} \\ 2B_{\mathrm{NVC},y}^{\mathrm{I}}(d-0)=\dfrac{4\pi}{c}J_{\mathrm{SF},x}\ , \end{cases} \tag{18}$$

(refer to eq. (7)) and $\boldsymbol{B}_{\mathrm{NVC}}^{\mathrm{M}}(d-0)$ is calculated by

$$\boldsymbol{B}_{\mathrm{NVC}}^{\mathrm{M}}(d-0)=\boldsymbol{B}(d-0)-\boldsymbol{B}_{\mathrm{NVC}}^{\mathrm{I}}(d-0)\ . \tag{19}$$

Figures 6a and 6b show the dependence of the ratio $B^{\mathrm{M}}/B^{\mathrm{I}}$ in eq. (17) upon λ_x $(=2\pi/k)$ for λ_y $(=2\pi/l)=1000$ km and $\lambda_y=100$ km, respectively. The dip angle of 77° employed in these figures corresponds to the geomagnetic latitude of 65°, which lies in the auroral zone. It is found in these figures with $\Sigma_{\mathrm{H}}/\Sigma_{\mathrm{P}}=1$ that the magnitude of $\boldsymbol{B}_{\mathrm{NVC}}^{\mathrm{M}}$ is at most about 20% of that of $\boldsymbol{B}_{\mathrm{NVC}}^{\mathrm{I}}$ for longer wave-periods ($T=2\pi/\omega\gtrsim 100$ s). Plotted in Fig. 7 is the result taken with $\Sigma_{\mathrm{H}}/\Sigma_{\mathrm{P}}=10$, which sometimes appears in the midnight

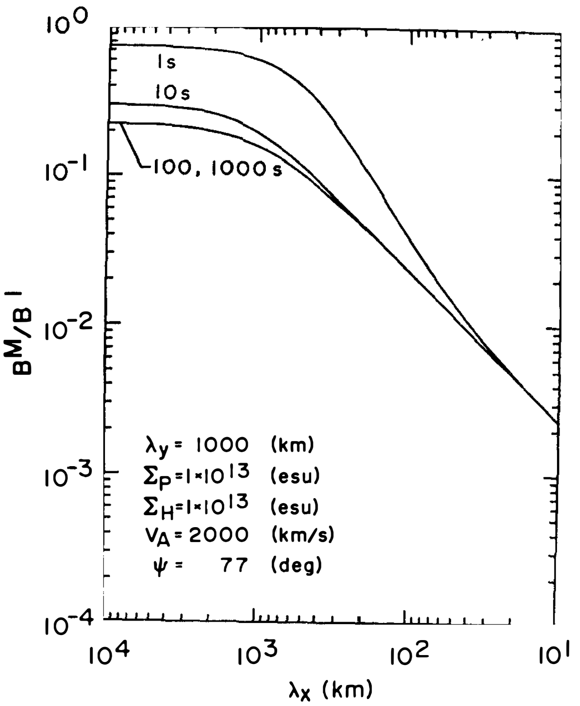

Fig. 6a. Dependence of magnitude of the ratio B^M/B^I just below the ionosphere upon λ_x for $\lambda_y=1000$ km in the case of $\psi=77°$ and $\Sigma_P=\Sigma_H$. The presence of the earth is neglected.

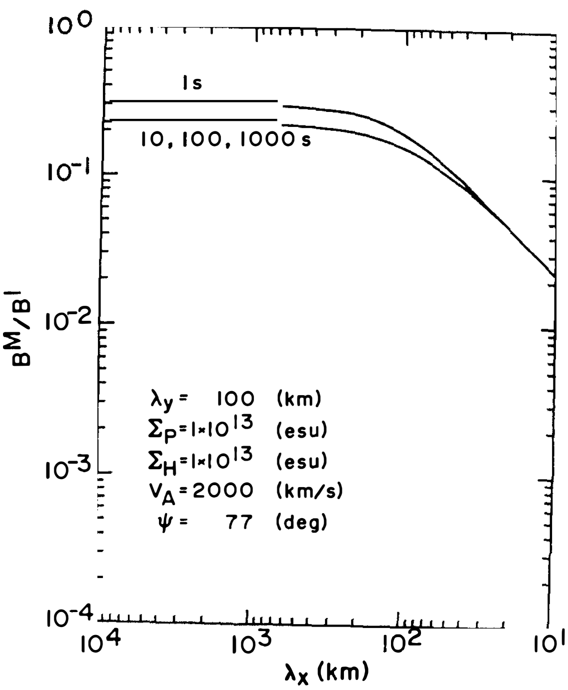

Fig. 6b. Same as Fig. 6a but for $\lambda_y=100$ km.

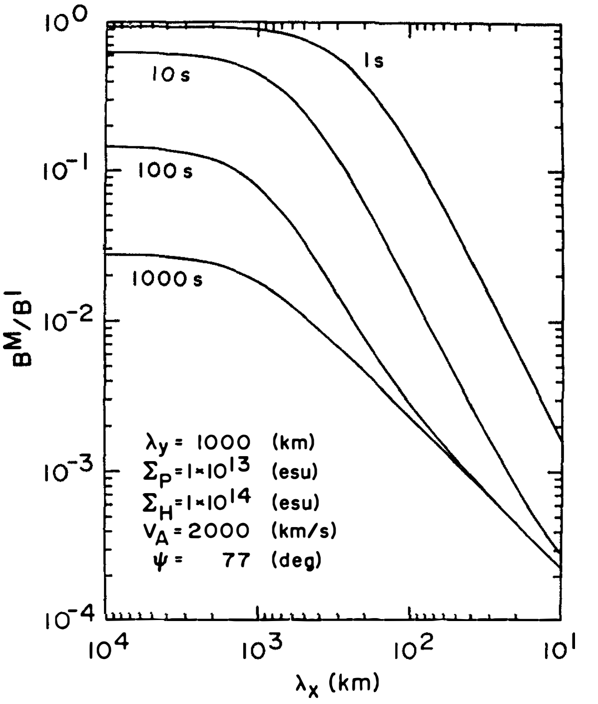

Fig. 7. Dependence of magnitude of the ratio B^M/B^I just below the ionosphere upon λ_x in the case of $\psi=77°$ and $\Sigma_P \neq \Sigma_H$. The presence of the earth is neglected.

sector during polar substorms (KAMIDE and BREKKE, 1977). By comparing Fig. 7 with Figs. 6a and 6b, it is seen that the ratio which $\boldsymbol{B}^{\mathrm{M}}_{\mathrm{NVC}}$ bears to $\boldsymbol{B}^{\mathrm{I}}_{\mathrm{NVC}}$ becomes smaller with an increase of $\Sigma_{\mathrm{H}}/\Sigma_{\mathrm{P}}$ for longer wave-periods ($T \gtrsim 100$ s). This result for longer wave-periods is plausible since the increase of $\Sigma_{\mathrm{H}}/\Sigma_{\mathrm{P}}$ appears to mean the enhancement of the Hall current, accordingly, the ionospheric source-free current $\boldsymbol{J}_{\mathrm{SF}}$. However, the inverse result is obtained for shorter wave-periods ($T \lesssim 10$ s). Hence, the relationship between the ratio of $\boldsymbol{B}^{\mathrm{M}}_{\mathrm{NVC}}$ to $\boldsymbol{B}^{\mathrm{I}}_{\mathrm{NVC}}$ and that of Σ_{H} to Σ_{P} is not so straightforward as we suppose. It is found in these figures that the ratio of $\boldsymbol{B}^{\mathrm{M}}_{\mathrm{NVC}}$ to $\boldsymbol{B}^{\mathrm{I}}_{\mathrm{NVC}}$ grows large as λ_y (wave-length in the east-west direction) shortens compared with λ_x (wave-length in the north-south direction). Further, the ratio also increases with a decrease of the geomagnetic latitude (Fig. 8). In general, the ratio is larger for shorter wave-periods ($T \lesssim 10$ s) than for longer wave-periods ($T \gtrsim 100$ s) but the dependence of that upon T is not monotonous (see Fig. 8).

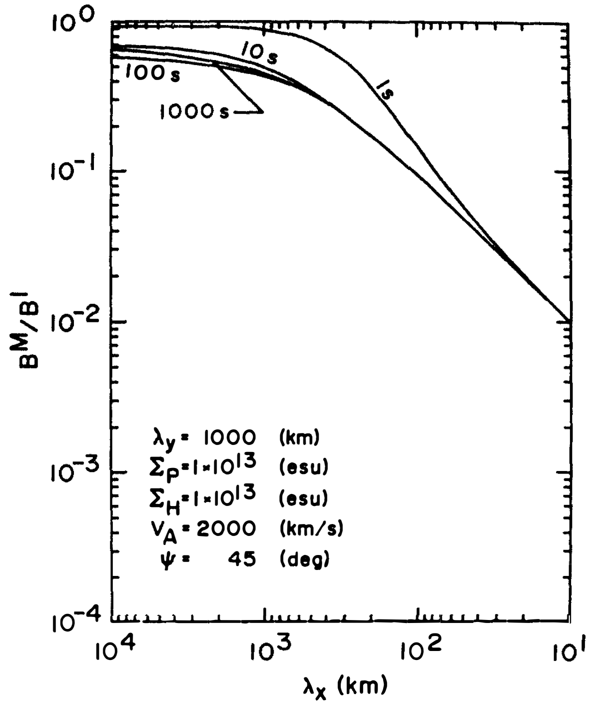

Fig. 8. Dependence of magnitude of the ratio $B^{\mathrm{M}}/B^{\mathrm{I}}$ just below the ionosphere upon λ_x in the case of $\psi = 45°$. The presence of the earth is neglected.

5. Summary and Discussion

As far as the height-integrated Pedersen and Hall conductivities, Σ_{P} and Σ_{H}, are at least greater than 10^{11} esu (~ 0.1 mho) and the ratio of Σ_{H} to Σ_{P} lies in the range of 10^{-2} to 10^{2}, the theorem that no ground magnetic effect is produced by the combination of the vertical current and the associated ionospheric irrotational current (closing current via the vertical current) holds in the ULF range which is prescribed by wave-period of $T \gtrsim 1$ s and horizontal wave-length of $\lambda \lesssim 10000$ km as well as in the static case when the ionosphere is uniform. The validity of the theorem does not depend on whether the ambient magnetic field is vertical or not. Further, the theorem is independent of the value of σ (earth conductivity) and is insensitive to the value of V_A (Alfvén velocity in the magnetosphere) as far as it lies at least in the range of 1000 to 10000 km/s (such examples are not exhibited in the present paper). The theorem means that the magnetic field observed on the ground is that with no vertical current, $\boldsymbol{B}_{NVC}$, and hence the displacement current in the atmosphere is neglected. Up to this time, neglect of the displacement current in the atmosphere for low-frequency variations has been made unconditionally but as a matter of fact its validity depends on the ratio of the irrotational to divergence-free ionospheric electric fields as seen from eq. (11). Of course, as indicated by our results, this ratio for realistic parameters never makes that of $\boldsymbol{B}_{VC}$ to $\boldsymbol{B}_{NVC}$ in the atmosphere approach the unity. However, it should be noted that it is in principle possible that the ratio of $\boldsymbol{B}_{VC}$ to $\boldsymbol{B}_{NVC}$ in the atmosphere approaches the unity, that is, the displacement current in the atmosphere is significant even in the low-frequency variations. If one were extremely greater than the other in Σ_P and Σ_H, $\boldsymbol{B}_{VC}$ would be no longer neglected compared with $\boldsymbol{B}_{NVC}$ in the atmosphere (see eqs. (12) and (13)).

In general the magnetic field observed on the ground consists of the part produced by the magnetospheric current closing two-dimensionally in the horizontal plane, $\boldsymbol{B}_{NVC}^{M}$, and that produced by the ionospheric source-free current, $\boldsymbol{B}_{NVC}^{I}$, except for that due to the induction current within the earth. In the oblique magnetic field line problem with the slow (Alfvén) wave incidence the ratio which $\boldsymbol{B}_{NVC}^{M}$ bears to $\boldsymbol{B}_{NVC}^{I}$ has been investigated. Consequently, it has been found that for longer wave-periods ($T \gtrsim 100$ s) the contribution of $\boldsymbol{B}_{NVC}^{M}$ to the ground magnetic field is not so much important in the auroral zone except for the case of λ_y (wave-length in the east-west direction) less than λ_x (wave-length in the north-south direction) since Σ_H is usually greater than Σ_P in that zone. When $\lambda_y < \lambda_x$, the magnitude of $\boldsymbol{B}_{NVC}^{M}$ is about 20% of that of $\boldsymbol{B}_{NVC}^{I}$ for longer wave-periods ($T \gtrsim 100$ s) in the auroral zone.

Finally, we emphasize that since the horizontally stratified model with the uniform ambient magnetic field is assumed, our conclusions are applicable only to the phenomenon localized at a specific region such as the auroral zone, the Fourier transform of which with respect to x and y has been considered in this paper. Since the ionosphere is no longer regarded as an infinitely thin sheet for $\lambda = 10$ km, the curvature of the earth is significant for $\lambda = 10000$ km, and the ducted horizontal propagation in the ionospheric F-region appears for $T = 1$ s, these extreme values are meaningless in our simple model. However, the extreme values ($\lambda = 10$ km, $\lambda = 10000$ km, and $T = 1$ s) have been used in order to examine the trend before reaching those values.

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Appendix

In the ionosphere ($z=d$) horizontal components of the electric field $\boldsymbol{E}$ are continuous but those of the magnetic field $\boldsymbol{B}$ are changed by the sheet current $\boldsymbol{J}$ flowing therein. That is, the horizontal magnetic fields are connected by

$$\left\{\begin{aligned} B_x(d+0)-B_x(d-0)&=\frac{4\pi}{c}J_y \\ B_y(d+0)-B_y(d-0)&=-\frac{4\pi}{c}J_x\ , \end{aligned}\right. \tag{A-1}$$

where

$$\left\{\begin{aligned} J_x&=\frac{\Sigma_{\mathrm{P}}}{\sin^2\psi}E_x(d)+\frac{\Sigma_{\mathrm{H}}}{\sin\psi}E_y(d) \\ J_y&=-\frac{\Sigma_{\mathrm{H}}}{\sin\psi}E_x(d)+\Sigma_{\mathrm{P}}E_y(d)\ , \end{aligned}\right. \tag{A-2}$$

and the system of CGS gaussian units is employed.

In the following we assume a horizontal spatial and time variation of the form $\exp[-i(kx+ly-\omega t)]$. In the atmosphere ($0<z<d$) the equation which $\boldsymbol{E}(z)$ should satisfy becomes

$$\left[\frac{d^2}{dz^2}-\left(k^2+l^2-\frac{\omega^2}{c^2}\right)\right]E(z)=0\ . \tag{A-3}$$

In the magnetosphere ($z>d$) we employ a cold magnetized plasma. Then, the equation which $E_x(z)$ and $E_y(z)$ should satisfy becomes

$$\left\{\begin{aligned}\sin^2\phi\frac{d^2E_x}{dz^2}&=2ik\sin\phi\cos\phi\frac{dE_x}{dz}+il\sin\phi\cos\phi\frac{dE_y}{dz}\\&\quad+\left[k^2\cos^2\phi+l^2-\frac{\omega^2}{V_A^2}\right]E_x-kl\sin^2\phi\,E_y\\\sin^2\phi\frac{d^2E_y}{dz^2}&=il\sin\phi\cos\phi\frac{dE_x}{dz}\\&\quad-kl\sin^2\phi\,E_x+\left(k^2-\frac{\omega^2}{V_A^2}\right)\sin^2\phi\,E_y\ ,\end{aligned}\right. \tag{A-4}$$

and $E_z(z)$ is given by the condition that the electric field parallel to the ambient magnetic field vanishes because of an extremely high conductivity along it. As for the derivation of eq. (A-4), see NISHIDA (1964). Solving eq. (A-4), two fundamental modes, which are termed fast (compressional) and slow (Alfvén) modes, appear.

Solving eqs. (A-3) and (A-4), we can describe the left-hand side of eq. (A-1) in terms of $E_x(d)$ and $E_y(d)$ by using

$$\mathrm{rot}\,\boldsymbol{E}=-\frac{1}{c}\frac{\partial\boldsymbol{B}}{\partial t}\ . \tag{A-5}$$

Further, the right-hand side of eq. (A-1) is also described in terms of $E_x(d)$ and $E_y(d)$ by eq. (A-2). Thus, basic equations for $E_x(d)$ and $E_y(d)$ are obtained. For example, assuming that the ambient magnetic field is vertical ($\phi=90°$), the earth is a perfect conductor, and $k\neq 0$, those equations are as follows:

$$\left\{\begin{aligned}A_{11}E_x(d)+A_{12}E_y(d)&=F_1\\A_{21}E_x(d)+A_{22}E_y(d)&=F_2\ ,\end{aligned}\right. \tag{A-6}$$

where

$$\left\{\begin{aligned}A_{11}&=\frac{4\pi}{c}\Sigma_P+A\left[\frac{\gamma^{RF}c}{\omega}\left(\frac{l}{k}\right)^2+\frac{\gamma^{RS}c}{\omega}\right]-\frac{i\gamma c}{\omega}\left[1-\left(\frac{k}{\gamma}\right)^2\right]\coth(\gamma d)\\A_{12}&=\frac{4\pi}{c}\Sigma_H-A\frac{l}{k}\left(\frac{\gamma^{RF}c}{\omega}-\frac{\gamma^{RS}c}{\omega}\right)+\frac{i\gamma c}{\omega}\frac{kl}{\gamma^2}\coth(\gamma d)\\A_{21}&=-\frac{4\pi}{c}\Sigma_H-A\frac{l}{k}\left(\frac{\gamma^{RF}c}{\omega}-\frac{\gamma^{RS}c}{\omega}\right)+\frac{i\gamma c}{\omega}\frac{kl}{\gamma^2}\coth(\gamma d)\\A_{22}&=\frac{4\pi}{c}\Sigma_P+A\left[\frac{\gamma^{RF}c}{\omega}+\frac{\gamma^{RS}c}{\omega}\left(\frac{l}{k}\right)^2\right]-\frac{i\gamma c}{\omega}\left[1-\left(\frac{l}{\gamma}\right)^2\right]\coth(\gamma d)\\F_1&=-2\left(\frac{l}{k}B_x^{IF}+B_y^{IS}\right)\\F_2&=2\left(B_x^{IF}-\frac{l}{k}B_y^{IS}\right)\ ,\end{aligned}\right.$$

and

$$A=\left[1+\left(\frac{l}{k}\right)^2\right]^{-1}$$

$$\gamma=\sqrt{k^2+l^2-\frac{\omega^2}{c^2}}$$

$$\gamma^{\mathrm{RF}}=-i\sqrt{k^2+l^2-\frac{\omega^2}{V_{\mathrm{A}}^2}}$$

$$\gamma^{\mathrm{RS}}=\frac{\omega}{V_{\mathrm{A}}}\ .$$

B_x^{IF} is the x component of the magnetic field of the incident fast wave and B_y^{IS} the y component of the magnetic field of the incident slow wave. As for details of the derivation of eq. (A-6), see ITONAGA (1984).

As soon as $E_x(d)$ and $E_y(d)$ are obtained, the magnetic field in the atmosphere or the magnetosphere is calculated by use of eqs. (A-5) and (A-3) or (A-4). Further, j_z from

$$(\mathrm{rot}\,\boldsymbol{B})_z=\frac{4\pi}{c}j_z\ , \tag{A-7}$$

$\boldsymbol{J}_{\mathrm{C}}$ from

$$\mathrm{div}\,\boldsymbol{J}_{\mathrm{C}}=-j_z(d+0)\ , \tag{A-8}$$

and $\boldsymbol{J}_{\mathrm{SF}}$ from

$$\boldsymbol{J}_{\mathrm{SF}}=\boldsymbol{J}-\boldsymbol{J}_{\mathrm{C}} \tag{A-9}$$

are in turn taken, where $\boldsymbol{J}$ is calculated by eq. (A-2).