DETAILED CALCULATION OF MODIFIED RADAR EQUATION FOR DETECTING METEORITES BURIED WITHIN THE ICE BY RADIO ECHO SOUNDING

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Abstract: The radar equation is usually given in free space and the field far from the radar transmitted is only studied. However, using the radar sounding apparatus to detect meteorite buried within the ice, we should consider the field near the radar transmitted. In this paper, the scattered field near the radar transmitted is calculated exactly and a more accurate radar equation is derived.

Based on the detailed calculation of the modified radar equation the detectable domains indicate that the intensity of echo reflected from the meteorite buried in the ice is sufficient to be detected by the present radio echo sounding apparatus taking $G_0=2$ as the antenna gain and $P_r/P_i=10^{-10}$ as the maximum sensitivity of the back-scattered power from spherical meteorite.

The detectable domain for iron meteorite is larger than that for stony meteorite, indicating that if the diameter is identical, the detectable depth for iron meteorite must be deeper than that for stony meteorite. The detectable domain extends to a smaller diameter of meteorite and a larger depth using a higher frequency in the case of the Rayleigh scattering, but it should be noted that for iron meteorite, the frequency dependence on the detectable domain becomes reciprocal and shows resonance phenomena for the 400 MHz at the diameter larger than about 10 cm where the scattering aspect due to meteorite pieces dispersed in the ice changes from the Rayleigh scattering into the Mie scattering.

In the maximum frequency of diameter of meteorite, about 20 cm for the iron meteorite and 1 cm for the stony meteorite, the detectable domain shows resonance phenomena for the iron meteorite by the electromagnetic wave of 400 MHz at the depth of 10 m, while for the stony meteorite the detectable domain is within 1 m in depth. Therefore, a higher frequency should be applied to detect the stony meteorite.

1. Introduction

Recovery of a large number of the Yamato meteorites and the Allan Hills meteorites in Antarctica led us to postulate the presence of meteorite pieces still buried in the ice. It has been assumed, from the topographic situation near the Yamato Mountains and Allan Hills, that meteorite pieces found on the bare ice surface might have been conveyed by the ice masses coming up from the interior of the ice sheet and disclosed on the bare ice surface as the result of continuous ablation and scraping of ice. NISHIO and ANNEXSTAD (1980), and NARUSE (1978) observed the upward movements and the rate of ablation of ice on the bare ice surface and added significant data to ascertain the above assumption.

Since the meteorite pieces found on the bare ice surface near the Yamato Mountains and Allan Hills were concentrated in a comparatively small area, it might be assumed that meteorites buried deeply in the interior of the Antarctic ice field are continuously conveyed by the ice flow and accumulated in the limited area of the bare ice surface, while many other meteorite pieces may be still remaining within the ice. This suggests that meteorite pieces remaining within the ice may probably be on their way to come out onto the bare ice surface.

If meteorites buried within the ice could be detected by radio echo sounding, it may be significant to clarify the concentration mechanism of a large number of meteorites, and also if meteorites buried in the ice are taken out together with the surrounding ice, valuable information for dating both meteorite and ice will be obtained. The most significant point about the Antarctic meteorites buried within the ice is that they have been kept in the Antarctic ice, which has never been artificially contaminated, so that they are unaffected by weathering and fracturing. The meteorites within the ice can therefore be considered the least contaminated specimens, and this uncontaminated condition is extremely important in various aspects of meteorite research, such as the analyses of rare-gas or organic composition and remanent magnetization.

During the glaciological survey in the 1982–83 field season in the Meteorite Ice Field near the Yamato Mountains, it has been tried to locate meteorites buried within the ice in the bare ice area, using a radio echo sounder with a pulse radar operated at a frequency of 300 and 500 MHz, but the buried meteorite fragments could not be detected within the ice.

NISHIO *et al.* (1981) proposed the possibility of locating meteorites buried in the ice by means of radio echo sounding, assuming that meteorite pieces are spherical in shape, and that the dielectric constant of stony meteorite is equal to that of granite (8.0) and iron meteorite is a conductive sphere. For simplification, it is also assumed that a small number of meteorite pieces are dispersed uniformly in the ice in the isolated form and the strength of the electric field is not attenuated throughout the ice, and that the scattering mechanism will predominate over the Rayleigh scattering because most of the diameter of meteorite pieces is smaller than wavelength used by radio echo sounder.

However, we have to consider that the electromagnetic wave shall be attenuated in propagating through the ice. Also we should calculate in more detail the relation between the detectable depth and the diameter of meteorite since, when the electromagnetic wave of the higher frequency such as 400 MHz is used, the back scattering due to iron meteorites causes the Mie scattering.

In this paper it is, therefore, shown that the detailed calculation of the modified radar equation should be made for detecting meteorites buried within the ice.

2. Basic Theory to Derive the Modified Radar Equation

In this paper, targets on which electromagnetic wave impinges are assumed to be a perfectly conductive sphere for an iron meteorite or a dielectric sphere for a stony meteorite. Although incident wave is spherical wave, electromagnetic wave is assumed to be plane wave at the surface of spherical meteorite and then written as,

$$E_t = \exp(jkz).$$

When the electromagnetic wave impinges on the meteorite, the scattered field of electromagnetic wave from the meteorite are obtained in the following procedure.

Step 1: Expressed with wave number k and transmission coefficients T and T', the relative permittivity of ice (ϵ_i) and stony meteorite (ϵ_m) is determined as 3.17 and 8.0, respectively. The loss tangent ($\tan\delta$) in the ice is taken as 0.001 (JOHARI and CHARETTE, 1975). Then, wave number k is given by,

$$k^2 = \omega^2 \epsilon \mu_0,$$

$$\epsilon_{\text{ice}} = \epsilon_i (1 - j \tan\delta),$$

where ω is angular frequency, μ_0 the permeability in vacuum and ϵ_{ice} the complex permittivity of the ice with the loss tangent. When the wave travels a long distance from a transmitter to a receiver through the ice, attenuation of wave is caused by the loss tangent of the ice. Even though the loss tangent of the ice is small, the effect of attenuation by the ice could not be neglected. As the radio wave that antenna transmitted travels from air to ice and from ice back to air, the interface between air and ice must be considered. The effect of the interface can be explained by the transmission coefficients T and T'.

The transmission coefficients T and T' are the ratio of the incident field in the air to the field in the ice, and to the contrary the incident field in the ice to the field in the air. However, the multiple interaction between the meteorite and the interface is not considered here.

Step 2: The scattered field is obtained by a modified radar equation. The radar equation is usually given in free space and the field far from the radar transmitted is only studied.

However, using the radar sounding apparatus to detect meteorite buried within the ice, we should consider the field near the radar transmitted. In such a case, the scattered field should be calculated more exactly.

Step 3: Once the antenna gain and the sensitivity of the radar are determined, the detectable depth and the detectable diameter of meteorite depend on these parameters. The antenna gain is assumed to be 2 and the sensitivity of the radar of P_r/P_t is equal to 10^{-10} (100 dB). When these parameters are determined, the relation between the detectable depth and the detectable diameter of meteorite can be obtained for the electromagnetic wave of 80 and 400 MHz by the modified radar equation. Since the relation cannot be explicitly expressed as analytical solution, it must be numerically calculated by the Newton method to solve the modified radar equation.

3. Expression of the Scattered Fields from a Meteorite

We define the geometry used to represent the scattered field as shown in Fig. 1. Then, incident fields are written by,

$$\left.\begin{aligned} E_x^i &= E_0 \exp(-jkz), \\ H_y^i &= \frac{E_0}{\eta} \exp(-jkz), \end{aligned}\right\} \tag{1}$$

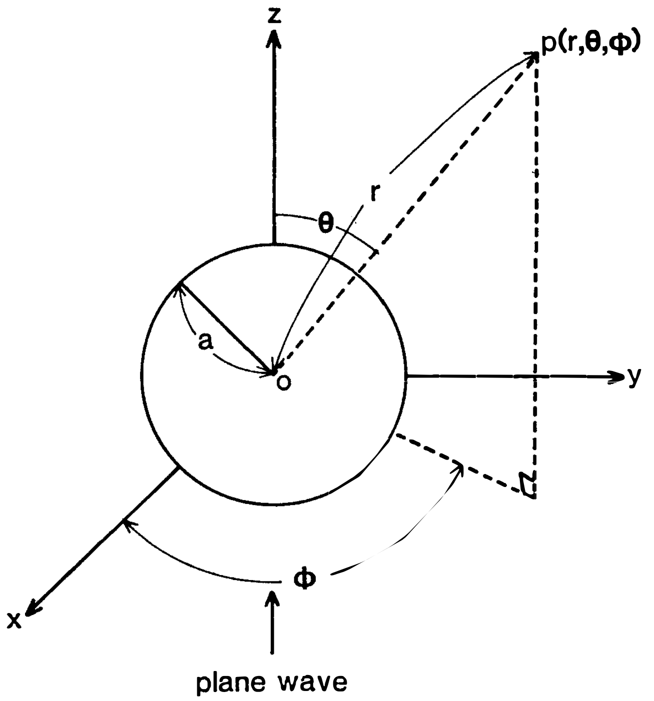

Fig. 1. Geometry used to obtain the scattered field.

where $\eta=\sqrt{\mu_0/\epsilon_{ice}}$ is the wave impedance for the ice. Incident wave of $\exp(jz)$ is expressed in terms of spherical wave as

$$\exp(jz)=\exp(jr\cdot\cos\theta)=\sum_{n=0}^{\infty}j^n(2n+1)\cdot J_n(r)\cdot P_n(\cos\theta). \qquad (2)$$

The r component of the incident field is obtained by eq. (1) as follows,

$$E_r^i=\cos\phi\cdot\sin\theta\cdot E_x^i=E_0\frac{\cos\phi}{jkr}\cdot\frac{\partial}{\partial\theta}\{\exp(-jkr\cdot\cos\theta)\}. \qquad (3)$$

Substituting eq. (2) into eq. (3), we obtain,

$$E_r^i=-\frac{jE_0\cos\phi}{(kr)^2}\sum_{n=1}^{\infty}j^{-n}(2n+1)\cdot\hat{J}_n(kr)\cdot P_n^1(\cos\theta), \qquad (4)$$

where $\hat{J}_n$ is the spherical Bessel function and P_n^1 the Legendre function of the first order.

Next we define the magnetic vector potential for incident wave as A^i and the electric vector potential for incident wave as F^i. All components of electromagnetic field for sphere are expressed by the r components of the magnetic and electric vector potentials (HARRINGTON, 1961). Therefore, the magnetic vector potential A_r^i and the electric vector potential F_r^i for the incident wave are given by,

$$\left.\begin{aligned} A_r^i&=\frac{E_0}{\omega\mu_0}\cos\phi\cdot\sum_{n=1}^{\infty}a_n\cdot\hat{J}_n(kr)\cdot P_n^1(\cos\theta),\\ F_r^i&=\frac{E_0}{k}\sin\phi\cdot\sum_{n=1}^{\infty}a_n\cdot\hat{J}_n(kr)\cdot P_n^1(\cos\theta),\end{aligned}\right\} \qquad (5)$$

where $a_n = j^{-n}(2n+1)/n(n+1)$.

We consider two special scattered bodies which are; 1) a perfectly conductive sphere for iron meteorite, and 2) a dielectric sphere for stony meteorite.

3.1. Scattered fields from a perfectly conductive sphere

The magnetic and electric vector potentials for the scattered fields which are written as A and F respectively are assumed to be,

$$\left.\begin{aligned} A_r^s &= \frac{E_0}{\omega\mu_0}\cos\phi\cdot\sum_{n=1}^{\infty} b_n\cdot\hat{H}_n^{(2)}(kr)\cdot P_n^1(\cos\theta), \\ F_r^s &= \frac{E_0}{k}\sin\phi\cdot\sum_{n=1}^{\infty} c_n\cdot\hat{H}_n^{(2)}(kr)\cdot P_n^1(\cos\theta), \end{aligned}\right\} \tag{6}$$

where b_n and c_n are determined by the boundary conditions on the sphere, that is, θ and ϕ components of electric field at $r=a$ should be vanished,

$$\left.\begin{aligned} b_n &= -a_n\frac{\hat{J}_n'(ka)}{\hat{H}_n^{(2)}(ka)}, \\ c_n &= -a_n\frac{J_n(ka)}{\hat{H}_n^{(2)}(ka)}. \end{aligned}\right\} \tag{7}$$

From eqs. (6) and (7), electric fields from a sphere are given by,

$$\left.\begin{aligned} E_r^s &= \frac{1}{j\omega\epsilon_{\text{ice}}}\left(\frac{\partial^2}{\partial r^2}+k^2\right)\cdot A_r^s, \\ E_\theta^s &= -\frac{1}{r\sin\theta}\frac{\partial E_r^s}{\partial\phi}+\frac{1}{j\omega r\epsilon_{\text{ice}}}\frac{\partial^2 A_r^s}{\partial r\cdot\partial\theta}, \\ E_\phi^s &= \frac{1}{r}\frac{\partial F_r^s}{\partial\theta}+\frac{1}{j\omega r\sin\theta\cdot\epsilon_{\text{ice}}}\frac{\partial^2 A_r^s}{\partial r\cdot\partial\phi}. \end{aligned}\right\} \tag{8}$$

The x component of the scattered field E_x^s is given by E_θ^s or E_ϕ^s as follows:

$$E_x^s = \frac{E_0}{2kr}\sum_{n=1}^{\infty} j^n(2n+1)\cdot\left[\frac{\hat{J}_n'(ka)}{j\hat{H}_n^{(2)\prime}(ka)}\cdot\hat{H}_n^{(2)\prime}(kr)\frac{\hat{J}_n(ka)}{\hat{H}_n^{(2)}(ka)}\cdot\hat{H}_n^{(2)}(kr)\right]. \tag{9}$$

The above expression is an exact equation for all perfectly conductive spheres. However, for small spheres ($ka \ll 1$), the Rayleigh approximation is sufficient and the ray-optical approximation is available for large spheres ($ka \gg 1$). For a medium size sphere, the exact expression of eq. (9) should be used.

3.2. Scattered fields from a dielectric sphere

Concerning a dielectric sphere, electric and magnetic fields can exist inside the dielectric sphere. Therefore, the electric and magnetic vector potentials should be different between the inside and the outside the sphere. We define the magnetic and electric vector potentials A_r^{s+} and F_r^{s+} for the outside of the sphere. A_r^{s+} and F_r^{s+} are written as,

$$\left.\begin{aligned} A_r^{s+} &= \frac{E_0}{\omega\mu_0}\cos\phi\cdot\sum_{n=1}^{\infty} b_n\cdot\hat{H}_n^{(2)}(kr)\cdot P_n^1(\cos\phi), \\ F_r^{s+} &= \frac{E_0}{k}\sin\phi\sum_{n=1}^{\infty} c_n\cdot\hat{H}_n^{(2)}(kr)\cdot P_n^1(\cos\phi). \end{aligned}\right\} \tag{10}$$

For the sphere's inside, A_r^{s-} and F_r^{s-} are written as follows:

$$\left.\begin{aligned} A_r^{s-} &= \frac{E_0}{\omega\mu_0}\cos\phi\sum_{n=1}^{\infty} d_n\cdot\hat{J}_n(k_j r)\cdot P_n^1(\cos\phi), \\ F_r^{s-} &= \frac{E_0}{k}\sin\phi\sum_{n=1}^{\infty} e_n\cdot\hat{J}_n(k_j r)\cdot P_n^1(\cos\phi), \end{aligned}\right\} \tag{11}$$

where b_n, c_n, d_n and e_n are determined by the continuity of the electric and magnetic field across the surface of sphere. Scattered fields outside the sphere is given by,

$$E_x = \frac{E_0}{2kr}\sum_{n=1}^{\infty} j^n(2n+1)\cdot\left[\frac{P}{j}\hat{H}_n^{(2)}(kr) - Q\cdot\hat{H}_n^{(2)}(kr)\right], \tag{12}$$

where P and Q are as follows:

$$\begin{aligned} P &= \frac{\sqrt{\epsilon\mu_0}\hat{J}_n(ka)\hat{J}'_n(k_d a) - \sqrt{\epsilon_m\mu_0}\hat{J}'_n(ka)\hat{J}_n(k_d a)}{\sqrt{\epsilon\mu_0}\hat{H}_n^{(2)}(ka)\hat{J}'_n(k_d a) - \sqrt{\epsilon_m\mu_0}\hat{H}_n^{(2)\prime}(ka)\hat{J}_n(k_d a)}, \\ Q &= \frac{\sqrt{\epsilon\mu_0}\hat{J}'_n(ka)\hat{J}_n(k_d a) - \sqrt{\epsilon_m\mu_0}\hat{J}_n(ka)\hat{J}'_n(k_d a)}{\sqrt{\epsilon\mu_0}\hat{H}_n^{(2)\prime}(ka)\hat{J}_n(k_d a) - \sqrt{\epsilon_m\mu_0}\hat{H}_n^{(2)}(ka)\hat{J}'_n(k_d a)}, \\ k_d^2 &= \omega^2\epsilon_m\mu_0. \end{aligned} \tag{13}$$

When a dielectric sphere for a stony meteorite is small in terms of wavelength, ka is very small. In such a case, only the first term of eq. (10) is sufficient. By this approximation, eq. (12) is reduced to,

$$E_x^s = \frac{jE_0\cos\phi}{r}\cdot k^2a^3\cdot\frac{\epsilon_r-1}{\epsilon_r+2}\hat{H}_1^{(2)\prime}(kr), \tag{14}$$

where $\epsilon_r=\epsilon_m/\epsilon_i$. For large kr, $\hat{H}_1^{(2)\prime}(kr)$ becomes

$$\hat{H}_1^{(2)\prime}(kr) = \exp(-jkr)\cdot\left(j+\frac{1}{kr}-\frac{j}{(kr)^2}\right). \tag{15}$$

This approximation is accurate up to $ka=0.1$. If ka is larger than 0.1, the exact expression should be used.

4. Modified Radar Equation

The radar equation is generally used for the field far from the scattered body and the Rayleigh scattering domain. However, the radar equation is not accurate for the resonant region and the field in the vicinity of the scattered body. In this section, more accurate radar equation will be introduced.

The radiated wave is assumed to be spherical wave and polarized in the x direction. Since the transmitter is located on the ice surface, the interface between ice and air should be introduced in the radar equation. We define the transmission coefficient T and T' as follows:

$$T = \frac{2}{\sqrt{\epsilon_r}+1}, \quad T' = \frac{2\sqrt{\epsilon_r}}{\sqrt{\epsilon_r}+1}, \tag{16}$$

where T is the transmission coefficient from air to ice and T' the one from ice to air.

As shown in Fig. 1, power density S_a at the meteorite is given by,

$$S_a=\frac{P_t}{4\pi r^2}\cdot\exp(-j2kr)\cdot G_0=C_1E_0^2, \tag{17}$$

where P_t is the transmitted power, r distance between the transmitter and the meteorite, E_0 the electric field at the meteorite and C_1 constant. Scattered power density S_0 at the transmitter is,

$$S_0=C_1(E_x^s)^2, \tag{18}$$

where E_x^s is the electric field at the transmitter scattered from the meteorite. Hence, the received power P_r is,

$$P_r=S_0\frac{\lambda^2G_0}{4\pi}(E_x^s)^2. \tag{19}$$

Substituting eq. (9) for iron meteorite or eq. (14) for stony meteorite into eq. (19), we obtain the modified radar equations in the following,

$$\frac{P_r}{P_t}=\frac{\lambda^2G_0^2\,|\exp(-j2kr)|}{64(\pi kr^2)^2}\cdot\left\{\sum_{n=1}^{\infty}j^n(2n+1)\left[\frac{\hat{J}_n'(ka)}{j\hat{H}_n^{(2)\prime}(ka)}\cdot\hat{H}_n^{(2)\prime}(kr)+\frac{\hat{J}_n(ka)}{\hat{H}_n^{(2)}(ka)}\cdot\hat{H}_n^{(2)}(kr)\right]\right\}^2,\ \text{for iron meteorite,} \tag{20}$$

$$\frac{P_r}{P_t}=\frac{\lambda^2G_0^2\,|\exp(-j2kr)|}{16\pi^2r^2}\cdot\left\{\frac{jk^2a^3}{r}\cdot\frac{\epsilon_r-1}{\epsilon_r+2}\hat{H}_1^{(2)\prime}(kr)\cdot(TT')^2\right\}^2,\ \text{for stony meteorite.} \tag{21}$$

5. Concluding Remarks

Based on the detailed calculation of the modified radar eqs. (20) and (21), Fig. 2 shows the criteria for detectable and undetectable meteorites as a function of diameter, depth and frequency of electromagnetic wave of 80 and 400 MHz. The detectable domains indicate that the intensity of echo reflected from the meteorite buried in the ice is sufficient to be detected by the present radio echo sounding apparatus taking $G_0=2$ as the antenna gain and $P_r/P_t=10^{-10}$ as the maximum sensitivity of the back-scattered power from spherical meteorite.

As shown in Fig. 2, the detectable domain for iron meteorite is larger than that for stony meteorite, indicating that if the diameter is identical, the detectable depth for iron meteorite must be deeper than that for stony meteorite. The detectable domain extends to a smaller diameter of meteorite and a larger depth using a higher frequency in the case of the Rayleigh scattering. However, it should be noted that for iron meteorite, the frequency dependence on the detectable domain becomes reciprocal and shows resonance phenomena for the 400 MHz at diameter larger than about 10 cm where the scattering aspect due to meteorite pieces dispersed in the ice changes from the Rayleigh scattering into the Mie scattering.

In the maximum frequency of diameter of meteorite, about 20 cm for the iron meteorite and about 1 cm for the stony meteorite, the detectable domain shows reso-

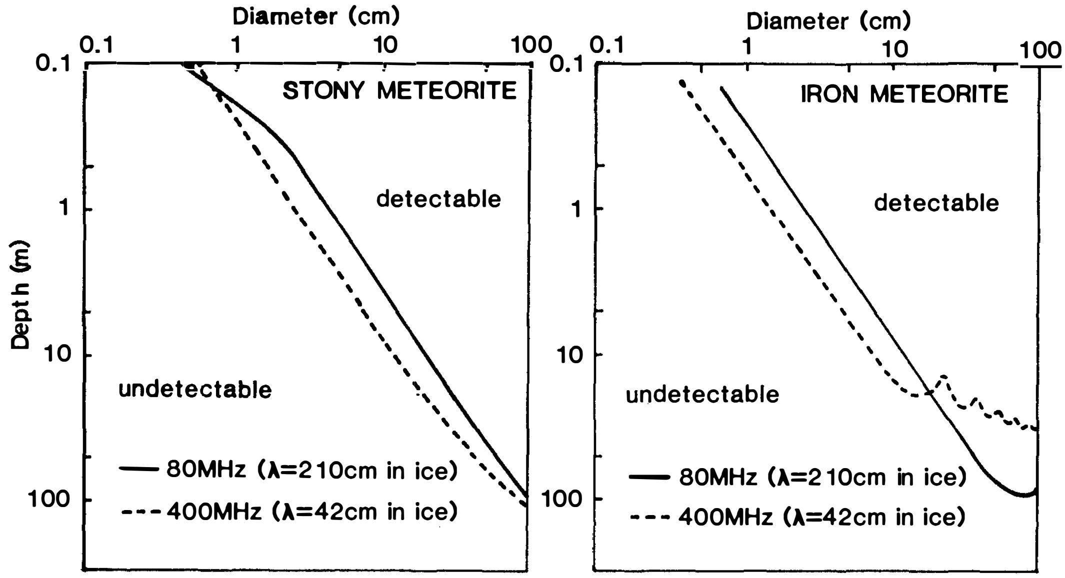

Fig. 2. Relation between the depth of ice and the diameter of spherical meteorite for the iron meteorites and the stony meteorites when electromagnetic wave having frequency of 80 MHz (λ=210 cm in ice) and 400 MHz (λ=42 cm in ice) are respectively radiated into the ice.

nance phenomena for the iron meteorite by the electromagnetic wave of 400 MHz at the depth of about 20 m, while for the stony meteorite the detectable domain is within 1 m in depth. Therefore, a higher frequency should be applied to detect the stony meteorite.

With reference to Fig. 2, the relation between the diameter of meteorite and the detectable depth is of no great difference from the one that was previously reported by NISHIO *et al.* (1981) though the attenuation of electromagnetic wave due to loss tangent in the ice was not considered.

The relation between the diameter and the depth for iron meteorite by the use of frequency 400 MHz indicates the oscillations as resonance phenomena in case of more than 10 cm in diameter and the detectable depth of iron meteorite is shallower than that by the use of frequency 80 MHz since the Mie scattering in 400 MHz but the Rayleigh scattering in 80 MHz. If the electromagnetic wave of frequency 80 MHz is used to detect an iron meteorite buried within the ice at great depth, the direction may be successful, but the resolution of the radio echo sounding apparatus may be deteriorated.

For lower frequency below 400 MHz, the electromagnetic wave is attenuated due to the loss tangent. However, if the electromagnetic wave of 1 GHz is used to detect such a stony meteorite as buried in the uppermost ice depth in the future, it may be attenuated by another mechanism as the high frequency of electromagnetic wave may be scattered by air bubbles in the ice and then propagation loss will be increased.

In this paper, it is investigated to detect the isolated meteorite buried within the ice and the electrical property of the ice is assumed to be uniform. However, the electrical property of the ice, that is, ϵ_{ice} and tan δ should be considered to be a function of depth and the scattered field from several meteorites should be considered.

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(Received June 16, 1984; Revised manuscript received October 1, 1984)