HYDROMAGNETIC EIGEN-OSCILLATIONS IN THE DIPOLAR FIELD

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***Abstract*:** MHD equations in the dipolar coordinates are analyzed for an idealized model in which the earth surface is assumed to be a perfect conductor. For the case of standing resonance waves, *i. e.*, the guided toroidal and the guided poloidal waves, the eigenperiods are determined mainly by the mass density near the equatorial plane in the magnetosphere. The eigenperiods of the standing waves importantly depend on the mass density of the heavy ions outside the plasmapause. The resonance oscillations are expected to occur only on a field line having L-values within a narrow range, since a displacement of MHD fluid in the equatorial plane is of the order of 0.1 R_E for a perturbed electric field ~ 2 mV/m near $L=10$. The relation of the electric field wave amplitude (δE) of the standing oscillation to the displacement ($\delta\xi_{eq}$) is represented by $|\delta E| = B_0 L^{-3} \cdot \delta\xi_{eq}$ near the magnetic equator. The magnetic perturbation fields of the standing oscillations in the equatorial plane are one order smaller than those in the ionosphere for a case of perfect reflection. The conditions of the coupling oscillations of an Alfvén wave and a compressional wave are theoretically discussed for the special case of highly asymmetric modes.

1. Introduction

In situ observations have recently revealed the existence of various modes of daytime Pc 3–5 magnetic pulsations in the geomagnetosphere (*cf.* the reviews of ARTHUR *et al.*, 1977; KOKUBUN, 1980; HUGHES, 1980). Most of Pc 3–5 pulsations in the magnetosphere can be classified into the two type, *i. e.*, the transverse and the compressional modes. A part of the transverse waves, *e. g.*, azimuthal Pc 5 pulsations in the morning sector, can be explained by a standing Alfvén wave resonance of the earth's magnetic field lines based on the idea of a resonance coupling between a shear Alfvén wave and a surface wave at the magnetopause (CHEN and HASEGAWA, 1974a, b; LANZEROTTI and FUKUNISHI, 1974; SOUTHWOOD, 1974, 1975, 1977; KOKUBUN *et al.*, 1976; TAMAO, 1965; SINGER and KIVELSON, 1979). On the other hand, satellite-ground correlations of Pc 4–5 pulsations have been reported that the correlations with ground observations near the subsatellite point existed only when the satellite was near the same L-shell and within a few hours (PATEL, 1965; BARFIELD *et al.*, 1971, 1972; HEDGECOCK, 1976; KOKUBUN *et al.*, 1976; SINGER and KIVELSON, 1979). KOKUBUN (1980) also noted an interesting tendency that azimuthally polarized transverse Pc 5 in space is better correlated with ground events than the compressional Pc 5. The existence of various modes of daytime pulsations in the magnetosphere excited by the solar wind energy implies that there should be various propagation mechanisms of input fluctuations from the solar wind. Then, the study of the standing oscillations of a local field line for the eigenperiods, the equatorial extent of the oscillation, the amplitudes of the perturbed magnetic and electric fields and the conditions of the coupling resonance, is indispensable to clarify the characteristics of the various modes of daytime magnetic pulsations and the transmission mechanisms of magnetic ULF energies in the geomagnetosphere from the solar wind, and to diagnose magnetospheric properties.

The basic magneto-hydro-dynamic (MHD) equations for the low-frequency have been used to derive an equation for the period and amplitude of transverse modes by numerous authors as a starting point for examining long-period ULF waves in the earth's magnetic field (WESTPHAL and JACOBS, 1962; DUNGEY, 1963; RADOSKI and CAROVILLANO, 1966; CUMMINGS *et al.*, 1969; ORR and MATTHEW, 1971; RADOSKI, 1974). The hydromagnetic wave theory was first applied to the magnetosphere by DUNGEY (1954), who restricted consideration to axisymmetric waves, with particular reference to the situation in a dipolar field. He showed the two basic solutions of the axisymmetric wave equation, later known as the toroidal and poloidal oscillations which in general will be coupled in a non-uniform magnetic field, but the coupled equations have not been solved analytically (LANZEROTTI and SOUTHWOOD, 1979). The toroidal mode is also known as a transverse, torsional or azimuthal Alfvén wave, and the poloidal as a radial and compressional or fast magnetosonic wave. In contrast to the axisymmetric case, DUNGEY (1963) and RADOSKI (1967) introduced a guided poloidal mode in a highly asymmetric case, where the azimuthal wave number is large and the toroidal field becomes small.

CUMMINGS *et al.* (1969) numerically determined the eigenfrequencies for the guided toroidal and poloidal modes on a field line at 6.6 R_E for a plasma distribution along the field line given by $n=n_0(r_0/r)^\gamma$, where γ varies from 0 to 6, n_0 is the proton number density at r_0, the geocentric distance to the equatorial crossing point of the field line. This approximation was also used successfully by ORR and MATTHEW (1971) to give a quantitative explanation of the latitude dependence of Pc 3–4 pulsation period at mid-latitudes ($<60°$). WARNER and ORR (1979) used the MEAD and FAIRFIELD (1975) magnetic field model and solved for wave periods by using the time of flight approximation (OBAYASHI and JACOBS, 1958), *i. e.*, the WKB approximation, to the toroidal wave equation for different tilt angles and Kp conditions. But the WKB approximation is not valid when the wave length of the oscillation is comparable to the scale size of the system, and is particularly poor for the fundamental mode oscillation of a field line (MIURA *et al.*, 1982), which will be represented in Section 2. SINGER *et al.* (1981) examined the effect on eigenfrequencies of only the field geometry and the diurnal variations of the eigenfrequencies, using the recent Olson-Pfitzer magnetospheric magnetic field model, by keeping density constant along all field lines.

The present numerical study extends and reconfirms the previous papers by CUMMINGS *et al.* (1969) and ORR and MATTHEW (1971) in three ways. First, additional effects of plasma density distribution and heavy ions outside the plasmapause on the eigenfrequencies are analyzed by using OGO-5 plasma density profile (CHAPPELL, 1972), which are useful to examine the L-value dependence of the observed pulsation periods. Second, relations between the perturbation fields at the earth surface and the equatorial extends of the resonance field line oscillation are examined to clarify the satellite-ground correlations. Finally, the special solution of the coupling oscillations between an Alfvén wave and a compressional wave will be theoretically discussed for the limiting case of highly asymmetric modes near the magnetic equator.

2. Eigenperiod of Alfvén Resonance Oscillation

Incompressible perturbations are analyzed in this section, because we are interested in the oscillation of the local field line, which is possible only in the guided waves. For the guided waves with a time dependence of the form $e^{-1\omega t}$ in which the Poynting vector is alway along the field line, eqs. (A9.1–A9.2) are no longer coupled. We obtain the guided toroidal and the guided poloidal wave equations (RADOSKI, 1967) as the following forms;

$$\partial^2 X_\nu/\partial\mu^2+\partial(\text{In } h_\phi/h_\nu h_\mu)/\partial\mu\cdot\partial X_\nu/\partial\mu+\omega^2 h_\mu^2 V_A^{-2}\cdot X_\nu=0\,, \tag{1.1}$$

and

$$\partial^2 X_\phi/\partial\mu^2+\partial(\text{In } h_\nu/h_\mu h_\phi)/\partial\mu\cdot\partial X_\phi/\partial\mu+\omega^2 h_\mu^2 V_A^{-2}\cdot X_\phi=0\,, \tag{1.2}$$

respectively. Equation (1.1) indicates the axially symmetric toroidal mode with torsional oscillations (b_ϕ) of an entire magnetic shell. Eigenmodes of the transverse Alfvén waves with wavelengths comparable to the background magnetic field lines have eigenfrequencies that are different for different shells. The guided poloidal mode with eq. (1.2) corresponds to the limiting case, where large azimuthal wave number $m\gg 1$, with magnetic oscillations in a meridian plane (b_ν) and represents incompressible perturbation of the entire magnetosphere. These eqs. (1.1–1.2) are in agreement with the previous results (DUNGEY, 1967; RADOSKI, 1967; CUMMINGS *et al.*, 1969; ORR, 1973).

Equations (1.1–1.2) of the guided toroidal and the guided poloidal modes can be rewritten as,

$$\partial^2 X_\nu/\partial z^2+\omega^2[r_E(1-z^2)^3(\mu_0\rho_0)^{1/2}B_E^{-1}]^2 X_\nu=0\,, \tag{2.1}$$

and

$$\partial^2 X_\phi/\partial z^2-6z(1+3z^2)^{-1}\partial X_\phi/\partial z+\omega^2[r_E(1-z^2)^3\sqrt{\mu_0\rho_0}\,B_E^{-1}]^2 X_\phi=0\,, \tag{2.2}$$

respectively, where $z=\cos\theta$, $B_E=B_0/L^3$; the field strength at r_E, and $r_E=LR_E$; the geocentric distance to the equatorial crossing point of the field line at L. In subsequent calculations, the eigenfrequencies of the guided toroidal and poloidal modes depend on the model of the number density distribution $n_0\ (r,\theta)$. The plasma density $n_0(L)$ on the equatorial plane in the dayside magnetosphere is given by a smoothed profile deduced from Ogo 5 spectrometer measurements as shown in Fig. 1 (CHAPPELL, 1972). CUMMINGS *et al.* (1969) and ORR and MATTHEW (1971) numerically determined the eigenfrequencies for a plasma distribution along the field line given by $n=n_0(r_E/r)^m$ (an isospherical model), where the density index m varies from 0 to 6, n_0 is a proton number density at r_E, and r is a geocentric distance to the position of interset on the field line. We numerically solved the eq. (2) for the gyrofrequency model of the plasma distribution along the field line given by,

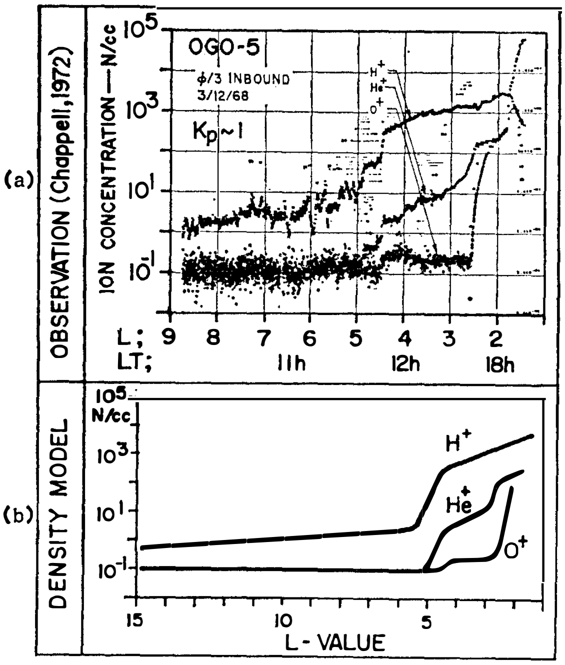

Fig. 1a. *The observed plasma density on the equatorial plane in the dayside magnetosphere* (*CHAPPELL, 1972*).

Fig. 1b. *A smoothed profile of the plasma density deduced from Ogo 5 spectrometer measurements, as a function of the L-value.*

$$n=n_0(L)\cdot(1+3\cos^2\theta)^{1/2}\sin^{-6}\theta\propto B\,, \tag{3}$$

where B is a background magnetic field and $n_0(L)$ is a number density at the equatorial crossing point of the field line. Recently MIURA *et al.* (1982) demonstrated that the ionospheric boundary conditions, *e. g.*, a finite height-integrated ionospheric conductivity and the steady electric field in the ionosphere, are important for electrodynamics in the ionosphere-magnetosphere coupling system. The eigenfrequency and damping (growth) rate of the eigen oscillations of the shear Alfvén wave are hardly dependent on the conditions in the daytime, although the perturbation fields are strongly dependent on those. Using the boundary condition that $X_\nu=X_\phi=0$ at $z=\pm\cos\theta_0$, *i. e.*, a case of perfect reflection at the surface of the earth, the eigenfrequencies of the guided, toroidal and poloidal modes in the dayside magnetosphere are computed by means of the Runge-Kutta-Gill method. Figures 2a and 2b illustrate the fundamental eigenperiods against the L-value for the gyrofrequency model and the isospherical model with $m=0$, respectively. Zero-order characteristic periods given by "time of flight" method (OBAYASHI and JACOBS, 1958) are also illustrated by the broken lines for both of the density models. The characteristic period of the oscillating magnetic lines of force is given by,

$$T=2\int\frac{ds}{V_A}=2\frac{\mu_0^{1/2}r_E}{B_0L^{-3}}\int\rho_0^{1/2}\sin^7\theta d\theta\,. \tag{4}$$

Many workers consider this "time of flight" (WKB) approximation of determining the pulsation periods in the magnetosphere. However, the fundamental periods by the "time of flight" method are $\sim$40% shorter than those of the fully developed decoupled mode theory for the isospherical model, that is consistent with the result of MIURA *et al.* (1982). This result implies that the time of flight approximation is not valid when

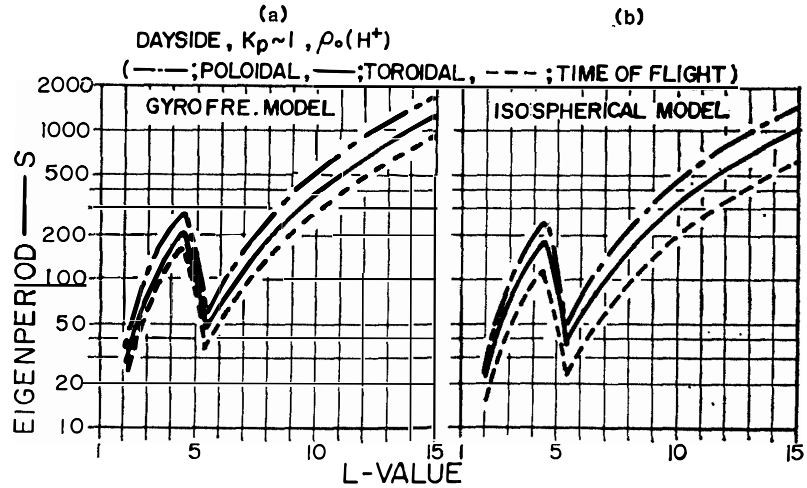

Fig. 2. Fundamental eigenperiods of the guided toroidal and the guided poloidal modes and the time of flight are represented by chain, solid and broken lines against the L-value for the gyrofrequency and the isospherical plasma density models, respectively. Where $\rho_0(H^+)$ *and Kp ~ 1 in the dayside magnetosphere.*

the wavelength of the oscillation is comparable to the scale size of the system, which has a nonuniform magnetic field, such as a dipolar field. The eigenperiods of the guided poloidal mode are ~30% longer than those of the guided toroidal modes, because of the "resistance" term in the meridional direction, *i. e.*, second term of eq. (2.2). The eigenperiods of the guided toroidal and poloidal modes for the gyrofrequency plasma density model are only ~11% longer than those of the isospherical plasma density model, though the number density for the gyrofrequency model is 10^2–10^3 times as large as that for the isospherical model near the high-latitude inner magnetosphere. CUMMINGS *et al.* (1969) showed that it is the equatorial mass density which is most important in determining the eigenperiods of a flux tube. It is confirmed that the eigenperiods are

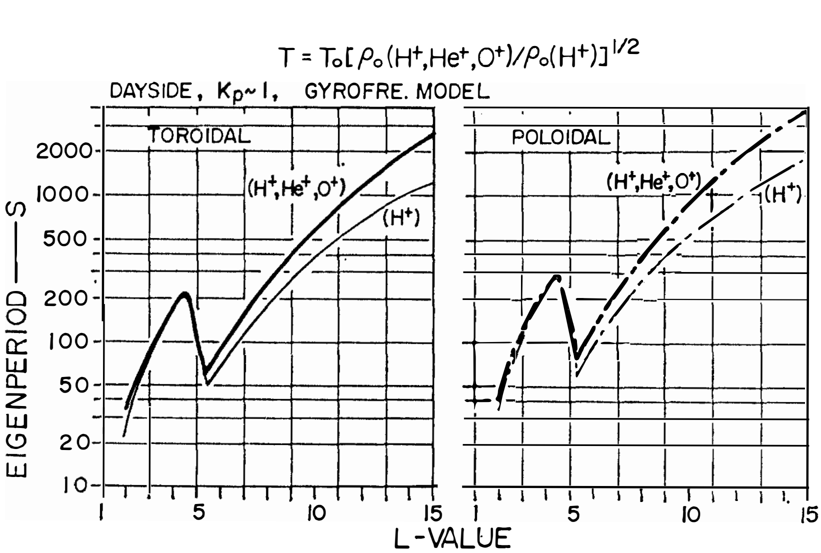

Fig. 3. Heavy ions effect on the eigenperiods in the dayside magnetosphere for the gyrofrequency model and Kp ~ 1.

determined mainly by the mass density near the equatorial plane in the magnetosphere.

Figure 3 indicates the effects of the heavy ions on the eigenperiods of the guided toroidal and poloidal modes for the gyrofrequency plasma model in the dayside magnetosphere. The thin and thick lines as a function of L-value express the eigenperiods for the only proton mass density and the total mass density of H^+-, H_e^+- and O^+-ions on the equatorial plane as shown in Fig. 1, respectively. The eigenperiods for any other mass density can be obtained by using the relation $T=T_0\cdot[\rho_0(H^+, H_e^+, O^+)/\rho_0(H^+)]^{1/2}$, where T_0 is the eigenperiod of the guided mode for the proton mass density and $\rho_0(H^+, H_e^+, O^+)$ is the observed mass density in unit of cm^{-3}. It is found that the eigenperiods importantly depend on the mass density of the heavy ions outside the plasmasphere. Therefore, the concentration of the heavy ions outside the plasmasphere is diagnosed by the observed periods (T_{obs}) of magnetic pulsations, *i.e.*, $\rho_0(H^+, H_e^+, O^+)=\rho_0(H^+)\cdot(T_{obs}/T_0)^2$. The first clear observational case of the heavy ions being important for a particular pulsation was recently reported by SINGER *et al.* (1979) who used the same formula as quoted in this paper.

Estimates of wave electric field, field line displacement and ratios of magnetic perturbations at different points on a field line have been made by NEWTON *et al.* (1978), ALLAN and KNOX (1979a, b, 1982) and WALKER (1980). Once X_ν and X_ψ are numerically determined as a function z, the magnetic perturbation (b_ψ, b_ν) and the displacement vector (ξ_ψ, ξ_ν) of the guided modes are obtained from equation (A10). The relations between the perturbations in the dipolar field can be rewritten as the following formulas;

$$b_\psi=+r_E^{-3}(1-z^2)^{-3/2}\frac{\partial X_\nu}{\partial z}\,, \tag{5.1}$$

$$b_\nu=-r_E^{-2}(1+3z^2)^{-1/2}(1-z^2)^{-3/2}\frac{\partial X_\psi}{\partial z}\,, \tag{5.2}$$

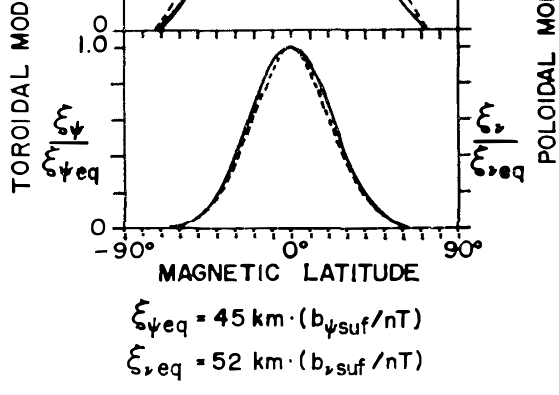

Fig. 4. Solutions of the toroidal and the poloidal wave equations for the gyrofrequency plasma model at L=10. The theoretical wave displacement vectors at any point along the field line can be obtained as a function of the magnetic latitude. The perturbation fields are normalized with the values at the equatorial plane.

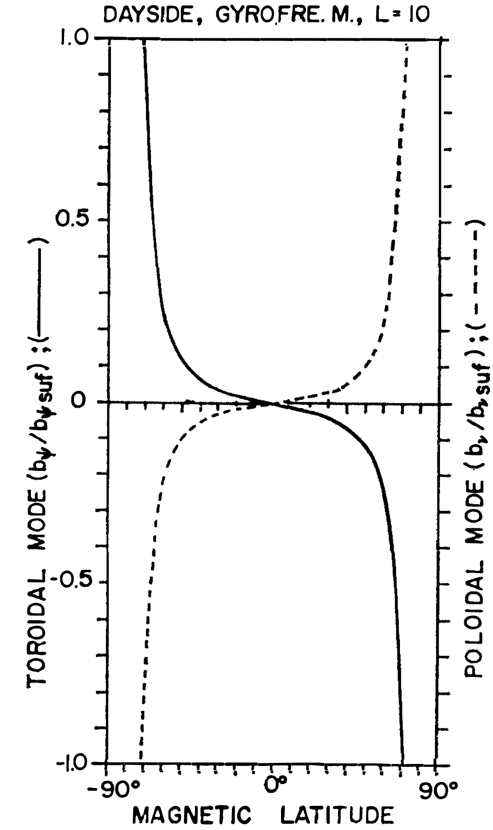

Fig. 5. Solutions of the toroidal and the poloidal wave equations for the gyrofrequency model at L=10. The theoretical wave magnetic field at any point along the field line are represented as a function of the magnetic latitudes. The perturbation fields are normalized with the reflecting surface of the perfect conductor ($\boldsymbol{b}_{\rm suf}$).

and

$$\xi_\phi = +LB_0^{-1}R_E^{-2}(1-z^2)^{3/2}X_\nu\,, \tag{6.1}$$

$$\xi_\nu = -L^2B_0^{-1}R_E^{-1}(1-z^2)^{3/2}(1+3z^2)^{-1/2}X_\phi\,. \tag{6.2}$$

The amplitudes (X_ν, X_ϕ), (ξ_ϕ, ξ_ν) and (b_ϕ, b_ν) of the fundamental modes are plotted in Figs. 4 and 5 as functions of magnetic latitude for the gyrofrequency plasma model, where the boundary condition is given by $X'(-\cos\theta_0) = -X'(+\cos\theta_0)$ at $z = \pm\cos\theta_0$. The perturbation fields $(\boldsymbol{X}, \boldsymbol{\xi})$ and $(\boldsymbol{b})$ are normalized with the values at the equatorial plane $(\boldsymbol{X}_{\rm eq}, \boldsymbol{\xi}_{\rm eq})$ and the reflecting surface of the perfect conductor $(\boldsymbol{b}_{\rm suf})$, respectively. For example, suppose an oscillation observed at the high-latitude in the daytime ($L \sim 10$) with $b_{\phi{\rm suf}} = b_{\nu{\rm suf}} = 10$ nT amplitude. If the oscillations represent the fundamental modes of the standing wave resonance, then $X_{\nu{\rm eq}} \sim 5.7\times10^{13}[TL^3]$ and $X_{\phi{\rm eq}} \sim 1.0\times10^6$ $[TL^2]$, and the displacement vector should be $\xi_{\phi{\rm eq}} \sim 450$ km, $\xi_{\nu{\rm eq}} \sim 520$ km near the equatorial plane. It is interesting to note that the equatorial extent of the standing wave resonance is estimated to be $\sim 0.1\ R_E$ for $b_{\rm suf} \sim 10$ nT and $L \sim 10$ in the magnetosphere. The amplitude of the wave electric field at any point along the field line can be determined from Figs. 4 and 5 by multiplying the displacement vector by the ambient magnetic field $B = B_0L^{-3}(1+3\cos^2\theta)^{1/2}\sin^{-6}\theta$, *i.e.*, $E_1 = \boldsymbol{B}\times\boldsymbol{\xi}$ and $|\delta E| = B\cdot\xi \sim B_0L^{-3}\cdot\delta\xi_{\rm eq}$. The peak electric field wave amplitudes of the toroidal and the poloidal modes should be $E_{1\nu} \sim 1.4$ mV/m and $E_{1\phi} \sim 1.6$ mV/m at the equatorial plane for $L \sim 10$ and $\delta\xi_{\rm eq} \sim 0.1\ R_E(b_{\phi{\rm suf}} = b_{\nu{\rm suf}} = 10$ nT), respectively.

Figure 6 indicates the amplitudes of the field line displacements (ξ_ϕ, ξ_ν) of the first

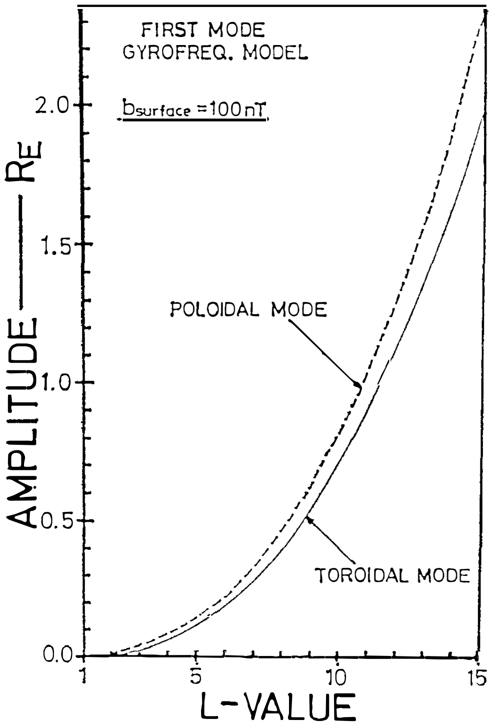

Fig. 6. The amplitude of the field line displacement of the first decoupled standing waves in the magnetic equatorial plane is shown as a function of the L-value for the magnetic perturbation b_{suf} $=100\,nT$ in the perfect conducting ionosphere and the electric fields $E^{eq}_{1\nu}\sim 14$ and $E^{eq}_{1\phi}\sim 16\ mV/m$.

toroidal and poloidal modes of standing resonance wave in the equatorial plane. The amplitudes are calculated as a function of the L-value for the equatorial electric fields $E^{eq}_{1\nu}\sim 14$ mV/m and $E^{eq}_{1\phi}\sim 16$ mV/m, which correspond to the magnetic perturbation $b_{suf}=100$ nT in the perfect conducting ionosphere, and the gyrofrequency plasma model in the dipolar background field. It is noteworthy that the radial extent of the displacement is no more than $\sim 0.5\ R_E$ in the inner magnetosphere, therefore, the coupling between field lines should be weak, allowing field lines within the resonance region to oscillate independently. Recently, HUGHES *et al.* (1978) inferred radial extents of the resonance region with $\sim 0.25\ R_E$ at Pc 3 frequency and $\sim 0.6\ R_E$ at Pc 4 frequency, using the synchronous satellite ATS 6 and SMS 2, which were separated by $\sim 0.1\ R_E$. The first identification of a spatially confined resonance structure was made possible with observations from the ISEE-1 and -2 satellites (SINGER *et al.*, 1979). SINGER *et al.* (1982) have examined four dayside Pc 4–5 pulsations, three of which are observed between $L=4$ and 7 within 10° of the magnetospheric equator. The resonant region widths of these three events range from ~ 0.2 to 1.6 L shells. These observational facts are well consistent with the calculated displacement of the fundamental waves in the dipolar background field with the gyrofrequency plasma model.

The second-harmonic and third-harmonic eigenperiods of the guided toroidal mode are illustrated in Fig. 7 against L-value for the gyrofrequency model in the dayside magnetosphere. The higher mode frequency (ω_2, ω_3, etc.) normalized by the first mode frequency ω_1 of the standing wave, *i. e.*, a sequence of frequency ratios, is obtained as a function of the mode number for the fully developed guided mode theory and the time of flight method, respectively (Fig. 8). The fact that the mode frequencies ω_2, ω_3, etc., given by means of the time of flight method consist of a sequence of harmonics of the lowest mode frequency ω_1 is a result of assumption that the field line is

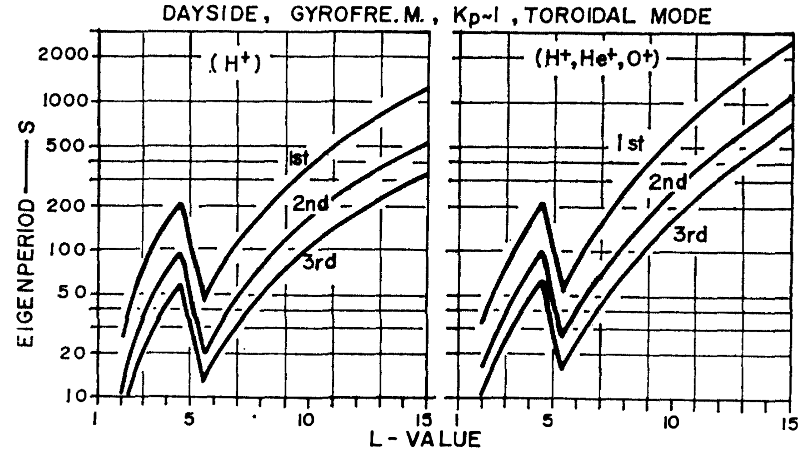

Fig. 7. Higher-harmonic eigenperiods of the guided toroidal mode vs. the L-value for the gyrofrequency plasma model at Kp~1 in the dayside magnetosphere.

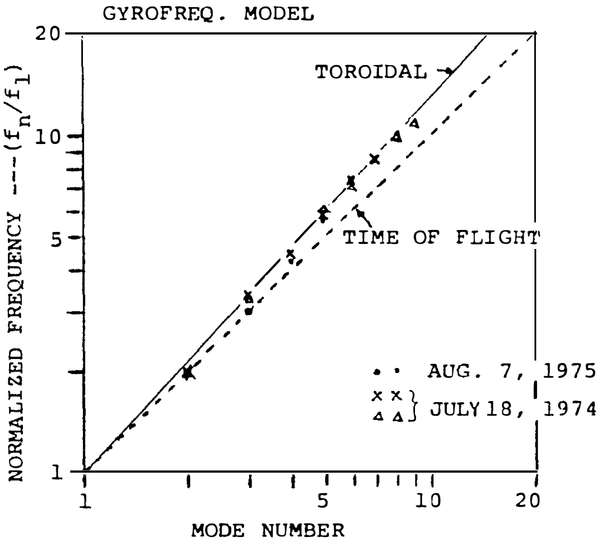

Fig. 8. Sequences of the frequencies ratio vs. mode numbers. The frequency obtained by the time of flight method indicates the harmonic relation. The plots, which present the frequencies of "harmonic events" in the Pc 3 range obtained by TAKAHASHI and MCPHERRON (1982), are nearly in the sequence of the toroidal standing wave in the dipolar ambient field with the gyrofrequency plasma model.

equivalently uniform and flexible. It is reasonable to note that the sequence of frequency ratios of the fully developed toroidal wave indicates the slope ~1.3, *i. e.*, the mode frequencies for a nonuniform field line such as a dipolar magnetic field line, do not form a sequence of harmonics of the fundamental. TAKAHASHI and MCPHERRON (1982) demonstrated that at least 10–30% of Pc 3 pulsations observed at synchronous orbit by the ATS-6 satellite show several spectral peaks with roughly harmonic frequency ratios. The sequence of frequency ratios of the second mode ω_2 illustrated in Figs. 3 and 5 of TAKAHASHI and MCPHERRON (1982) is also represented in Fig. 8. The observational sequence of the frequencies in the Pc 3 range at ATS-6 (L~6.6) is found to be nearly consistent with the slope of the toroidal standing wave in the dipolar

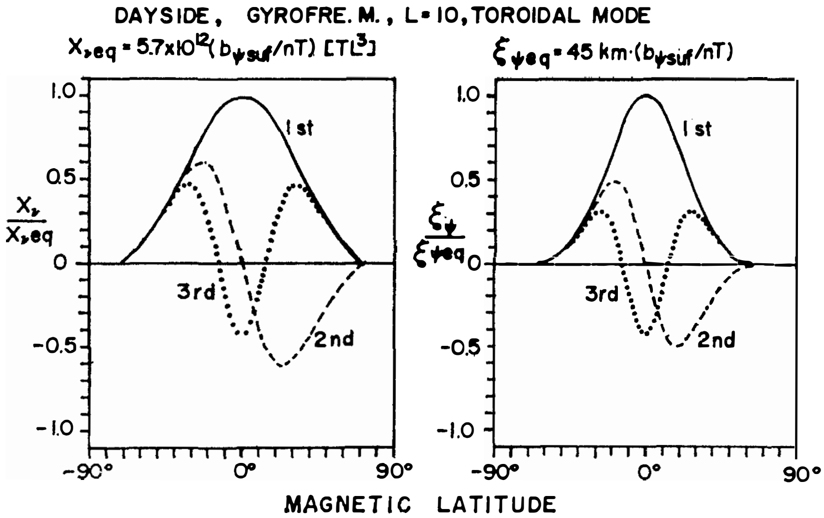

Fig. 9. Higher harmonic amplitudes of the perturbation field (X, ξ) at any point along the field line against the magnetic latitude for the gyrofrequency model at $L=10$ in the dayside magnetosphere. The perturbation fields are normalized with the fundamental wave fields at the equatorial plane.

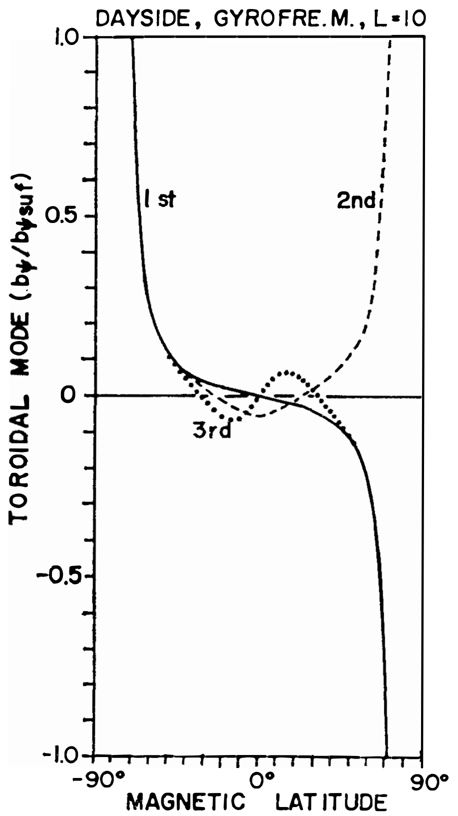

Fig. 10. Higher harmonic magnetic wave fields of the toroidal mode at any point along the background field line against the magnetic latitude for the gyrofrequency model at $L=10$ in the dayside magnetosphere. The perturbation fields are normalized with the fundamental fields at the reflecting surface.

background field with the gyrofrequency plasma model. Therefore, the approximations of the dipolar ambient field and the gyrofrequency plasma density are concluded to be good ones of the plasma parameters in the real inner magnetosphere. We have plotted in Figs. 9 and 10 the "second-harmonic" and "third-harmonic" amplitudes of

(X_ν, ξ_ϕ) and b_ϕ at $L=10$ for the gyrofrequency model. In Figs. 9 and 10 we have normalized the wave fields to the fundamental values that would be observed in the equatorial plane ($X_{\nu\mathrm{eq}}^{1\mathrm{st}}$, $\xi_{\phi\mathrm{eq}}^{1\mathrm{st}}$) and the reflecting surface of the perfect conductor ($b_{\phi\mathrm{suf}}^{1\mathrm{st}}$). When the oscillations are considered for the second and third harmonic of the standing wave resonances, the peak displacement vectors should occur along the field line at $\theta_{2\mathrm{nd}}-90°=\pm 19°$ and $\theta_{3\mathrm{rd}}-90°=0°$, $\pm 27°$, and should be $\xi_{\phi\mathrm{MAX}}^{2\mathrm{nd}} \sim 0.5\, \xi_{\phi\mathrm{eq}}^{1\mathrm{st}}$ and $\xi_{\phi\mathrm{MAX}}^{3\mathrm{rd}} \sim 0.42\, \xi_{\phi\mathrm{eq}}^{1\mathrm{st}}$, respectively. Then, the magnetic wave amplitudes near the magnetic equator should be $\lesssim 0.5$ nT at $\theta_{2\mathrm{nd}}-90°=0°$ and $\theta_{3\mathrm{rd}}-90°=\pm 15°$ for $E_{1\nu}^{\mathrm{eq}} \sim 1.4$ mV/m, $b_{\phi\mathrm{suf}}=10$ nT and $L=10$. It is noteworthy that the magnetic wave amplitude observed at the reflecting surface will be one order larger than the magnetic wave amplitude in the equatorial plane for the case of perfect reflection.

In this section MHD equations are analyzed for an idealized model in which the earth surface is assumed to be a perfect conductor. The ambient magnetic field is dipole, and the gyrofrequency and the isospherical plasma density models are considered. For the case of standing resonance waves, the guided waves, *i. e.*, the toroidal and poloidal wave equations are uncoupled. It is found that the eigenperiods of the guided toroidal and poloidal waves are determined mainly by the mass density near the equatorial plane in the magnetosphere. The heavy ions outside the plasmasphere have an effect on the eigenperiods of the guided toroidal and poloidal modes. The resonance oscillations are expected to occur only on field lines having L values within a narrow range ($\sim 0.1\ R_{\mathrm{E}}$), so that all the field lines in this range would have approximately the same eigenperiod. The magnetic perturbation field observed in the equatorial plane will be one order smaller than the magnetic perturbation field near the ionosphere. Therefore, the difficulty of the satellite-ground correlations are easily understood. Further coordinated researchs are needed with respect to the spatical and temporal resolutions of satellite data.

3. A Special Solution of Coupling Oscillation

We consider the coupling equations of an Alfvén and a fast magnetosonic modes in the magnetosphere for a special case. From eq. (A9) we obtain

$$\frac{\partial^2 X_\nu}{\partial \mu^2} - 2\frac{\partial}{\partial \mu}(\mathrm{In}\, h_\nu)\frac{\partial X_\nu}{\partial \mu} - \left(\frac{h_\mu^2}{v_{\mathrm{A}}^2}\right)\frac{\partial^2 X_\nu}{\partial t^2} = -h_\nu^2 \frac{\partial}{\partial \phi}\left(\frac{\partial X_\nu}{\partial \phi} - \frac{\partial X_\phi}{\partial \nu}\right), \tag{7.1}$$

and

$$\frac{\partial^2 X_\phi}{\partial \mu^2} - 2\frac{\partial}{\partial \mu}(\mathrm{In}\, h_\phi)\frac{\partial X_\phi}{\partial \mu} - \left(\frac{h_\mu^2}{v_{\mathrm{A}}^2}\right)\frac{\partial^2 X_\phi}{\partial t^2} = h_\phi^2 \frac{\partial}{\partial \nu}\left(\frac{\partial X_\nu}{\partial \phi} - \frac{\partial X_\phi}{\partial \nu}\right), \tag{7.2}$$

where

$$\left(\frac{h_\mu}{B}\right)\frac{\partial}{\partial \nu}\left(\frac{B}{h_\mu}\right) + 2\frac{1}{h_\mu}\frac{\partial h_\mu}{\partial \nu} = \frac{\partial}{\partial \nu}\mathrm{In}(Bh_\mu) = 0\,,$$

and in a cold plasma. The right-sides of these equations indicate that the toroidal and the poloidal modes are coupling with each other. Though these equations have not been exactly solved even in a dipole background field, we can analyze the limiting case. Only modes with the same oscillating exponentials on both sides of eq. (7) can exist for a long time in the magnetosphere, since the phase mixing of modes without the same time-exponentials should occur and the modes will be damping. Therefore, we can readily obtain the indispensable condition of the long-lived coupling oscillation given by

$$\frac{1}{X_\nu}\frac{\partial^2 X_\nu}{\partial t^2}=\frac{1}{X_\phi}\frac{\partial^2 X_\phi}{\partial t^2}=-\omega^2_{\mathrm{comp}}\,. \tag{8}$$

In the limiting case of $\partial X/\partial\mu \ll \partial X/\partial\phi \sim \partial X/\partial\nu$, from eq. (7), we can obtain a wave equation of compressional wave which propagates obliquely to the ambient magnetic field as the following form;

$$\frac{\partial}{\partial\nu}\left(\frac{V_A^2}{h_\nu^2}\frac{\partial Y}{\partial\nu}\right)+\frac{V_A^2}{h_\phi^2}\frac{\partial^2 Y}{\partial\phi^2}+\omega^2_{\mathrm{comp}}Y=0\,, \tag{9}$$

where

$$Y\equiv\left(\frac{\partial X_\nu}{\partial\phi}-\frac{\partial X_\phi}{\partial\nu}\right)=(-h_\mu b_\mu)\,.$$

We consider the perturbation fields being harmonics in the azimuthal direction with azimuthal wave number m, *i.e.*, $Y\sim e^{-i\omega t}e^{im\phi}$. Equation (9) becomes

$$\frac{h_\phi^2}{V_A^2}\frac{\partial}{\partial\nu}\left(\frac{V_A^2}{h_\nu^2}\frac{\partial Y}{\partial\nu}\right)+\left(\omega^2_{\mathrm{comp}}\frac{h_\phi^2}{V_A^2}-m^2\right)Y=0\,, \tag{10}$$

and can be rewritten as,

$$\frac{\partial^2 Y}{\partial L^2}+\left[2\frac{\partial}{\partial L}(\mathrm{In}\,V_A)-\frac{2}{L}\right]\frac{\partial Y}{\partial L}+R_E^2\left(\frac{\omega^2_{\mathrm{comp}}}{V_A^2}-\frac{m^2}{R_E^2L^2}\right)Y=0\,, \tag{11}$$

in the equatorial plane, where $\partial/\partial\nu|_{\theta=90°}=-R_E L^2(\partial/\partial L)$. If $\rho(L)=\rho_0 L^{-4}$, we obtain the wave equation of compressional wave in the equatorial plane given by

$$\frac{\partial^2 Y}{\partial L^2}-\frac{4}{L}\frac{\partial Y}{\partial L}+\left(a^2L^2-\frac{m^2}{L^2}\right)Y=0\,, \tag{12}$$

where

$$a^2=R_E^2\mu_0\rho_0\omega^2_{\mathrm{comp}}/B_0^2=R_E^2/L^2\cdot(\omega^2_{\mathrm{comp}}/V_A^2(L))\ \text{with}\ V_A^2(L)=B_E^2/\mu_0\rho(L)\,.$$

Note that there are the two separate cases of high and low frequencies (ω_{comp}).

3.1. An oscillating solution with the high frequency

If $a^2>m^2L^{-4}$, *i.e.*, $\omega^2_{\mathrm{comp}}>V_A^2(L)\cdot(m/LR_E)^2$, we can obtain an oscillating solution of the coupling eq. (12) as the following form;

$$Y=CL^\alpha Z_\nu(\beta L^\gamma)\,, \tag{13.1}$$

with $\alpha=5/2$, $\beta=a/2$ and $\nu=\pm(25+4m^2)^{1/2}/4$. The magnetic perturbation field in the equatorial plane becomes

$$b_\mu=(-Y/h_\mu)|_{\theta=90°}\sim CL^{-1/2}Z_\nu(\beta L^\gamma)\,, \tag{13.2}$$

where $Z_\nu(\beta L^\nu)$ is a modified Bessel function.

3.2. An evanescent solution with the low frequency

For $\omega^2_{\rm comp} < V^2_{\rm A}(L)\cdot(m/LR_{\rm E})^2$, the evanescent solution of coupling oscillation is given by,

$$Y \sim CL^\alpha \cdot Z_\nu(i\beta L^\nu) \propto I_\nu(\beta L^\nu) \text{ or } K_\nu(\beta L^\nu)\,, \tag{14.1}$$

and

$$b_\mu = CL^{-1/2}\cdot Z_\nu(i\beta L^\nu)\,, \tag{14.2}$$

where I_ν and K_ν are modified Bessel functions of first and second kinds. It is interesting to note that the coupling oscillations of the Alfvén and the fast magnetosonic waves with the high frequency $[\omega^2_{\rm comp} > V^2_{\rm A}(L)\cdot(m/LR_{\rm E})^2]$ and the low one $[\omega^2_{\rm comp} < V^2_{\rm A}(L)\cdot(m/LR_{\rm E})^2]$ should be the oscillating and the evanescent modes near the magnetic equatorial plane for $\partial/\partial\mu \ll \partial/\partial\phi \sim \partial/\partial\nu$, respectively.

The linear coupling of the guided standing wave and the compressional wave which has a $\boldsymbol{k}$-vector normal to the ambient magnetic field occurs only when the resulting dispersion laws are satisfied as the following equation in the magnetosphere,

$$\omega^2_{\rm comp} = V^2_{\rm A}(k^2_\parallel + k^2_\perp) \sim (2\pi n/T_{\rm eigen})^2, \text{ with } n \gtrsim 1\,, \tag{15}$$

where $k_\parallel$ and $k_\perp$ are mean wave numbers of the compressional waves parallel and normal to the ambient magnetic field and $T_{\rm eigen}$ is the eigenperiod of the toroidal mode (*cf.* Fig. 2). Therefore, the theory predicts that the linear coupling oscillations of the fundamental standing wave and the compressional propagating waves with the real larger azimuthal wave number never occur in the magnetosphere, since $(2\pi/\omega_{\rm comp}) \approx \{V_{\rm A}(m/2\pi 10R_{\rm E})\}^{-1} \sim 20\ {\rm s} \ll T_{\rm eigen}(L\sim 10) \sim 400\ {\rm s}$ for $m=10$ and $V_{\rm A} \sim 2000$ km/s near $L=10$ and $(2\pi/\omega_{\rm comp}) \sim 4\ {\rm s} \ll T_{\rm eigen}(L\sim 4) \sim 20\ {\rm s}$ for $m=10$ and $V_{\rm A} \sim 4000$ km/s near $L=4$ (*cf.* Fig. 2 and Fig. 3). Only the compressional evanescent waves which have a imaginary part of $k_\perp(m)$ can couple with the fundamental standing waves in the magnetosphere. If the real and imaginary parts of the larger azimuthal wave number of the compressional waves are same order and Real $(k^2_\perp) = m^2_{\rm Real} - m^2_{\rm Imag} \approx 0$, the condition of the linear resonance oscillation, *i.e.*, eq. (15), can be satisfied in the magnetosphere. We can obtain the coupling oscillations of the fundamental standing wave and the compressional evanescent wave with $m_{\rm Real} \approx m_{\rm Imag}$ and $\exp[(im_{\rm Real} - m_{\rm Imag})\phi]$. The higher harmonic waves $(n \gtrsim 2)$ of the standing oscillation can be coupled with the propagating compressional wave $(k^2_\perp \gg k^2_\parallel)$ in the outer magnetosphere.

SOUTHWOOD (1977) discussed localized compressional field line resonance-like signals to be possible in a hot inhomogeneous plasma like the magnetospheric ring current though as in a cold inhomogeneous plasma the hydromagnetic equations are generally coupled. A general requirement is that total pressure $(\delta p + \boldsymbol{B}_0\cdot\delta\boldsymbol{B}_\parallel/2\mu_0)$ perpendicular to $\boldsymbol{B}_0$ balances in the wave, *i.e.*, energy flux of the wave does not cross $\boldsymbol{B}_0$. We can distinguish the localized compressional wave from the propagating compressional wave, except the trapped oscillation of the fast magnetosonic wave (TAMAO, 1978), with respect to the energy flux across $\boldsymbol{B}_0$.

On the other hand, the propagating compressional wave in the outer magnetosphere can be coupled with the collective eigen oscillation at the plasmapause (CHEN and HASEGAWA, 1974b) and the fundamental standing oscillation in the plasmatrough by means of the nonlinear resonance (YUMOTO and SAITO, 1982). The condition of the nonlinear resonance is given by

$$\omega_{\text{comp}}=\omega_{\text{r}}+\omega_2^{\text{fs}}\,, \tag{16}$$

where ω_{comp}, ω_2^{fs} and ω_{r} are the frequencies of the propagating compressional wave in the outer magnetosphere, one component of HM noises near the plasmapause and collective eigen oscillation at the plasmapause or the fundamental standing oscillation in the plasmatrough, respectively.

In recent times many workers have investigated the azimuthal wave number of long-period magnetic pulsations. HUGHES *et al.* (1978) demonstrated that Pc 3–4 pulsations at the synchronous orbit are usually coherent over longitude separations of up to 20° and have various azimuthal wave numbers ($m\leq 10$). OLSON and ROSTOKER (1978) found a relationship of the form $m=(1.4\pm 0.4)f+0.26$, where f is the frequency in millihertz and m is the azimuthal wave number of Pc 4–5 magnetic pulsations. Some of these Pc 3–4 pulsations with longitudinal localization and larger azimuthal wave number at $L=6.6$ correspond to the linear coupling oscillations of the higher harmonic standing waves and the propagating compressional waves. TAKAHASHI and MCPHERRON (1982) recently demonstrated that at least 10–30% of Pc 3 pulsations at synchronous orbit can be statistically classified as harmonic events of many discrete harmonic frequencies. Long-period pulsations with small azimuthal wave number observed in the magnetosphere correspond to the guided standing oscillations analyzed in the previous section.

4. Summary and Discussions

In this paper, MHD equations in dipolar coordinates are analyzed for an idealized model in which the earth surface is assumed to be a perfect conductor. The dampling rates are strongly dependent on the ionospheric conductivity. NEWTON *et al.* (1978) found that a typically normalized damping rate, γ/ω, is $\sim$0.1 for nightside values of conductivity and $\sim$0.01 for the dayside, *i.e.*, although the joule dissipation in the nightside ionosphere is an important source of damping of pulsation, the damping is weak for typical dayside ionosphere ($\Sigma_{\text{p}}>10^{12}$ esu). ALLAN and KNOX (1979a, b, 1982) found that ionospheric coupling is small compared with magnetospheric coupling for any single non-axisymmetric mode; however, ionospherically coupled axisymmetric modes should be necessary components of the Fourier sum of modes required to model any real pulsation of low to moderate azimuthal wave number. The plasma density in the magnetosphere is considered to be the gyrofrequency and the isospherical models. The characteristics of the eigen oscillation of a dipolar field line may be summarized as follows;

(1) For the case of standing resonance waves, *i.e.*, the guided toroidal and the guided poloidal waves, the eigenperiods are determined mainly by the mass density near equatorial plane in the magnetosphere as shown in Fig. 2.

(2) The eigenperiods of the standing waves importantly depend on the mass

density of the heavy ions outside the plasmasphere (Fig. 3). The eigenperiods for any other mass density can be obtain by using the relation $T=T_0[\rho_0(H^+, H_e^+, O^+)/\rho_0(H^+)]^{1/2}$, where T_0 is the eigenperiods of the decoupled mode for the proton mass density and $\rho_0(H^+, H_e^+, O^+)$ is the observed mass density in units of cm^{-3}.

(3) The resonance oscillations are expected to occur only on field lines having L-values within a narrow range. The equatorial extents of the standing resonance waves are estimated to be $\delta\xi_{eq} \sim L^3 B_0^{-1} \cdot \delta E_{eq} \sim 0.1\ R_E$ for $\delta E_{eq} \sim 2$ mV/m at $L=10$, where $\delta\xi_{eq}$ is a displacement of MHD fluid and δE_{eq} is an electric perturbation at the equatorial plane. The electric field wave amplitudes of the standing waves are estimated to be $|\delta E| = B\xi \sim B_0 L^{-3} \delta\xi_{eq} \sim 1.5$ mV/m at the equatorial plane for $L \sim 10$ and $\delta B_{suf} \sim 10$ nT.

(4) The magnetic perturbation fields of the standing resonance waves in the equatorial plane will be one order smaller than those near the ionosphere for the case of perfect reflection.

(5) The compressional evanescent waves, which have a imaginary part of $k_\perp$ and Real $(k_\perp^2) = m_{Real}^2 - m_{Imag}^2 \approx 0$, and the propagating compressional waves $(m_{Imag}^2 = 0)$ can couple with the fundamental standing waves in the magnetosphere by means of the linear and the nonlinear resonances, respectively. The propagating compressional waves also couple with the higher "harmonic" waves of the standing oscillations in the magnetosphere.

The observational sequence of the frequencies in the Pc 3 range at ATS-6 ($L \sim 6.6$) demonstrated by TAKAHASHI and MCPHERRON (1982) is nearly consistent with the slope of the frequency ratios of the toroidal standing wave in the dipolar background field with the gyrofrequency plasma model (*cf.* Fig. 8). It is concluded that the approximations of the dipolar ambient field and the gyrofrequency plasma density model are good ones of the realistic inner dayside-magnetosphere ($L \lesssim 7$). The comparison between the calculated periods of the guided toroidal mode and those of Pc 5 waves having predominant azimuthal polarization in the dawn-side magnetosphere, which was demonstrated by YUMOTO *et al.* (1983), is presented as a function of L-value in Fig. 11.

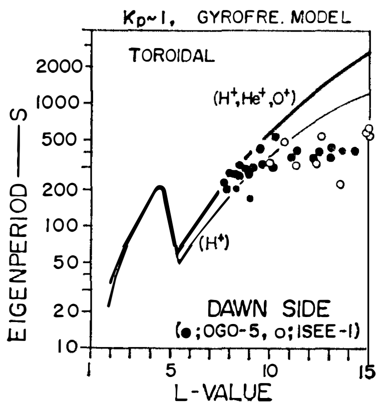

Fig. 11. Comparison between the calculated periods of the toroidal mode and those of azimuthal Pc 5 in the dawn-side magnetosphere.

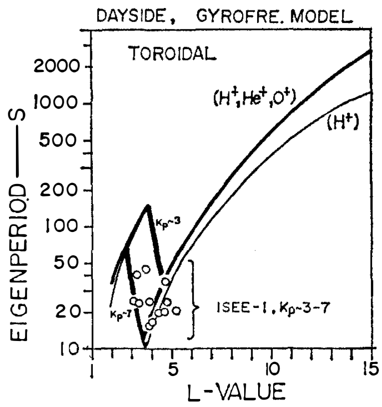

Fig. 12. Comparison between the calculated periods of the toroidal mode and those of Pc 3 pulsation observed by MOE et al. (1980) inside and near the plasmapause.

The thick and thin lines indicate the eigenperiods of the toroidal standing oscillations for the gyrofrequency plasma models (Fig. 1) of heavy ions (H^+, H_e^+, O^+) and proton (H^+), respectively. The black and white circles express the Pc 5 periods observed by OGO-5 and ISEE-1, respectively. It is clearly found that the observed periods of daytime Pc 5 pulsations in the inner magnetosphere ($L<10$) are in agreement with the calculated periods of the fundamental toroidal waves in the dipolar background field. The difference between the calculated and the observed periods in the outer magnetosphere ($L>10$) reduces to the distortion from a dipole field geometry and the effect of the mass density in the outer magnetosphere, which are recently discussed by SINGER *et al.* (1981).

The toroidal periods for $Kp\sim 3$ and $Kp\sim 7$ are also calculated for comparison with Pc 3 periods observed by ISEE-1 (MOE *et al.*, 1980) inside and near the plasmapause as shown in Fig. 12. The frequency of the collective eigen mode of the surface wave at the plasmapause is theoretically given by CHEN and HASEGAWA (1974b) as follows;

$$\omega_{ce}=\sqrt{2}\,k_{\parallel}V_{\mathrm{A}}^{\mathrm{II}}\,, \tag{17}$$

where $V_{\mathrm{A}}^{\mathrm{II}}$ is the Alfvén velocity inside the plasmapause. The value of $k_{\parallel}$ is decided by the length of the field line l, $k_{\parallel}=n\pi/l$ with $n\gtrsim 1$. The observed Pc 3 periods inside and near the plasmapause are clearly inconsistent with that ($T_{ce}\sim 140$ s for $n=1$) of the fundamental mode of the collective surface wave at the plasmapause. The higher harmonic oscillation of the collective eigen-mode excited by the propagating compressional wave at the plasmapause can be explained by the nonlinear resonance theory (YUMOTO and SAITO, 1982). The period of the trapped oscillation in the plasmasphere (TAMAO, 1978) is approximately given by

$$T_{\mathrm{trap}}\lesssim(2\varDelta L/V_{\mathrm{g}\perp})\sim 2\varDelta L/V_{\mathrm{A}}\,, \tag{18}$$

where $\varDelta L$ is the characteristics length between the two peaks of Alfvén velocity and $V_{\mathrm{g}\perp}$ is the group velocity of a fast magnetosonic wave normal to the ambient magnetic field. The period of the trapped oscillation is $\lesssim 40$ s for $\varDelta L\sim 3\,R_{\mathrm{E}}$ and $V_{\mathrm{g}\perp}\sim V_{\mathrm{A}}\sim 600$ km/s. These Pc 3 magnetic pulsations observed by ISEE-1 inside and near the plasmapause must be either the higher harmonic wave of the collective modes at the plasmapause or the trapped oscillation of the fast magnetosonic wave in the plasmasphere (and/or other occurrence mechanism). However, observational studies in space are needed further for the ground-satellite correlations and the wave characteristics of Pc 3 magnetic pulsations.

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Appendix. MHD Equations in Dipolar Coordinates

Equation of lines of force of the dipole field in the spherical coordinates (r, θ, ϕ) is

$$r = r_E \sin^2 \theta , \tag{A1}$$

where r_E is the equatorial distance of the field line and θ is a co-latitude. Now we take dipolar coordinates (ν, μ, ϕ), *i.e.*, orthogonal curvilinear coordinates, where Π_ν is the unit vector in the outward normal in the meridional plane, Π_μ the tangential direction to the field line and Π_ϕ the westward direction;

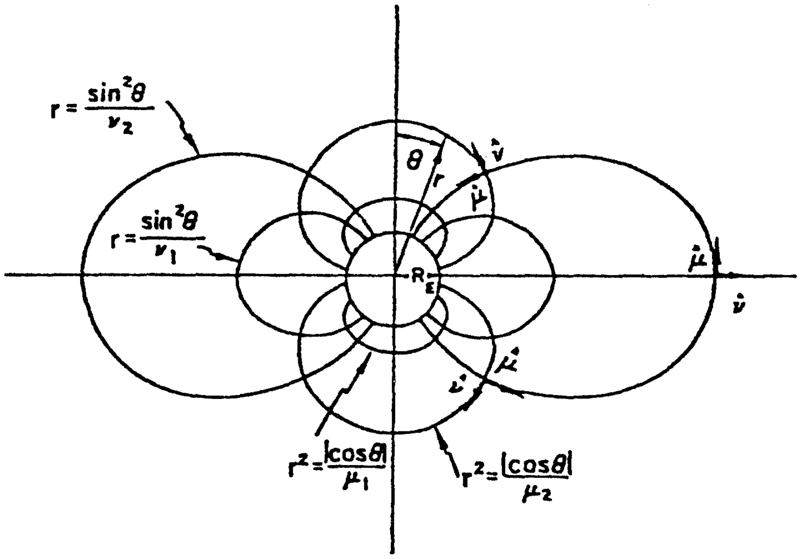

Fig. A1. An illustration of the orthogonal dipole coordinate system. The curves of constant ν are the dipole field lines and the curves of constant μ are the orthogonal trajectories to the field lines. The unit vectors μ and ν are shown for three positions.

$$\nu=\sin^2\theta/r\ , \tag{A2.1}$$

$$\mu=\cos\theta/r^2\ , \tag{A2.2}$$

$$\phi=\phi\ . \tag{A2.3}$$

Thus, ν is constant on the field line, while lines of μ=constant are equipotential line (*i.e.*, $\Phi=-M\cos\theta/r^2$ with the earth's magnetic dipole moment M as shown in Fig. A1). The matrices of dipolar coordinates are given by,

$$h_\nu=r^2(\sin\theta\sqrt{1+3\cos^2\theta})^{-1}\ , \tag{A3.1}$$

$$h_\mu=r^3(1+3\cos^2\theta)^{-1/2}\ , \tag{A3.2}$$

$$h_\phi=r\sin\theta\ . \tag{A3.3}$$

The linearized MHD equation is given by

$$\rho_{0\mathrm{m}}\partial^2\boldsymbol{\xi}/\partial t=-\nabla[p_1+(\boldsymbol{b}\cdot\boldsymbol{B}/\mu_0)]+1/\mu_0[(\boldsymbol{b}\cdot\nabla)\boldsymbol{B}+(\boldsymbol{B}\cdot\nabla)\boldsymbol{b}]\ , \tag{A4}$$

and

$$\boldsymbol{b}=\nabla\times(\boldsymbol{\xi}\times\boldsymbol{B})\ , \tag{A5}$$

where $\boldsymbol{\xi}$ is the displacement vector defined by

$$\partial\boldsymbol{\xi}/\partial t=\boldsymbol{v}\ , \tag{A6}$$

and $\boldsymbol{v}$ is the perturbed fluid velocity. In eq. (A4), $\rho_{0\mathrm{m}}$ and p_1 are mass density and the perturbed plasma pressure. The perturbed fields in the dipolar coordinates become;

$$\rho_{0\mathrm{m}}\partial^2\xi_\nu/\partial t^2=-\frac{1}{h_\nu}\frac{\partial}{\partial\nu}(p_1+b_\mu B/\mu_0)+\frac{1}{\mu_0}\left[\frac{B}{h_\nu h_\mu}(b_\nu\partial h_\nu/\partial\mu-b_\mu\cdot\partial h_\mu/\partial\nu)\right.$$

$$+\left(\frac{B}{h_\mu}\partial b_\nu/\partial\mu - b_\mu B/h_\nu h_\mu\cdot\partial h_\mu/\partial\nu\right)\Bigg], \tag{A7.1}$$

$$\rho_{0\mathrm{m}}\partial^2\xi_\mu/\partial t^2 = -\frac{1}{h_\mu}\partial p_1/\partial\mu\ , \tag{A7.2}$$

$$\rho_{0\mathrm{m}}\partial^2\xi_\phi/\partial t^2 = -\frac{1}{h_\phi}\partial(p_1+b_\mu B/\mu_0)/\partial\phi + \mu_0^{-1}[Bb_\phi/h_\phi h_\mu\cdot\partial h_\phi/\partial\mu + B/h_\mu\partial b_\phi/\partial\mu]\ , \tag{A7.3}$$

and

$$b_\nu = (h_\mu h_\phi)^{-1}\partial(h_\phi\xi_\nu B)/\partial\mu\ , \tag{A8.1}$$

$$b_\mu = -(h_\nu h_\phi)^{-1}[\partial(h_\nu\xi_\phi B)/\partial\phi + \partial(h_\phi\xi_\nu B)/\partial\nu]\ , \tag{A8.2}$$

$$b_\phi = (h_\nu h_\mu)^{-1}\partial(h_\nu\xi_\phi B)/\partial\mu\ . \tag{A8.3}$$

It is interesting to note tha tthe momentum equation in the μ-direction, *i. e.*, the tangential direction to the ambient field, is a function of only the gradient of the perturbed plasma pressure in the μ-direction.

Substituting eqs. (A8) into (A7), we obtain

$$\begin{aligned}&\partial^2X_\nu/\partial\mu^2 + \partial(\mathrm{In}\ h_\phi/h_\nu h_\mu)/\partial\mu\cdot\partial X_\nu/\partial\mu - (h_\mu^2/V_\mathrm{A}^2)\cdot\partial^2X_\nu/\partial t^2\\ &\quad = +B^{-1}\mu_0 h_\mu h_\nu^2\partial[p_1 + B(\mu_0 h_\nu h_\phi)^{-1}(\partial X_\phi/\partial\nu - \partial X_\nu/\partial\phi)]/\partial\phi\ ,\end{aligned} \tag{A9.1}$$

and

$$\begin{aligned}&\partial^2X_\phi/\partial\mu^2 + \partial(\mathrm{In}\ h_\nu/h_\mu h_\phi)/\partial\mu\cdot\partial X_\phi/\partial\mu - (h_\mu^2/V_\mathrm{A}^2)\cdot\partial^2X_\phi/\partial t^2\\ &\quad = -\mu_0 B^{-1}h_\mu h_\phi^2\partial[p_1 + B(\mu_0 h_\nu h_\phi)^{-1}(\partial X_\phi/\partial\nu - \partial X_\nu/\partial\phi)]/\partial\nu\\ &\quad\quad + 2h_\phi/h_\nu(\partial X_\nu/\partial\phi - \partial X_\phi/\partial\nu)\partial h_\mu/\partial\nu\ ,\end{aligned} \tag{A9.2}$$

where $X_\nu = h_\nu E_\nu = +h_\nu\xi_\phi B$, $X_\phi = h_\phi E_\phi = -h_\phi\xi_\nu B$ and $V_\mathrm{A}^2 = B^2/\mu_0\rho_{0\mathrm{m}}$. E_ν, E_ϕ and V_A stand for the perturbed electric fields and the Alfvén speed, respectively. Equations (A9.1–A9.2) are the coupled toroidal and poloidal wave equations. If X_ν and X_ϕ were numerically determined, it is possible to obtain b_ν and b_ϕ from eq. (A8), *i.e.*,

$$b_\nu = -(h_\mu h_\phi)^{-1}\cdot\partial X_\phi/\partial\mu\ , \tag{A10.1}$$

$$b_\phi = +(h_\nu h_\mu)^{-1}\cdot\partial X_\nu/\partial\mu\ . \tag{A10.2}$$

In the axisymmetric case ($\partial/\partial\phi = 0$), there are two decoupled modes. Equations (A9.1) and (A9.2) express the Alfvén and the fast magnetosonic modes, respectively. The Alfvén mode is a guided toroidal wave in which Poynting vector is always along the field line. The fast magnetosonic mode is a propagating wave of which energy flux crosses the ambient field. We can obtain another guided poloidal wave, which was introduced by RADOSKI (1967), where the azimuthally asymmetric terms dominate and the relation ($\partial X_\nu/\partial\phi - \partial X_\phi/\partial\nu = 0$) is constrained. Hence, the μ-component of the perturbation field along the field line becomes

$$b_\mu = -(h_\nu h_\phi)^{-1}\cdot(\partial X_\nu/\partial\phi - \partial X_\phi/\partial\nu) = 0\ .$$

When this condition is imposed, eqs. (A9.1–A9.2) are clearly no longer coupled.